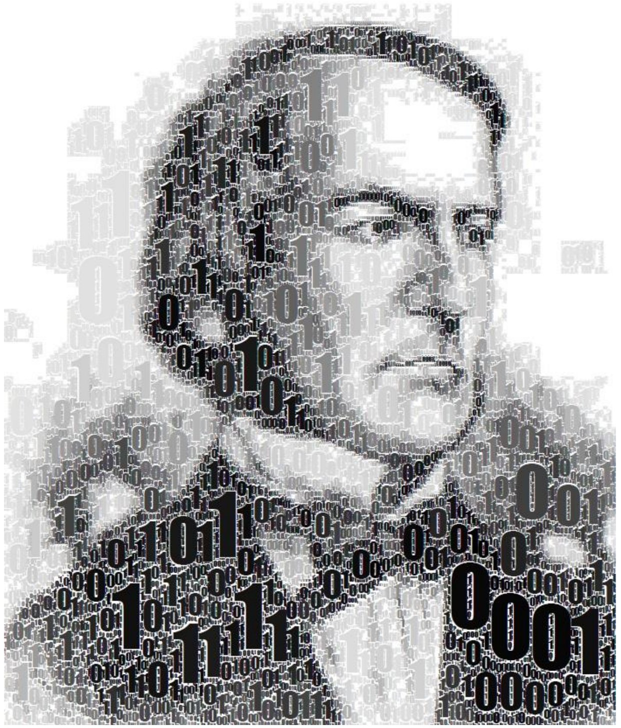
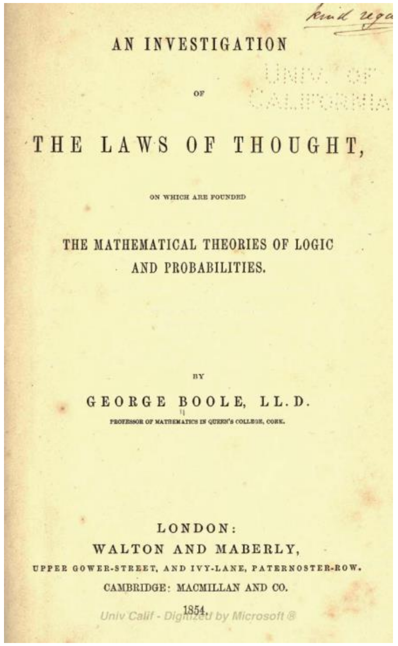
Historical Perspectives

George Boole (1815-1864)

- Mathematician and philosopher
- Invented symbolic / Boolean logic
- Invented Boolean algebra, i.e. "calculus of reasoning"
- A founder of computer science
- "An Investigation into the Laws of Thought"
- Influenced De Morgan, Schröder, Shannon
- All modern computers, electronics, phones, data transmission, rely on Boolean principles





AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

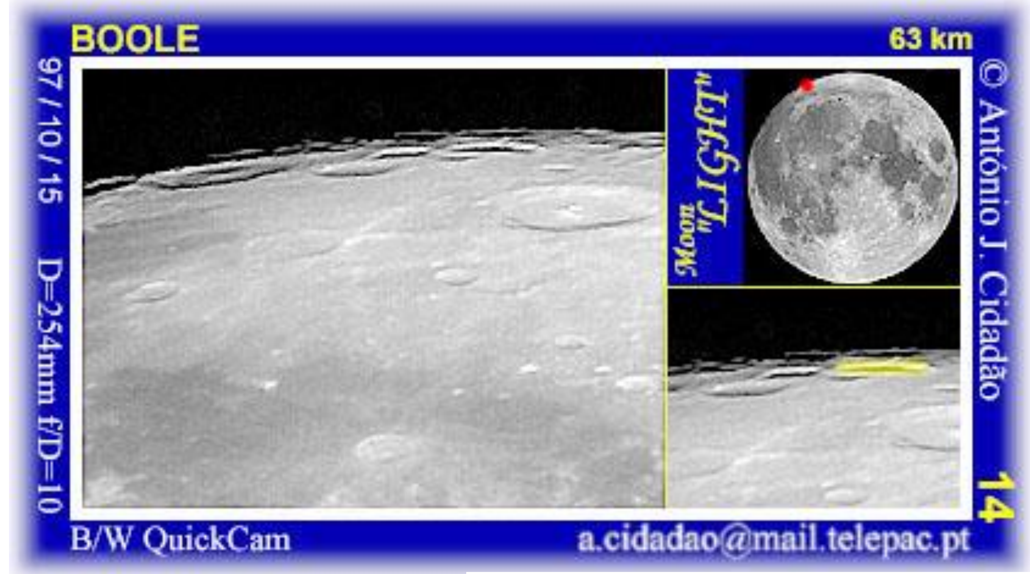
BY

GEORGE BOOLE, LL.D.

PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

LONDON:
WALTON AND MABERLY,
UPPER GOWER-STREET, AND IVY-LANE, PATERNOSTER-ROW.
CAMBRIDGE: MACMILLAN AND CO.
1854.


GEORGE BOOLE
LL.D. D.C.L. F.R.S.
1815 – 1864
GEORGE BOOLE, FATHER OF MODERN ALGEBRA, AUTHOR OF "THE LAWS OF THOUGHT" AND FIRST PROFESSOR OF MATHEMATICS AT UNIVERSITY COLLEGE, CORK, WAS BORN IN LINCOLN AND ESTABLISHED AN ACADEMY IN THIS HOUSE C.1840.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97 / 10 / 15 D=254mm f/D=10
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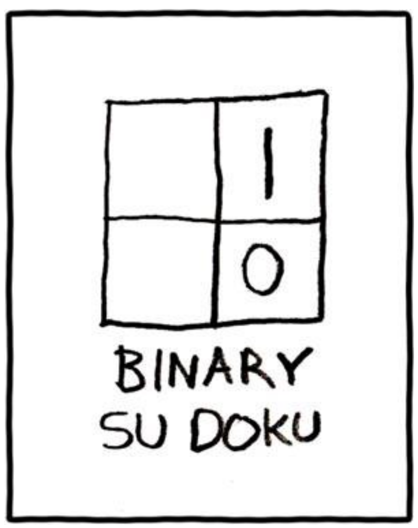
a.cidadao@mail.telepac.pt


