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Auditing Information Leakage for Distance Metrics

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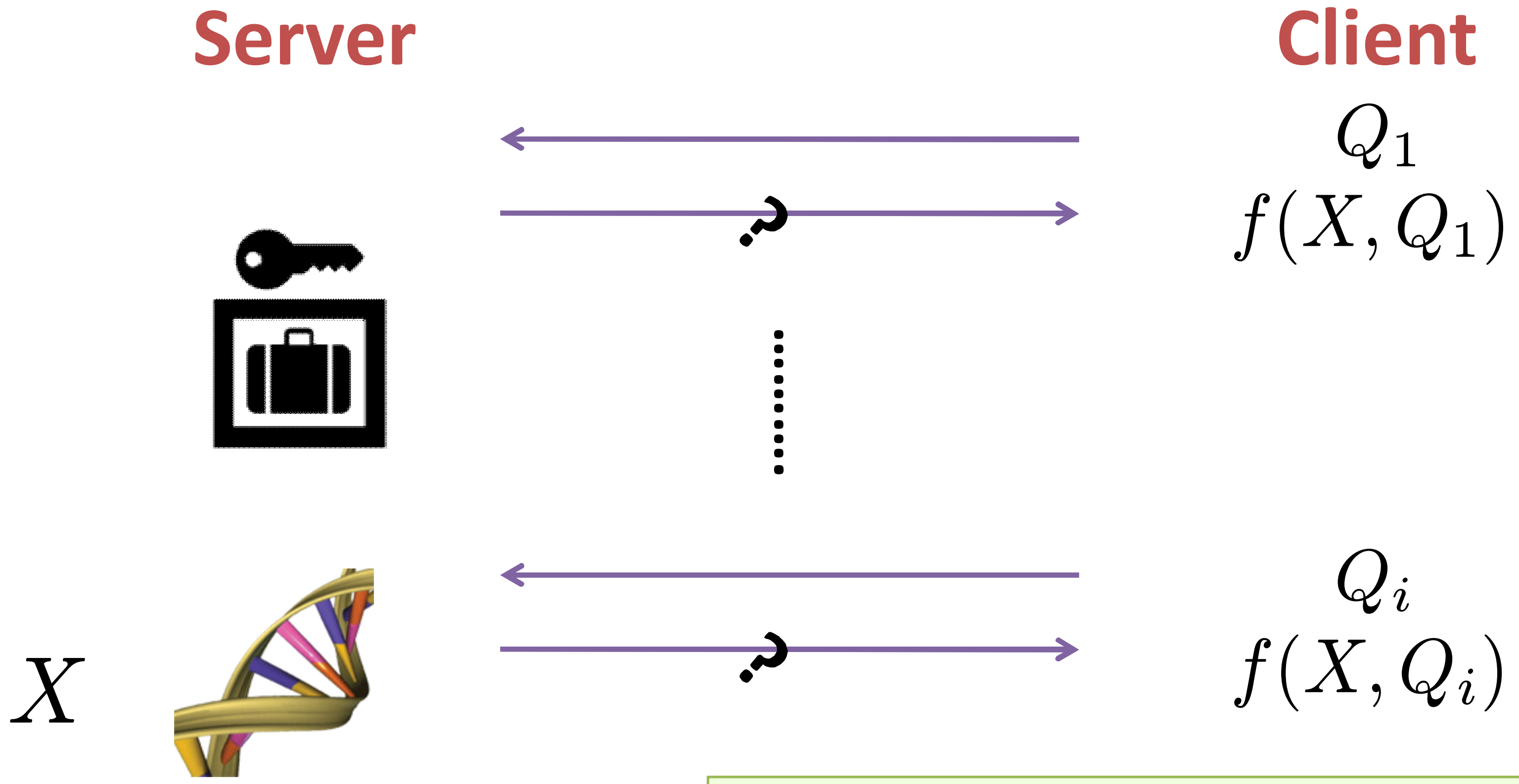
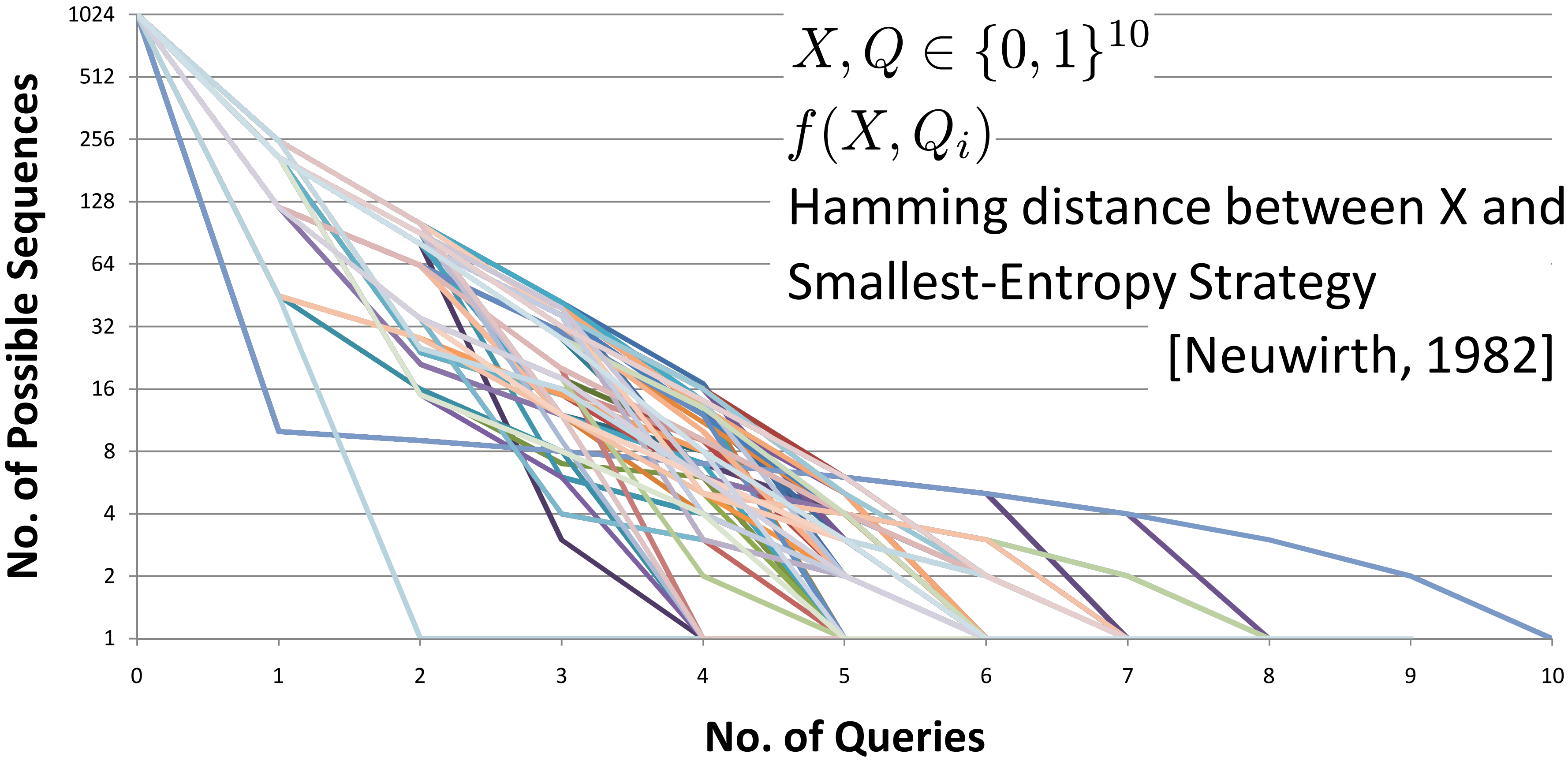
Motivating Scenario



