Name: ____ E-mail ID: ________@virginia.edu

Pledge: ____

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Signature: ____

There are 180 minutes (3 hours) for this exam and 180 points on the test; don't spend too long on any one question!

You may use scrap paper. However, all answers must be on these ten exam pages.

Part I	_____/40
Part II	_____/45
Part III	_____/20
Part IV	_____/30
Part V	_____/35
Part VI	_____/10
Total	_____/180

Part I: Mathematical reasoning

1. (40 points total) Consider the following four statements. Note that, for this example, the negation of "richly colored" is "dully colored", and "small" means the same as "not large".

All hummingbirds are richly colored
No large birds live on honey
Birds that do not live on honey are dull in color
Hummingbirds are small

a. (10 points) Express each of the four statements using quantifiers and propositional functions. Clearly label what your propositional functions represent, as well as your universe of discourse. Your universe of discourse must be the same for all the quantified statements.

b. (5 points) If they are not already, convert the quantified statements to use only one universal quantifier per statement (for example, $\forall x(P(x) \rightarrow Q(x))$). If you don't remember how to convert existential quantified statements into universally quantified statements, then try rephrasing the above sentences using only universal quantifiers.

c. (5 points) As all the statements in (b) now use one universal quantifier, we convert them to use only propositions. For example, $\forall x(P(x) \rightarrow Q(x))$ would become $p \rightarrow q$. Express these statements using only propositions. If you did not get part (b), try restating the sentences using propositions.

d. (20 points) If you are given that the first three statements from part (c) are true, can you conclude the fourth statement? Prove this using logical equivalences and rules of inference. Clearly label what rule you are using on each step.

Part II: Constructing proofs and mathematical theorems

2. (45 points total) Prove or disprove each of the following statements. Clearly state which proof method you are using.

a. (15 points) There are three consecutive odd integers that are primes, that is, odd primes of the form p, $p+2$, $p+4$.

b. (15 points) n^2-1 is composite whenever n is a positive integer greater than 1.

c. (15 points) If n is a positive integer such that the sum of its divisors is $n+1$, then n is prime.

Part III: Problem solving

3. (20 points) The value of a 5-card poker hand is inversely related to is rarity. Thus, a more rare hand (such as a royal flush) will beat a less rare hand (such as a pair). In a poker game using only one deck of cards, one person has a full house (a pair of one face value and a triple of another face value) and another person has a regular flush (all cards of the same suit). Who won? And what is the probability of getting each hand? We are assuming that you can't exchange cards (you are dealt the cards once only). You can leave the answer in combination notation.

Part IV: Discrete data structures

4. (15 points total) Define each of the following in TWENTY words or less.

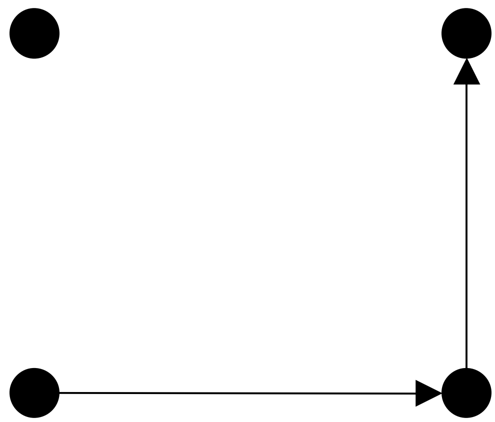
a. (5 points) Recursion.

b. (5 points) Onto (in other words, what the onto property means for a function).

c. (5 points) One-to-one (in other words, what the one-to-one property means for a function).

5. (15 points total) The answers for the following questions should be in the same form as the question (i.e. a graph for part (a), a matrix for part (b), etc.)

a. (5 points) Add the minimum number of edges required to make the following relation an equivalence relation. You may only remove a edge if it prevents the relation from becoming an equivalence relation.



b. (5 points) Consider the relation represented by the following matrix. Add or remove the minimum number of entries to make this relation a partial order. You may only remove an entry if it prevents the relation from becoming a partial order.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

c. (5 points) Consider the relation R on the set S where $S = \{1, 2, 3, 4, 5\}$ and $R = \{ (1,2), (2,3), (3,4), (5,2) \}$. Find the transitive closure of R.

Part V: Solving one problem in multiple ways

6. (35 points total) Assume that you only have 5 cent and 7 cent stamps.

a. (5 points) What is the minimum value of postage for which you can create that amount and all greater amounts of postage? In other words, find n such that you can create n or greater amounts of postage using the provided stamps.

b. (15 points) Prove this using weak mathematical induction. Clearly label the three steps.

c. (15 points) Prove this using strong mathematical induction. Clearly label the three steps. This answer must use strong induction.

Part VI: The End

7. (5 points) Find four numbers congruent to 5 modulo 17.

8. (5 points) What your answer is to the following question will not affect your grade; as long as it is answered, you will get full credit for this question. If you feel uncomfortable answering it (it's obviously not anonymous because it is stapled to the rest of your test), please circle N/A, and you will still get full credit for this question.

How helpful were the following parts of the course in terms of helping you *understand the course material*

	Very Unhelpful	Unhelpful	Neutral	Helpful	Very Helpful	Not Applicable
Giving the in-class presentations	1	2	3	4	5	N/A
Listening to the in-class presentations	1	2	3	4	5	N/A