36
РЕПУБЛИКА МАКЕДОНИЈА
САМИТ ЗА ИНФОРМАТИЧКО ОПШТЕСТВО
ПОШТА
2004 СКЕНПОИНТ


LOGIC LANE


GEORGE BOOLE
DIED DEC


1
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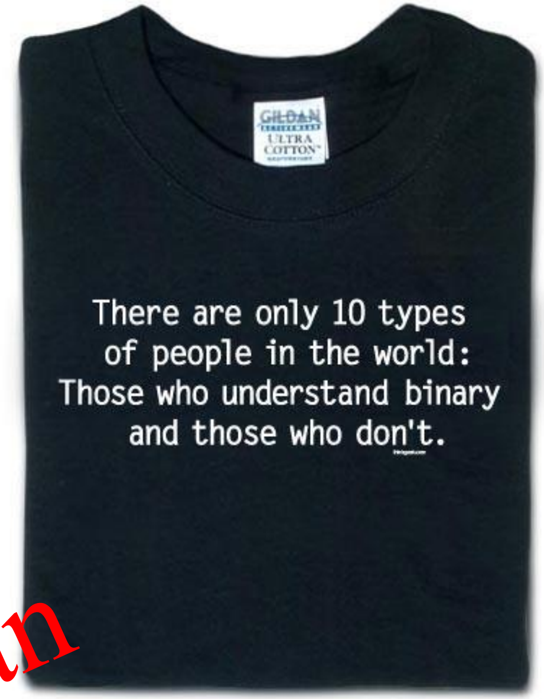
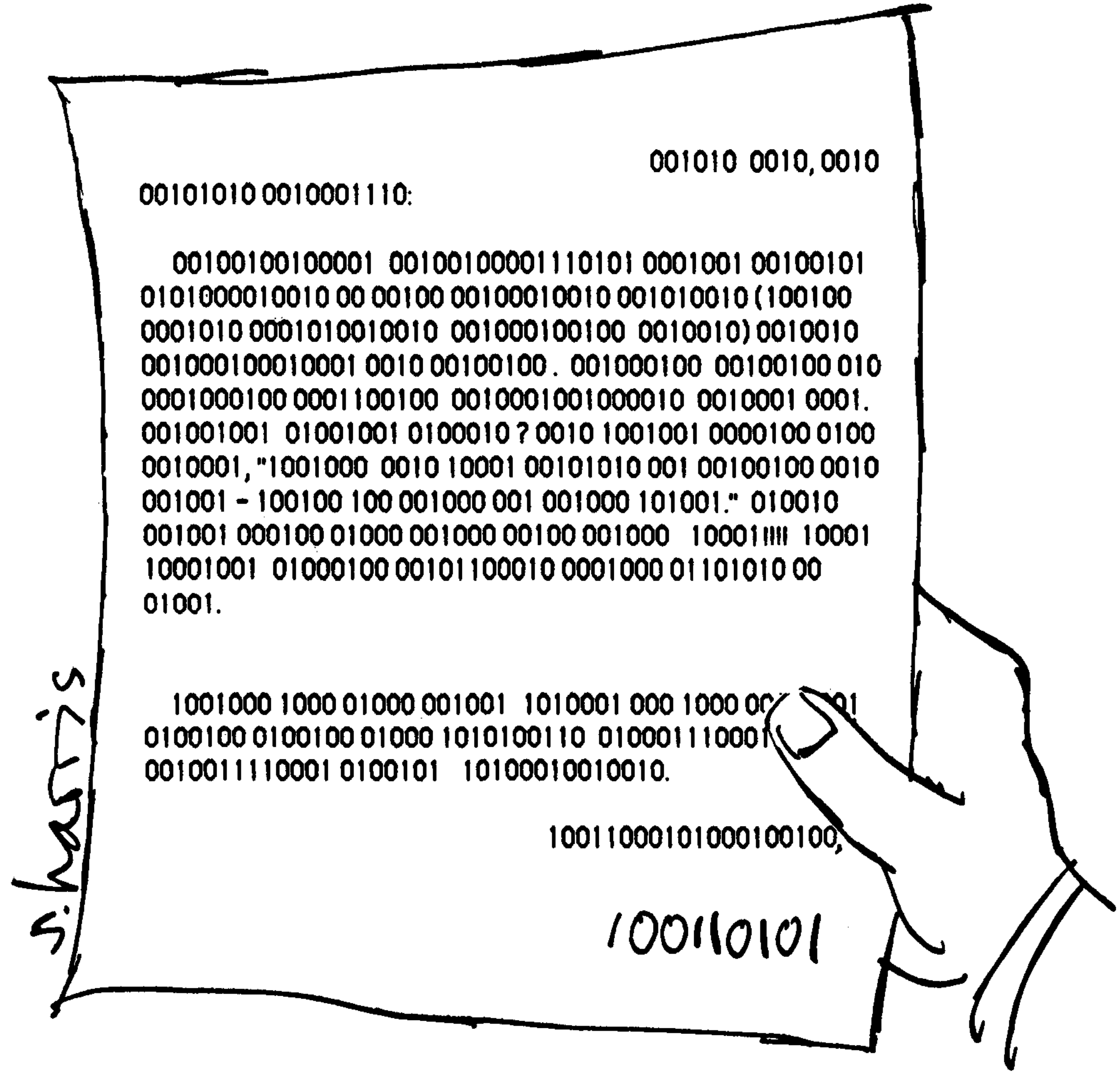
BINARY
SU DOKU


All cats have four legs.
I have four legs.
Therefore, I am a cat.
S. Harris

Boolean humor





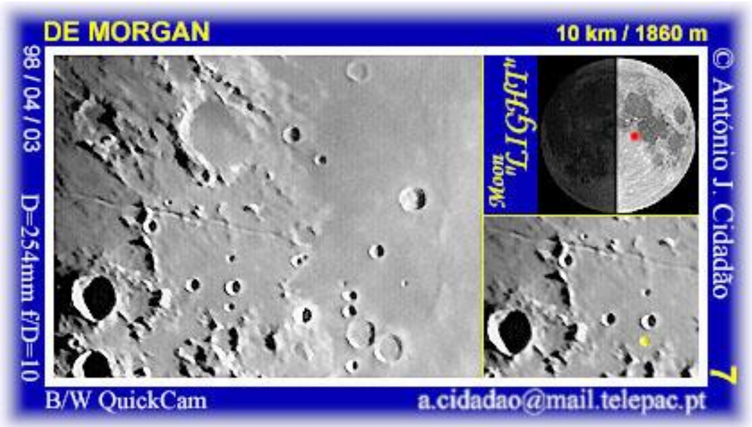
BINARY LETTER FROM GRANDMA

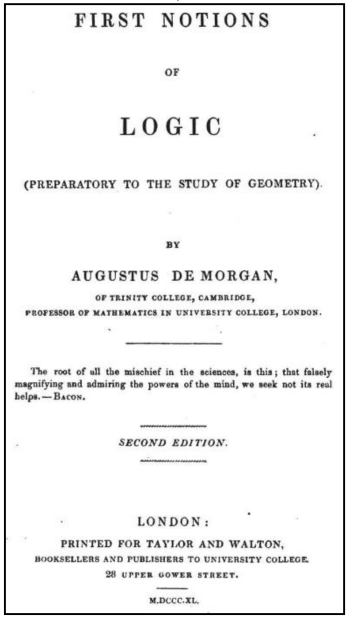
Mozart writing the digital version of his symphony No. 38
in D major.

Historical Perspectives

Augustus De Morgan (1806-1871)

- Mathematician and logician
- Developed logic & mathematical induction
- De Morgan's Laws in logic & set theory
- Invented relational algebra
- Corresponded extensively with Hamilton
- Influenced Russell, Whitehead, and Tarski
- Studied paradoxes




FIRST NOTIONS

OF

LOGIC

(PREPARATORY TO THE STUDY OF GEOMETRY).

BY

AUGUSTUS DE MORGAN,

OF TRINITY COLLEGE, CAMBRIDGE,

PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

The root of all the mischief in the sciences, is this; that falsely magnifying and admiring the powers of the mind, we seek not its real helps.—BACON.

SECOND EDITION.

LONDON:

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28 UPPER GOWER STREET.

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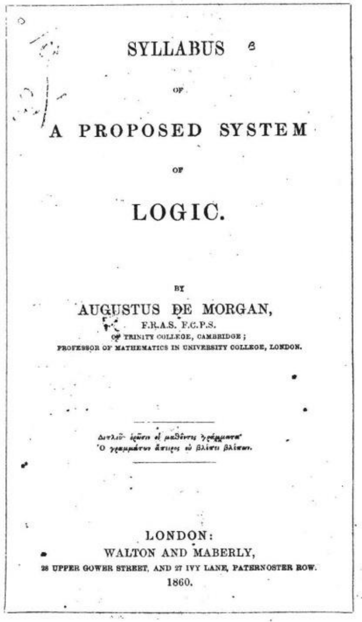
SYLLABUS

OF

A PROPOSED SYSTEM

OF

LOGIC.

BY

AUGUSTUS DE MORGAN,

F.R.A.S. F.C.P.S.

OF TRINITY COLLEGE, CAMBRIDGE;

PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

LONDON:

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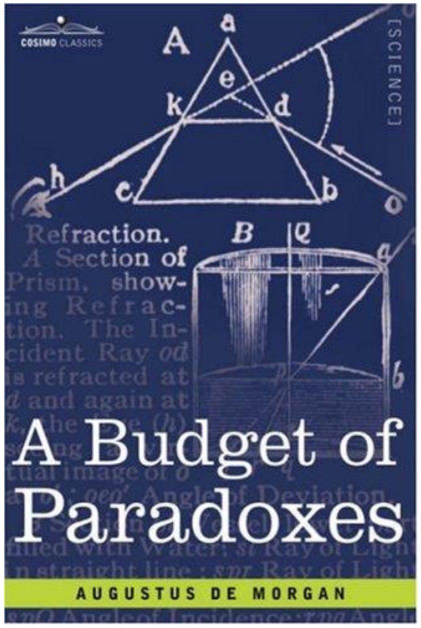
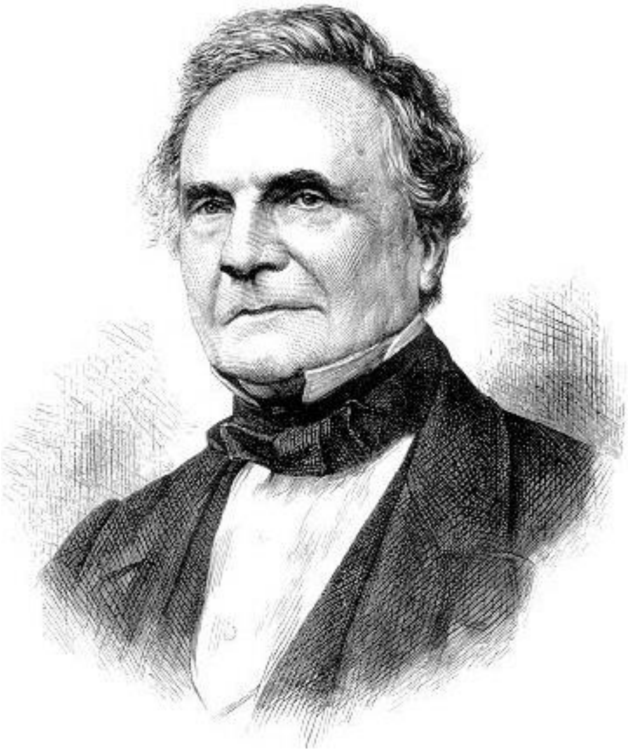
28 UPPER GOWER STREET, AND 27 IVY LANE, PATERNOSTER ROW.