How to determine whether it is safe to release the response?

From the Attacker's View



$X, Q \in \{0,1\}^{10}$

$f(X, Q_i)$

Hamming distance between X and Q

Smallest-Entropy Strategy

[Neuwirth, 1982]

Contribution

Target: class of distance functions

An **efficient query auditor** to estimate maximum information leakage

Technique for **single bit** information auditing

Strategy

Determine **# of secrets** consistent with all queries

– For a special class of functions:

  Reduce to *0-1 integer programming*

Too expensive to compute exhaustively

– Use heuristics [find a meaningful bound]

Assumption

– clients are authenticated by the server

– clients cannot share information with each other

Additively Decomposable Functions

Definition

A function, f (A, B), where A, B $\in \Sigma^N$, is *additively decomposable* if:

$$\forall i \in [0, N], f(A, B) = f(A_{0:i}, B_{0:i}) + f(A_{i:N}, B_{i:N})$$

Hamming distance

$|i \in N : A_i \neq B_i|$

Squared Euclidean distance

$\sum (A_i - B_i)^2$

Manhattan distance

$\sum |A_i - B_i|$

Edit distance

Ins,del and sub

Chebyshev distance

$max(|A_i - B_i|)$

Lee distance

$min(|A_i - B_i|, k - |A_i - B_i|)$

Influence Function

Influence function for any sequence *X'*: $g(\Delta i)$

Δi : changes made on *i*th bit of X

For *additively decomposable* functions, $g(\Delta i)$ for each bit is *independent* and for any consistent sequence over a set of queries:

$$\sum g(\Delta i) = 0$$

Hamming distance: $g(\Delta i) = \begin{cases} 1, & X_i = Q_i \\ -1, & X_i \neq Q_i \end{cases}$

Reduction for Hamming Distance

Convert a Hamming distance sequence leakage problem to:

***AK*=0**

where $$A_{ij} = \begin{cases} 1, & X_j = Q_{ij} \\ -1, & X_j \neq Q_{ij} \end{cases} \qquad K \in \{0,1\}^N$$

Example: ***X*=1111** ***Q*₁=1100** ***Q*₂=1110**

$$k_1 + k_2 - k_3 - k_4 = 0$$

$$k_1 + k_2 + k_3 - k_4 = 0$$

find the number of possible 0-1 solutions for *K*

Computing Number of Solutions

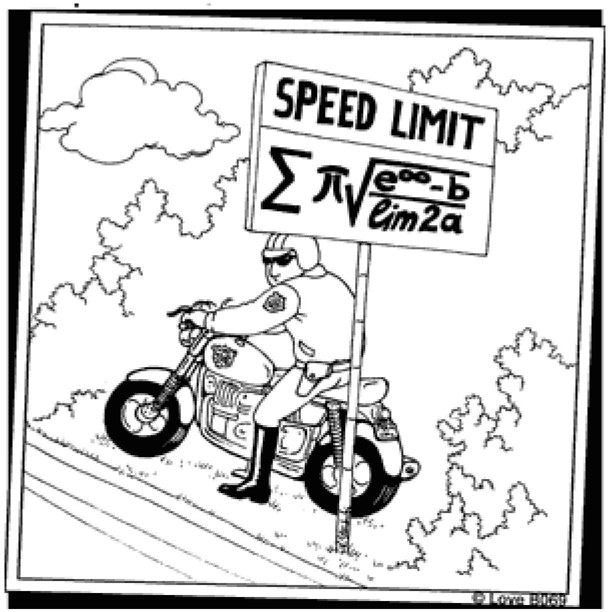
Exact methods: exponential growth

Exhaustive Search

Gröbner Basis ([Bertsimas, 2000])

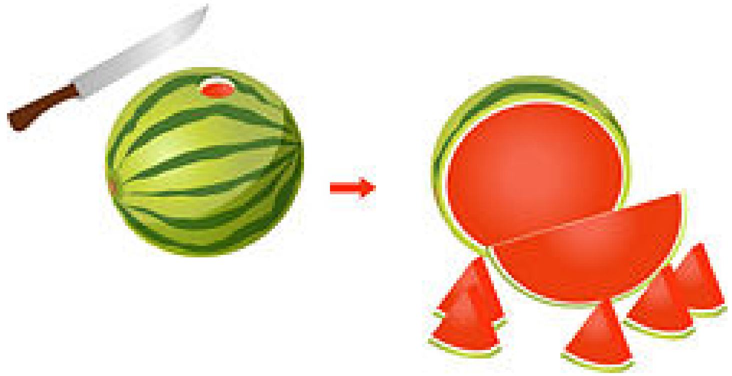
Lower bound is good enough!

Divide-and-merge algorithm



Divide-and-merge Algorithm

$X, \{Q_i\}$ Find # of 0-1 solutions for the equation set: $AK=0$



Divide and exhaustive search

- Divide matrix A into small blocks
- Analyze each output exhaustively

Sort and select

- Sort output
- Select r best combinations

Merge

- Merge adjacent blocks

X=11111111 Q_1=01001000 Q_2=00111100

$$A=\begin{bmatrix}-1 & 1 & -1 & -1 & 1 & -1 & -1 & -1\\ -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1\end{bmatrix}$$

$r = 3$

$AK=0$

$$\begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}k_1\\ k_2\end{bmatrix} \qquad k_1, k_2 \in \{0,1\}$$

$$\begin{bmatrix}0 & 1 & -1 & 0\\ 0 & -1 & -1 & -2\end{bmatrix}\begin{bmatrix}0 & -1 & -2\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}0 & 1 & -1 & 0\\ 0 & 1 & 1 & 2\end{bmatrix}\begin{bmatrix}0 & -1 & -2\\ 0 & 1 & -2\end{bmatrix}$$

[10 [3 1 2 3 3 1 9 … 9 …]

Complexity

linear to N
linear to r^2

Evaluation

Implementation experiment settings

Matlab R2010b, 4 GB RAM 2.13 GHz CPU

Experiment settings

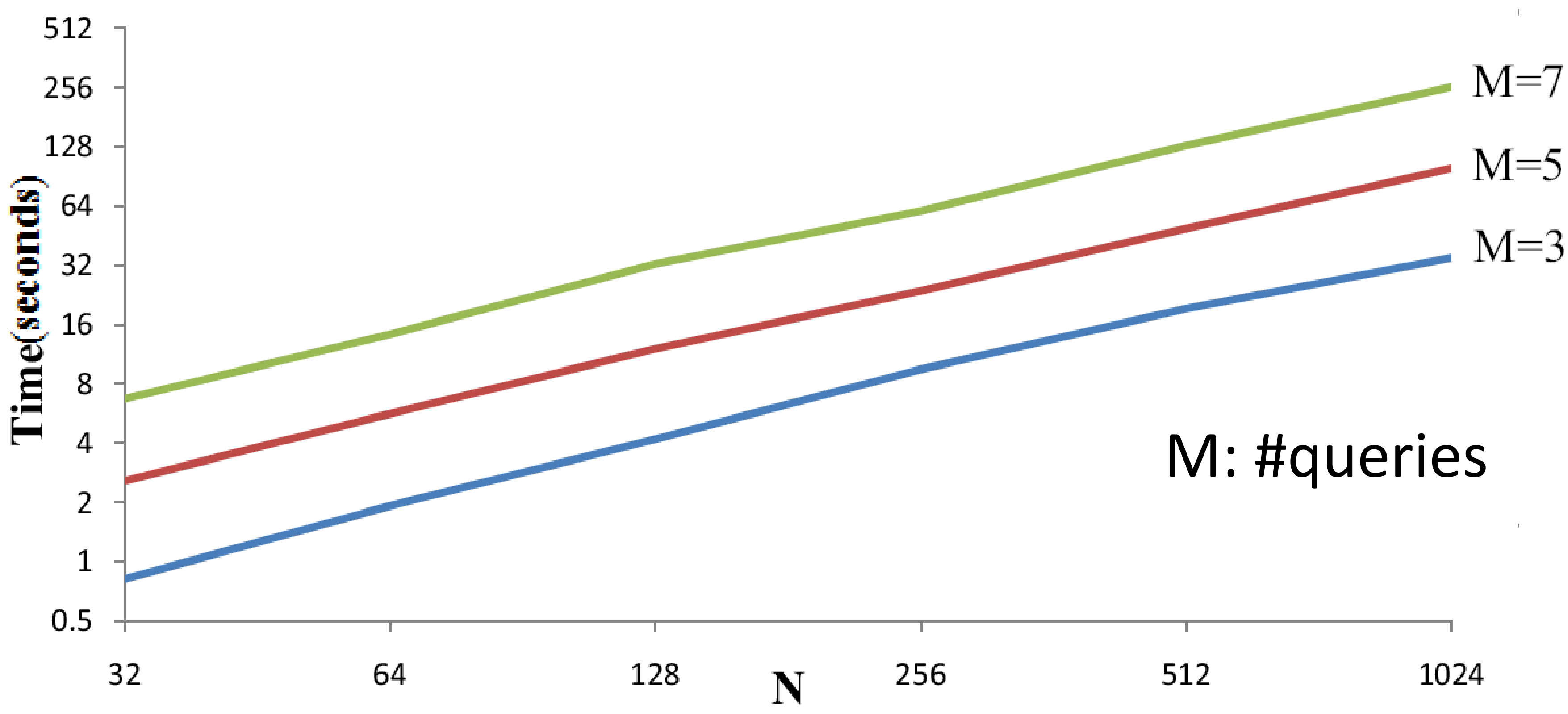
Averaging from 5 experiments

Scalability

Tightness

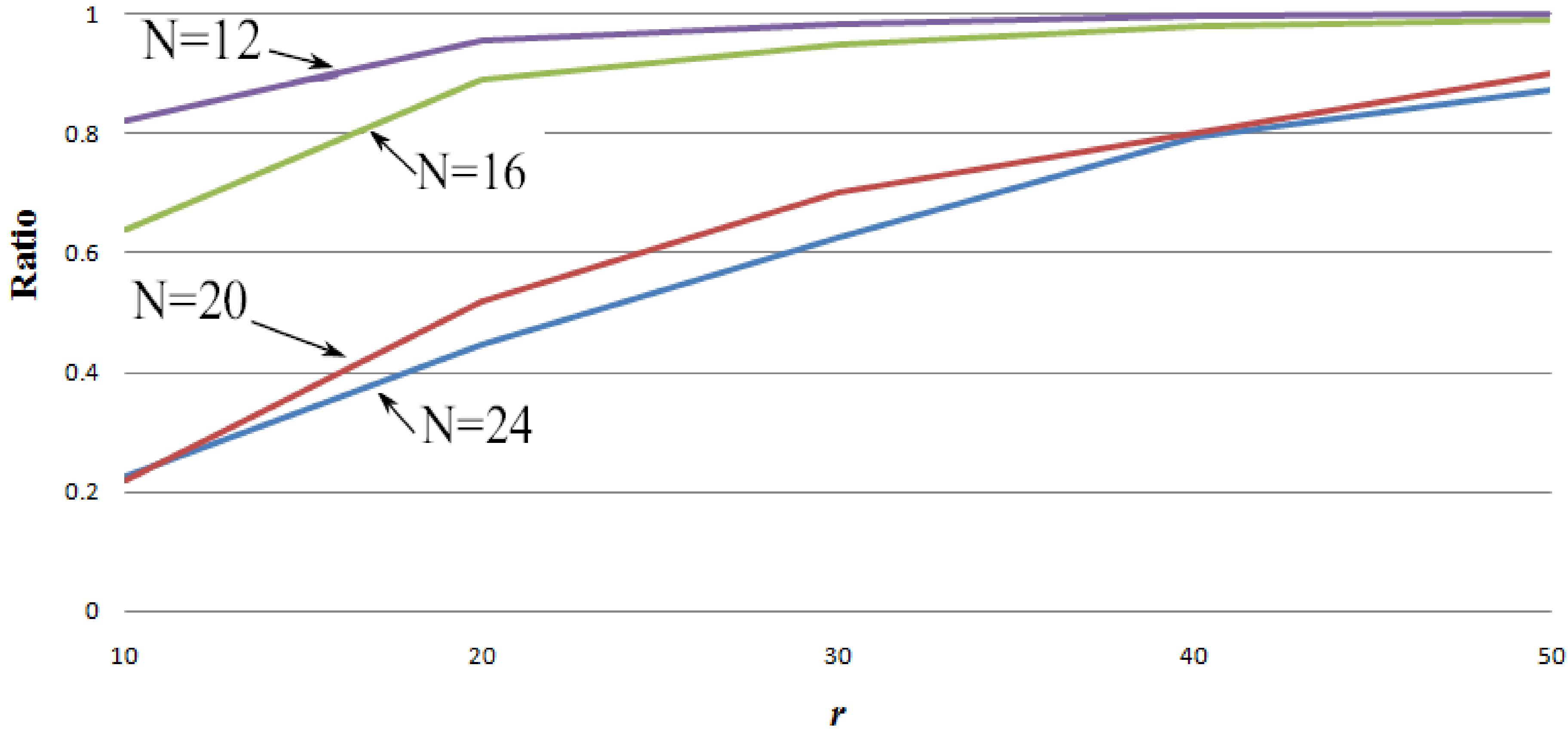
Real Application Performance (Iris Recognition)