1860.

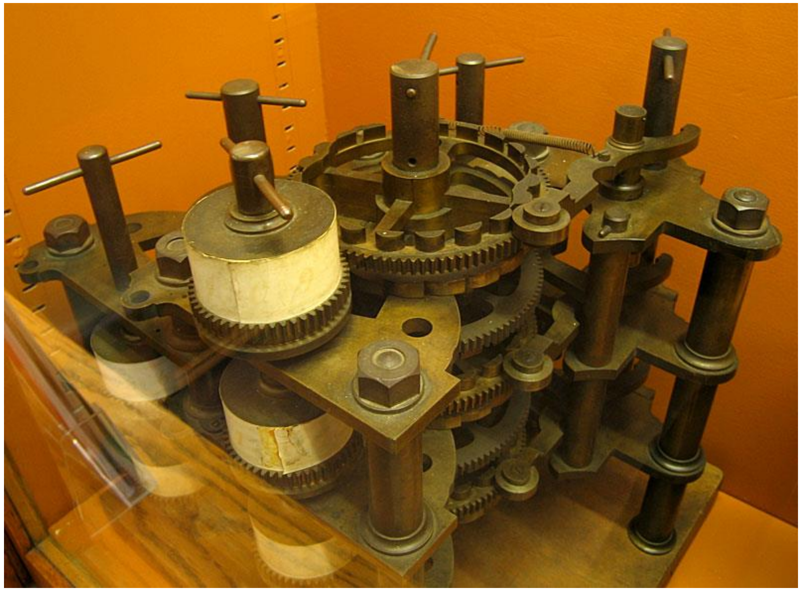


Historical Perspectives

Charles Babbage (1791-1871)

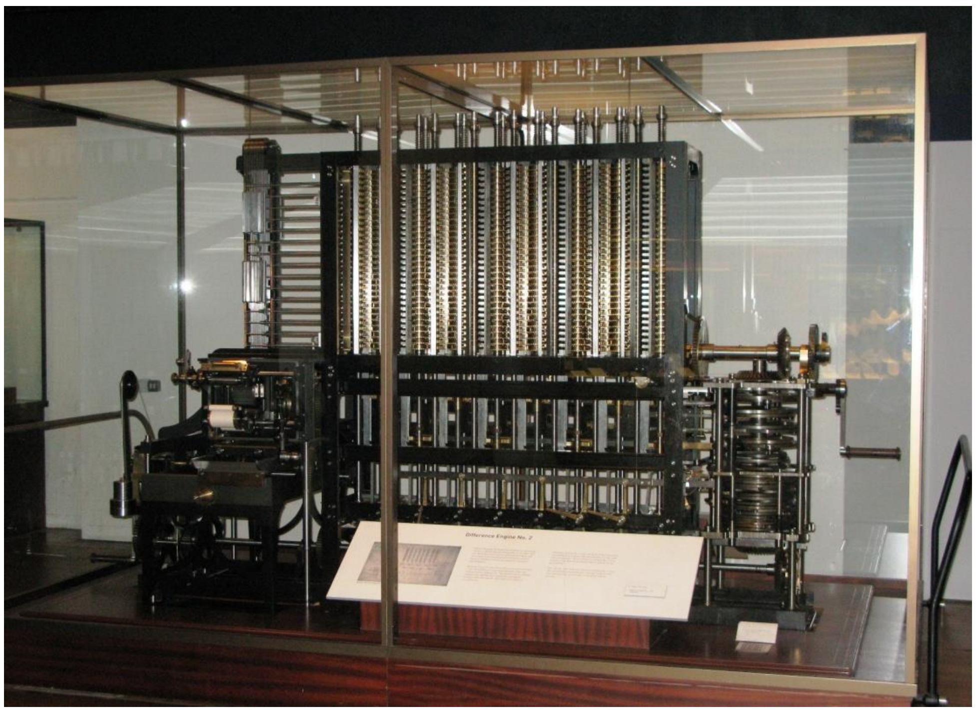


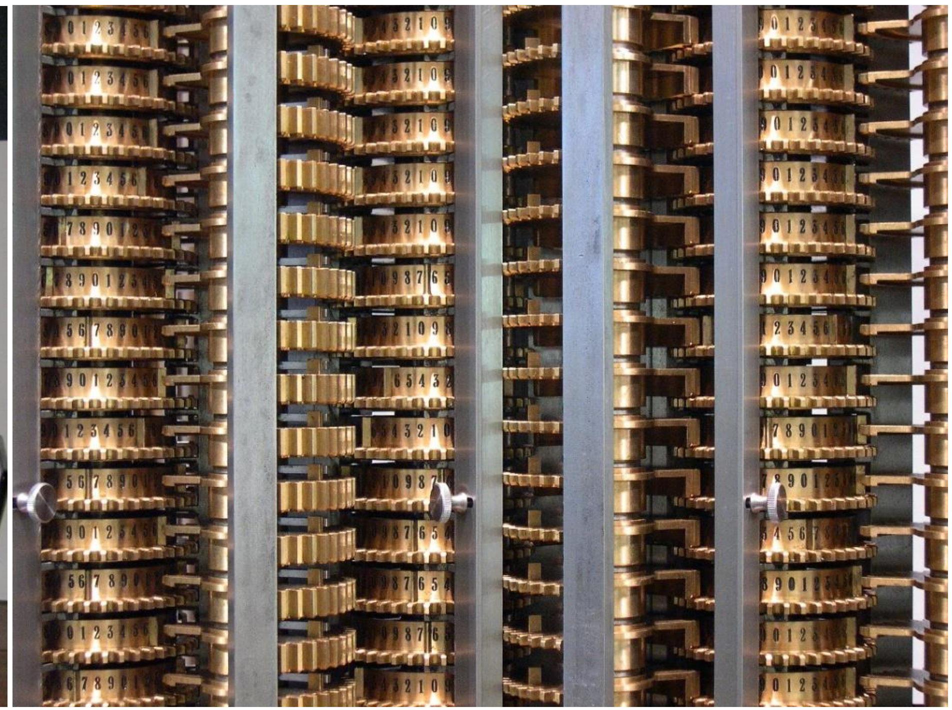
- Mathematician, philosopher, inventor mechanical engineer, and economist
- The father of computing
- Built world's first mechanical computer
  - the "difference engine" (1822)
- Originated the programmable computer
  - the "analytical engine" (1837)
- Worked in cryptography
- Developed Babbage's principle of division of labor



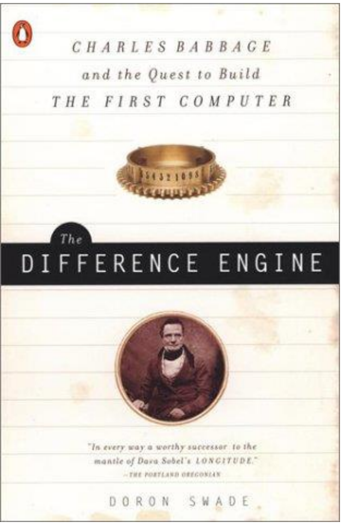
Babbage's Difference Engine

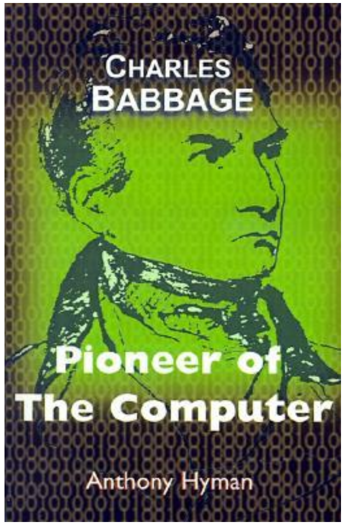
- World's first mechanical computer
- Designed in 1822, redesigned in 1847-1849
- 25,000 parts, 15 tons, 8ft tall, 31 digits of precision
- Tabulated polynomial functions, used Newton's method
- Approximated logarithmic and polynomial functions
- Used decimal number system and hand-crank