Evaluation



Scalability

Evaluation



Tightness

See the paper for detailed results and
performance on iris application

Related Work

Estimating Information Leakage

– No auditor ([Goodrich, Oakland 2009])

– Weak auditor ([Wang, Wang et al., CCS 2009])

Differential Privacy

– Add noise to protect secret information ([Dwork, ICALP 2006])

Auditing Aggregate Queries

– Auditing SUM, MEAN or MAX queries over a set of private entries in a database ([Elshiekh, 2008],[Nabar et al., VLDB 2006], [Wang, Li et al., CCS 2009])

Summary

- Query auditor for the server
  - Fast
  - Performance adjustable
  - Single-bit leakage (see paper)

- Challenges
  - Non-additive decomposable functions
  - Secure two-party computation protocol

Thanks!

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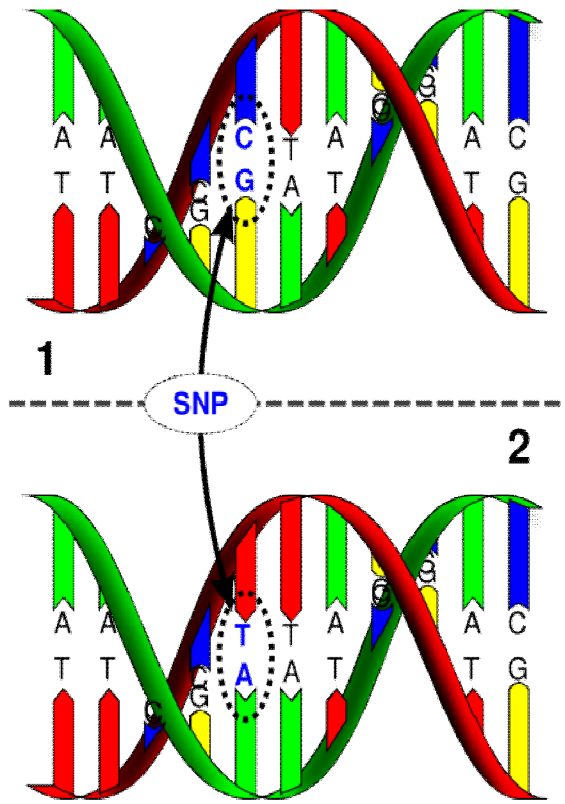
Single Bit Information Leakage

Motivation

One bit leakage can also be crucial:

SNP (single nucleotide polymorphism)



Assumptions

- The client knows nothing
- Any single bit information is leaked only when this bit is 100% determined

Single Bit Information Leakage

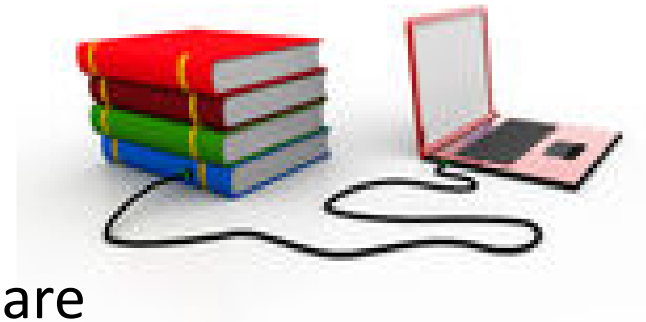
Straightforward idea

AK=0

Bit fully determined = corresponding k_i can only be 1

Check if there is a non-zero solution when k_i=0

Make it quicker……



Build pre-computed **libraries**
containing all possible combinations

Libraries can be pre-computer since they are
not related to sequence length (N), *X* and *Q*

Without libraries: $C^{6-1}_{1000-1} = 8.2 \times 10^{12}$ checks

With libraries: 9632 checks