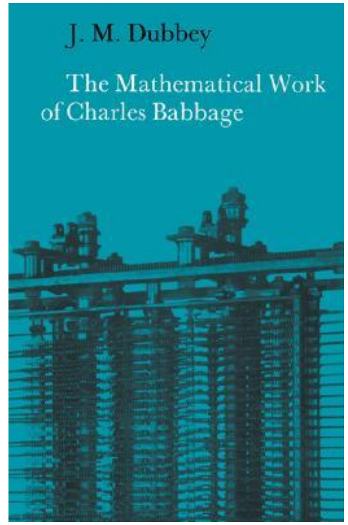


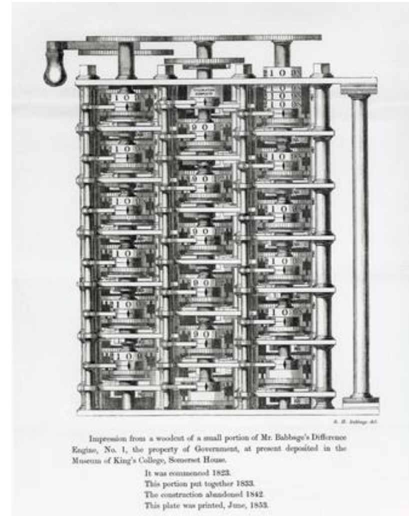


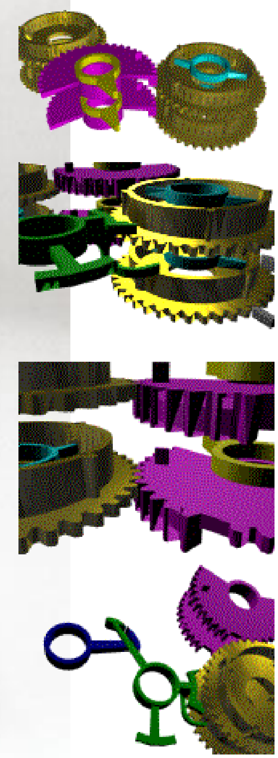
Babbage’s Difference Engine



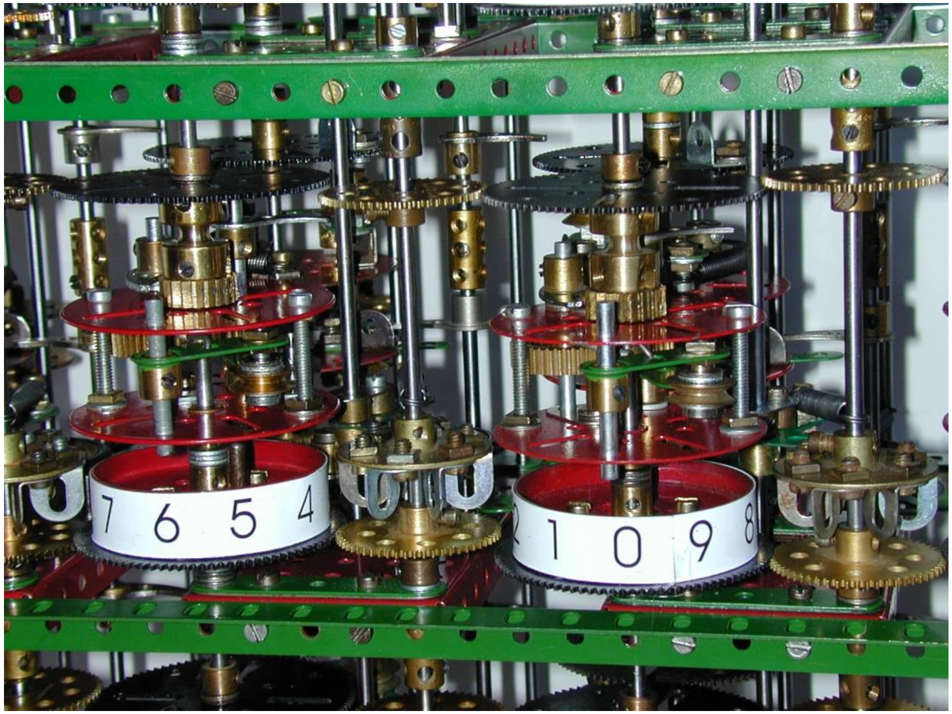


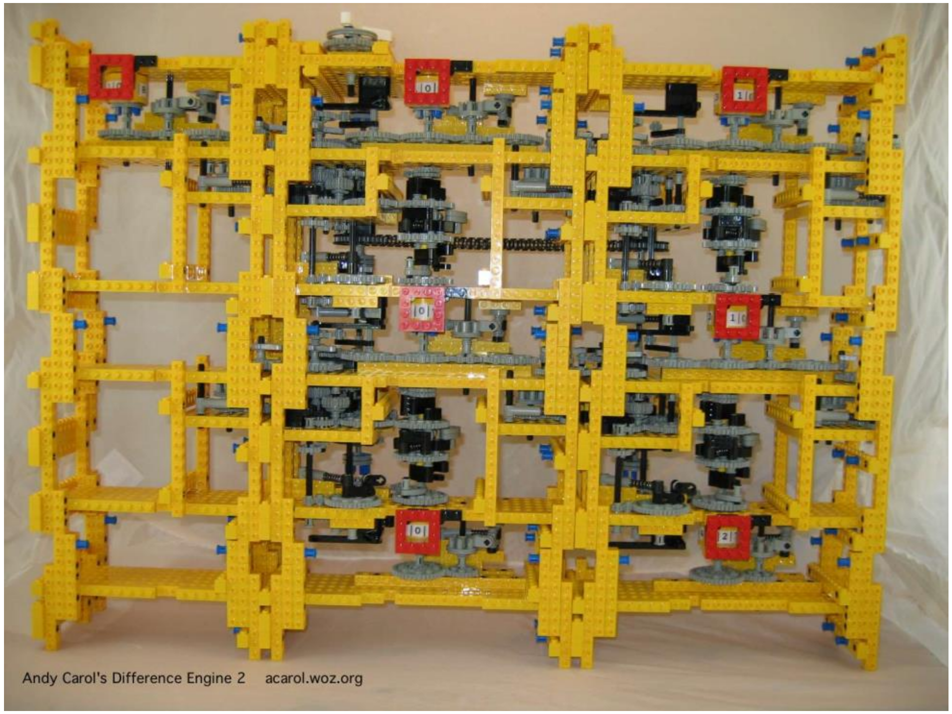






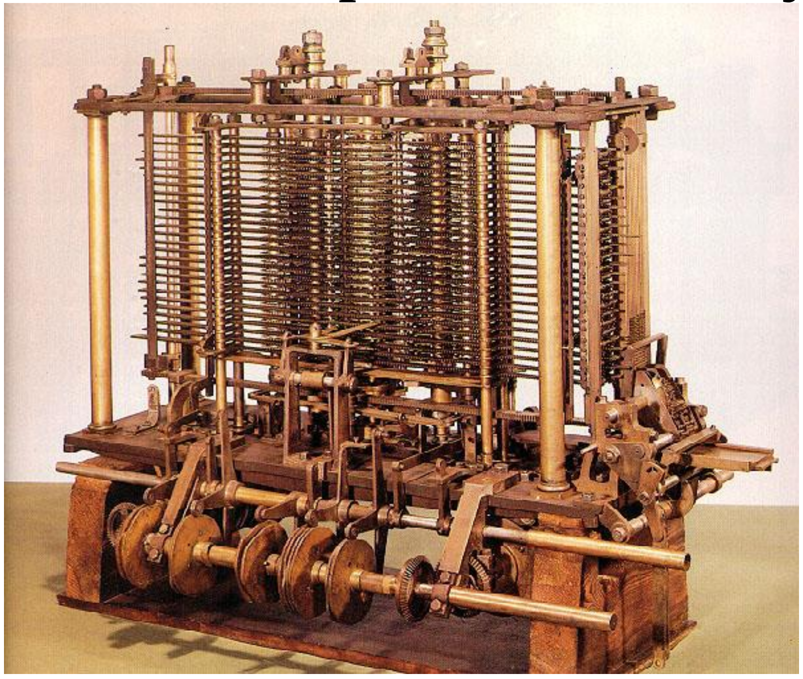
Babbage's difference engine built from Mechano and Lego

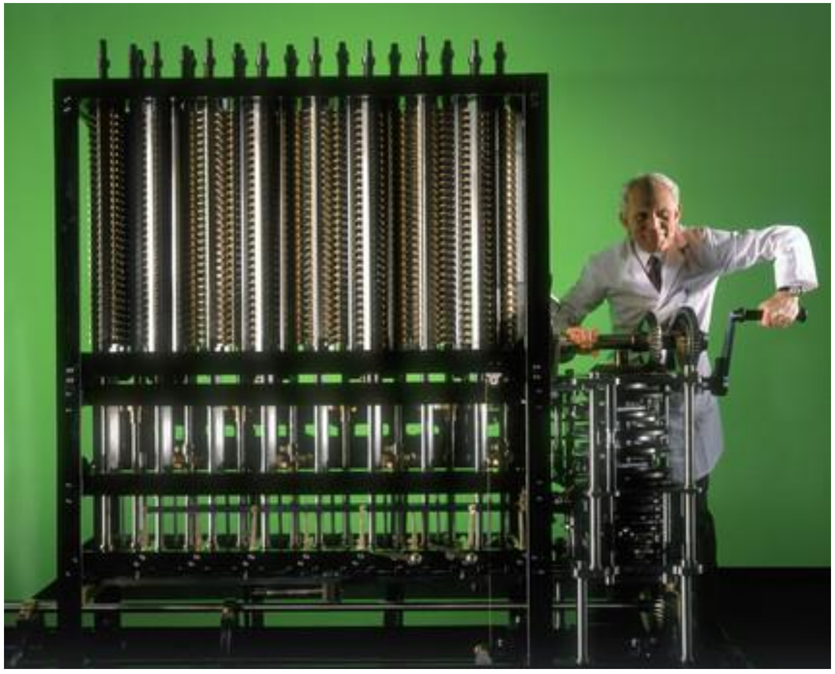




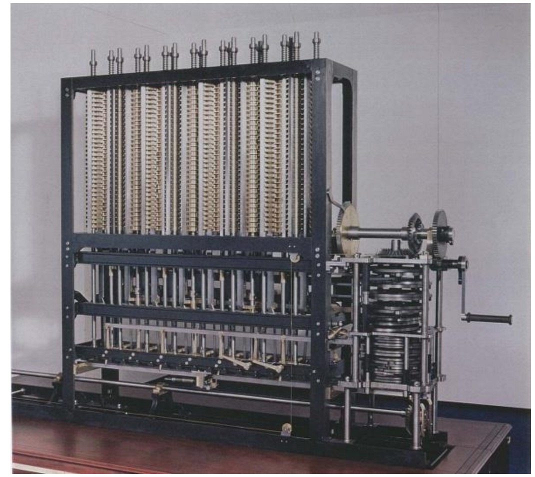
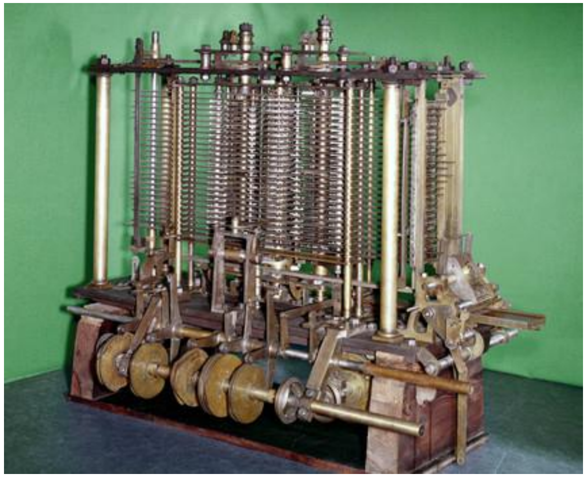
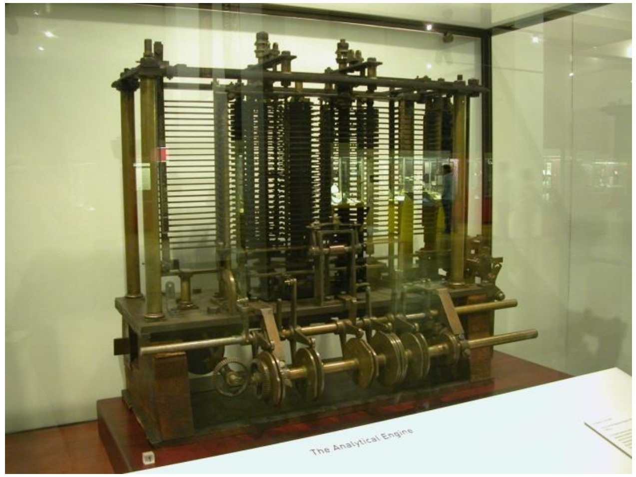
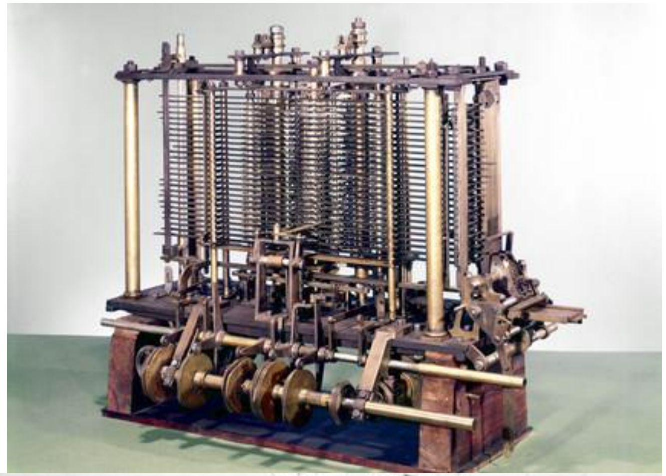
Babbage's Analytical Engine

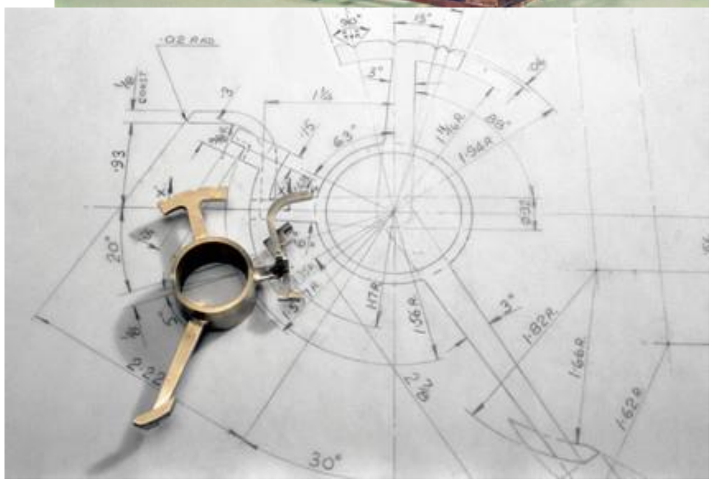
- World's first general-purpose computer
- Designed in 1837, redesigned throughout Babbage's life
- Turing-complete, memory: 1000x50 digits (21 kB)
- Fully programmable "CPU", used punched cards
- Featured ALU, "microcode", loops, and printer!
- Could multiply two 20-digit numbers in 3 min
- Few components built by Babbage; constructed in 1991

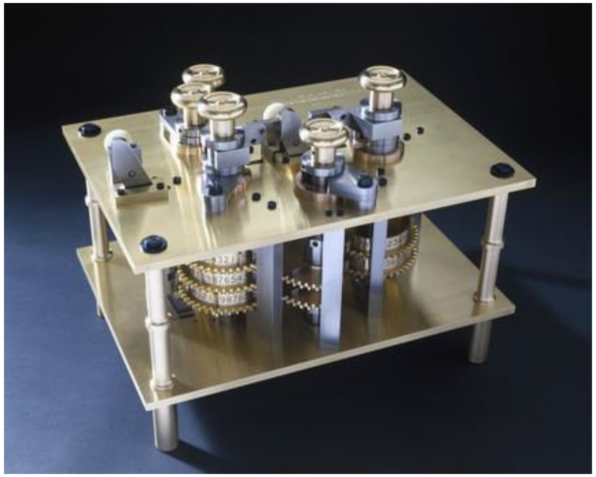
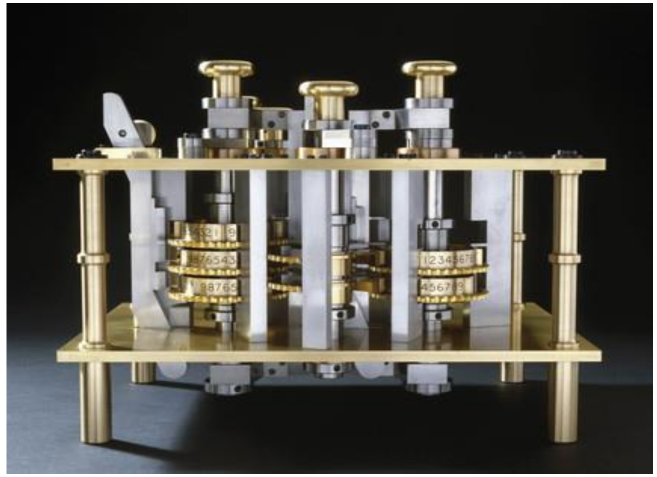


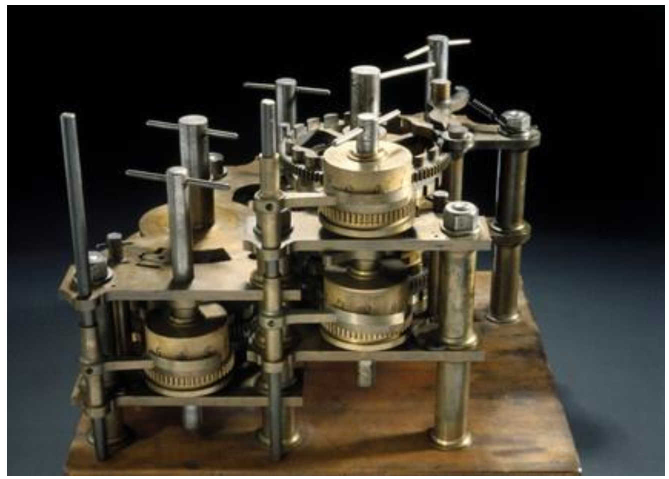




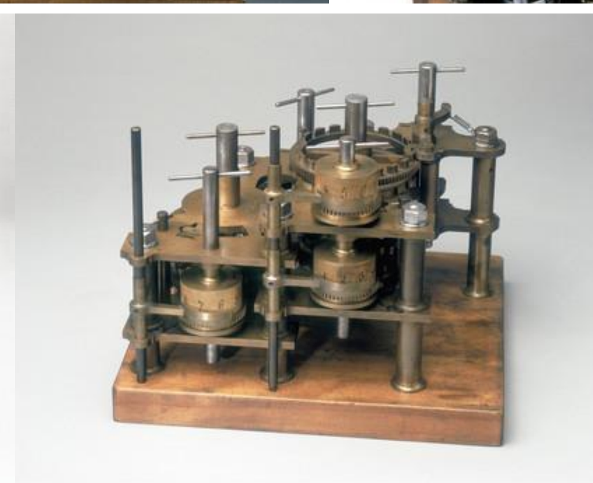




The Analytical Engine


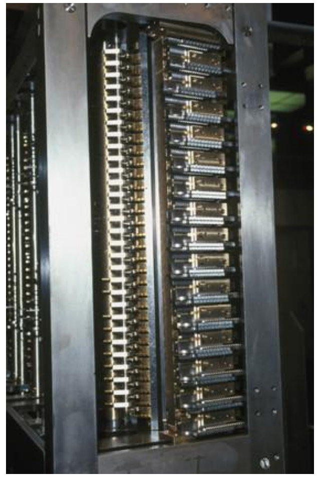



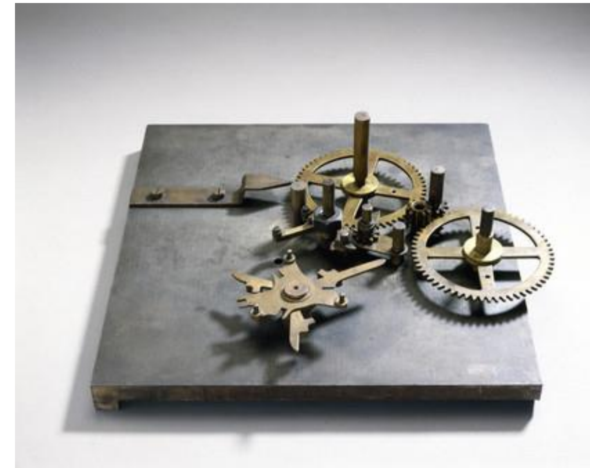
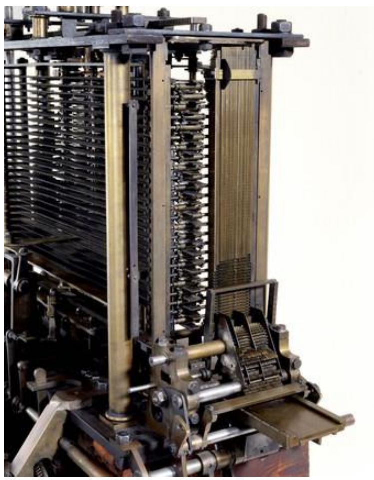



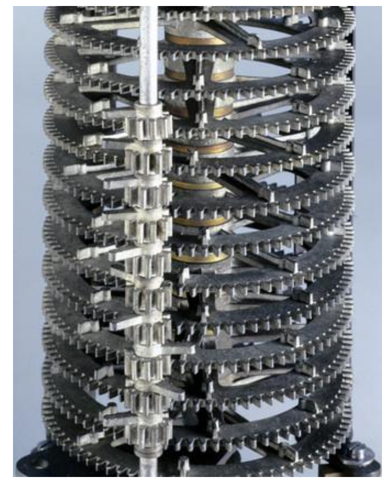




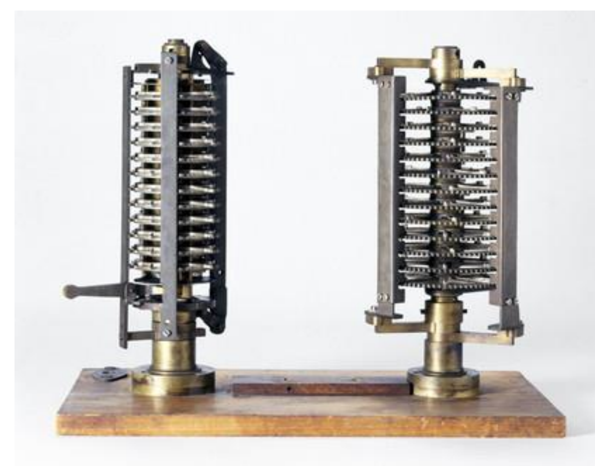


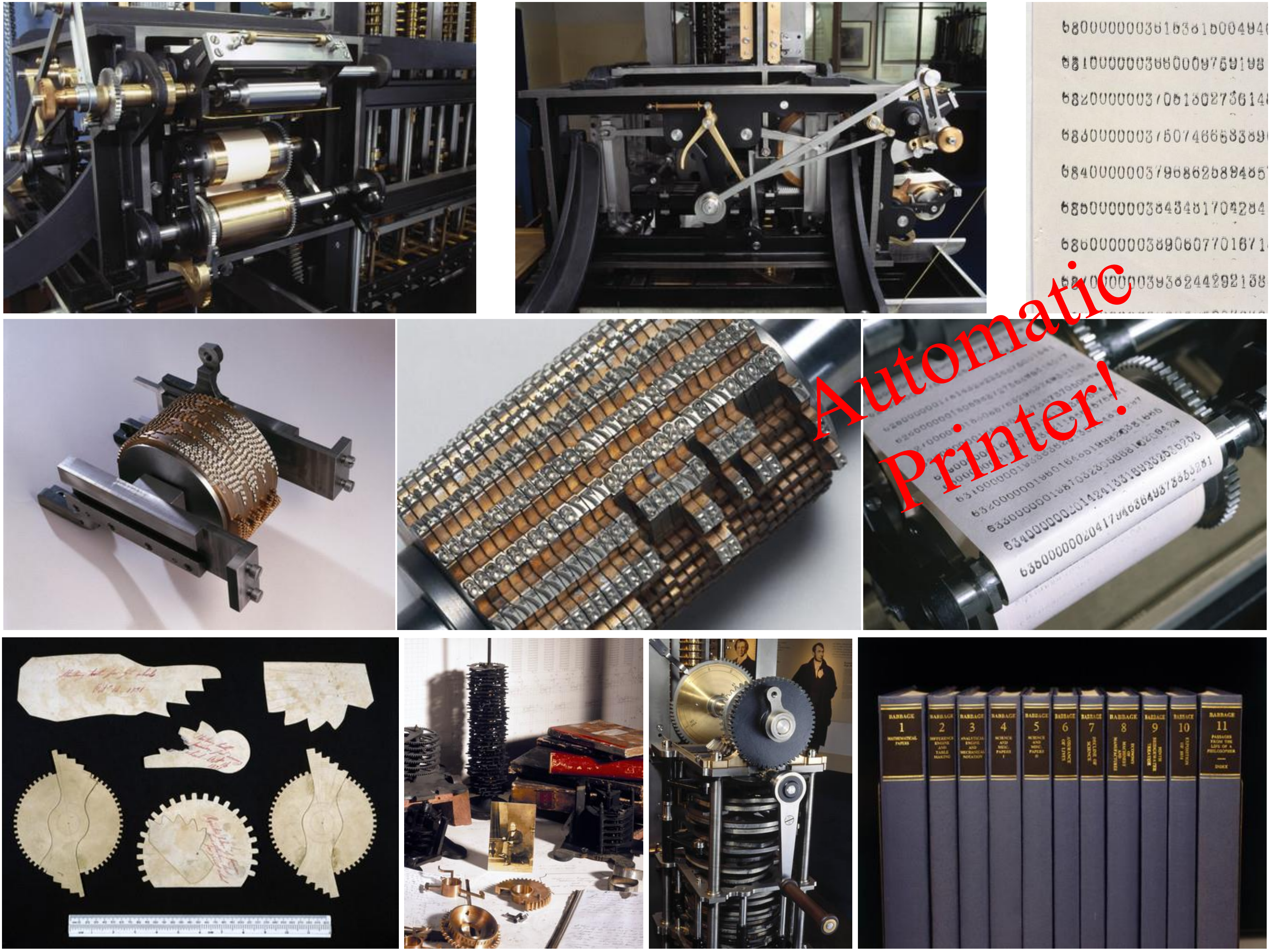




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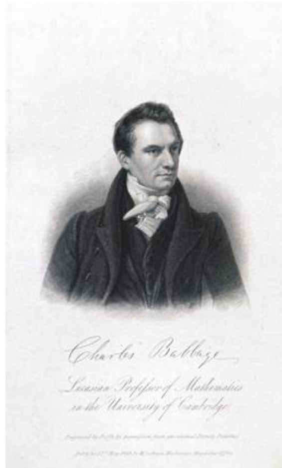
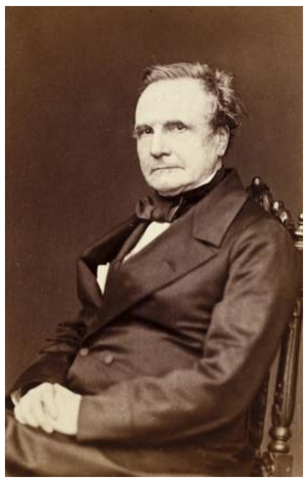

Charles Babbage
Lucasian Professor of Mathematics
in the University of Cambridge


Charles Babbage

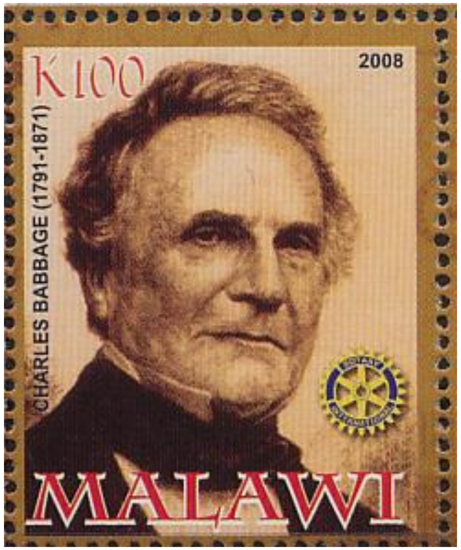


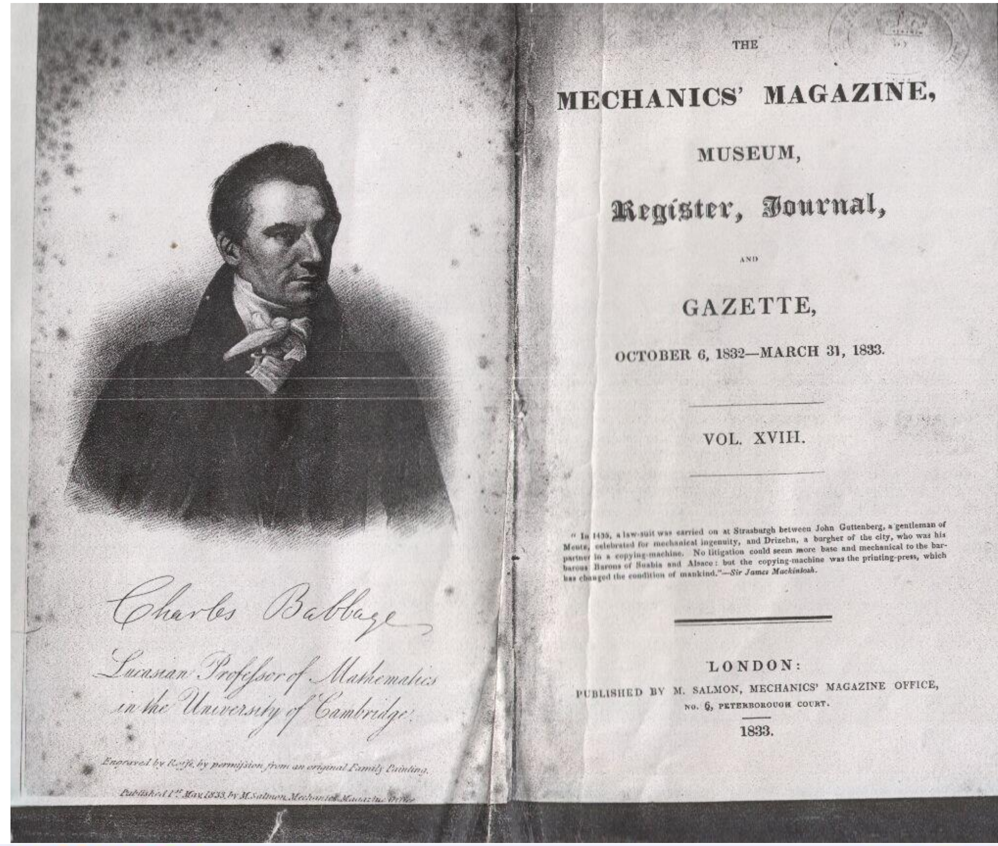

C. BABBAGE (MATHEMATICIAN)
Died Oct. 20, 1871, aged 79


Ever truly yours
C Babbage


Ada.
COUNTESS OF LOVELACE.
Daughter of the late Lord Byron


Lady Byron

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MECHANICS' MAGAZINE,

MUSEUM,

Register, Journal,

AND

GAZETTE,

OCTOBER 6, 1832—MARCH 31, 1833.

VOL. XVIII.

"In 1435, a law-suit was carried on at Strasburgh between John Guttenberg, a gentleman of Mentz, celebrated for mechanical ingenuity, and Drizehn, a burgher of the city, who was his partner in a copying-machine. No litigation could seem more base and mechanical to the barbarous Barons of Suabia and Alsace: but the copying-machine was the printing-press, which has changed the condition of mankind."—*Sir James Mackintosh.*

LONDON:

PUBLISHED BY M. SALMON, MECHANICS' MAGAZINE OFFICE,

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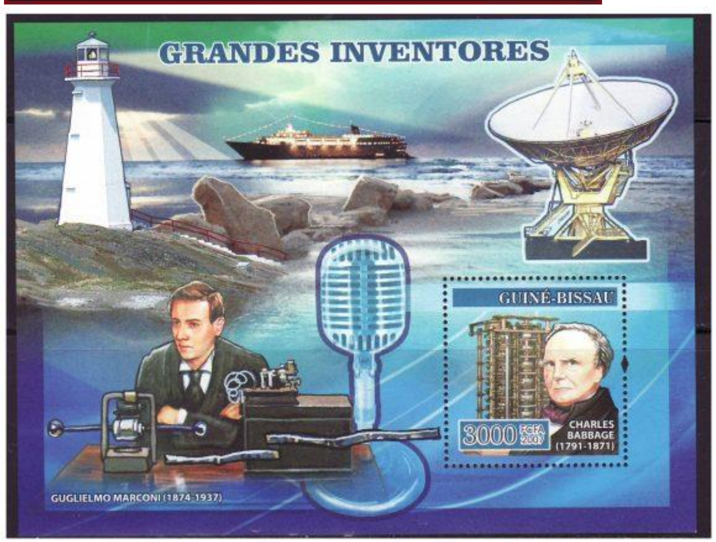
1833.



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- New *CBI Newsletter* (Spring 2009, Vol. 31:1)
- McDonald Named 2009-2010 Tomash Fellow
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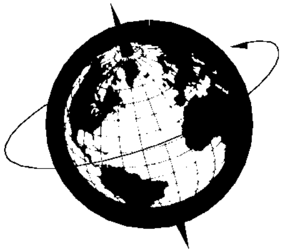
Historical Perspectives

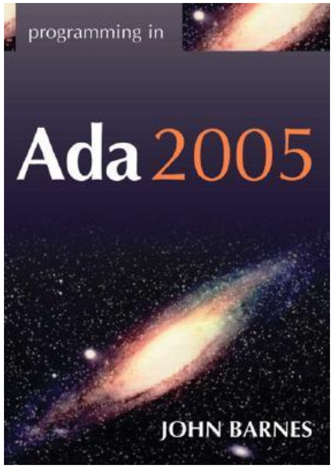
Countess Ada Lovelace (1815-1852)

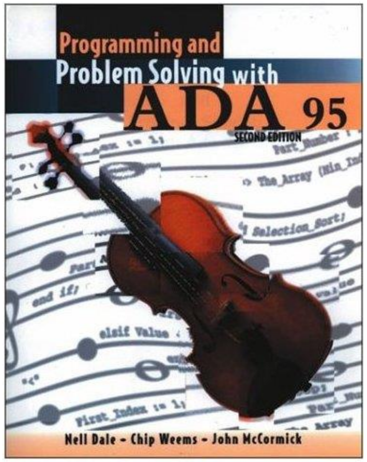
- Daughter of Lord Byron
- Tutored in math and logic by De Morgan
- Wrote the "manual" for Babbage's analytical engine, as well as programs for it
- World's first computer programmer!
- Foresaw the vast potential of computers
- Babbage: "The Enchantress of Numbers"
- DoD's Ada language "MIL-STD-1815"

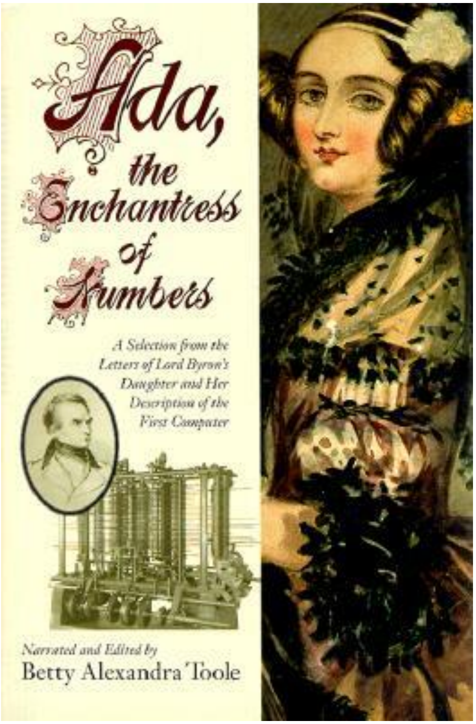


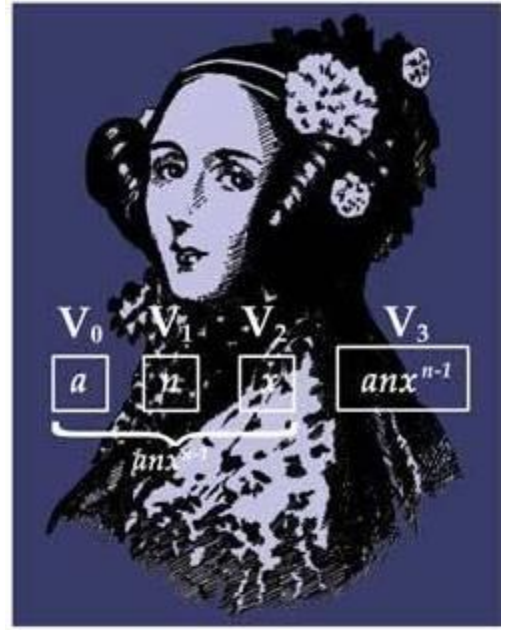
Ada

The International Language
for Software Engineering

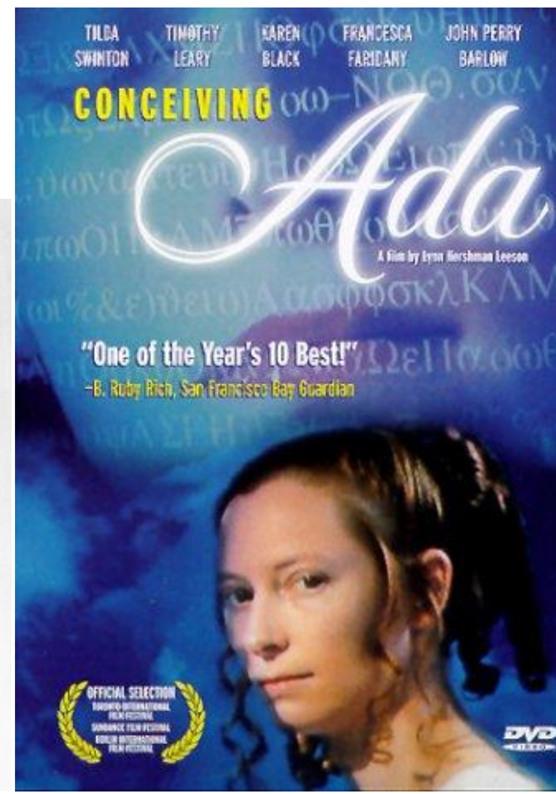








Ada Byron, Lady Lovelace
1815 - 1852

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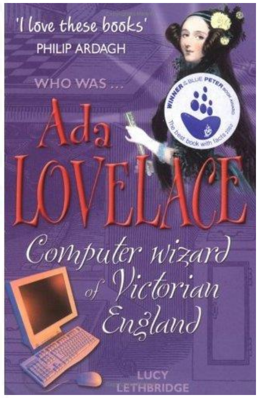
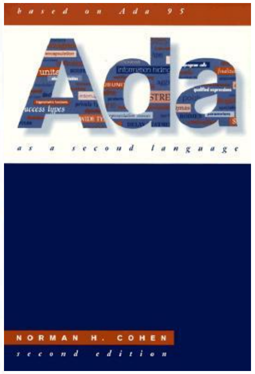
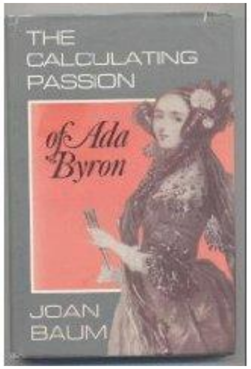
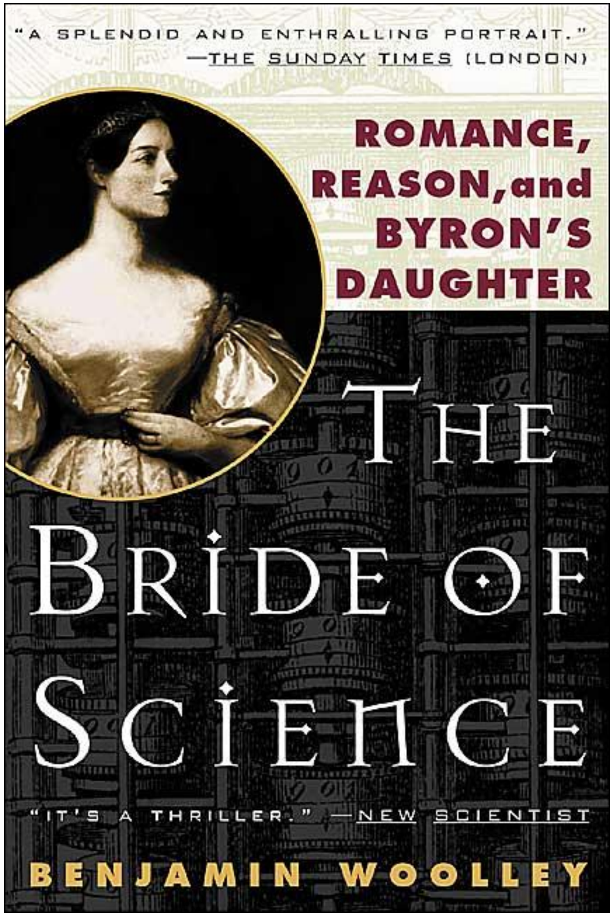








Lovelace Medal

Lovelace Medal

The Lovelace Medal is presented to individuals who have made a contribution which is of major significance in the advancement of Information Systems or which adds significantly to the understanding of Information Systems.

About the medal

Lovelace Medal 2009

2009 winner
The winner of the 2009 Lovelace Medal is Professor Yorick Wilks.

Previous Lectures

Video: A tribute to Karen Spärck Jones
The 2008 BCS Lovelace Medal lecture was a very special event dedicated to the memory of Karen Spärck Jones who was presented the award just weeks before she died last year. The lecture was delivered by Dr Ann Copestake and is now available to watch online.

2007 Lovelace Lecture - Sir Tim Berners-Lee
The Web is a technical and social creation, dependent on both technical protocols and social conventions. The origins and potential futures of this large scale, emergent phenomena were discussed by Sir Tim Berners-Lee in this year's BCS Lovelace Lecture - now available to watch via this website.

Previous winners

Previous winners of the Lovelace Medal have included:

2008 - Dr Tony Storey
2007 - Karen Sparck-Jones
2006 - Sir Tim Berners-Lee
2005 - Dr Nicholas McKeown

Ada Lovelace notes on “Sketch of the Analytical Engine Invented by Charles Babbage”, by L. F. Menabrea, 1843

Her notes (three times longer than the paper itself!) contain the world’s first computer program (for calculating Bernoulli numbers):

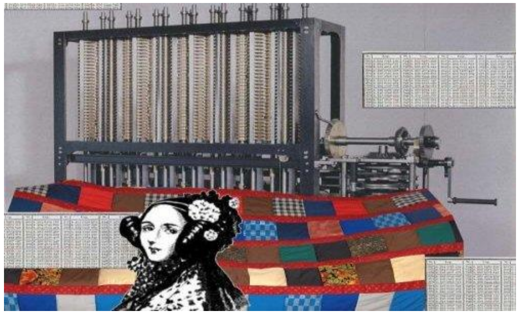
Number of Operations	Nature of Operations	Variables for Data						Working Variables									Variables for Results	
		$^1V_0$	$^1V_1$	$^1V_2$	$^1V_3$	$^1V_4$	$^1V_5$	$^0V_6$	$^0V_7$	$^0V_8$	$^0V_9$	$^0V_{10}$	$^0V_{11}$	$^0V_{12}$	$^0V_{13}$	$^0V_{14}$	$^0V_{15}$	$^0V_{16}$
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		$\boxed{m}$	$\boxed{n}$	$\boxed{d}$	$\boxed{m'}$	$\boxed{n'}$	$\boxed{d'}$										$\boxed{\frac{dn'-d'n}{mn'-m'n} = x}$	$\boxed{\frac{d'm-dm}{mn'-m'n} = y}$
1	×	m	....	....	....	n'	....	mn'										
2	×	....	n	....	m'	....	....	....	$m'n$									
3	×	....	....	d	....	....	....	....	....	dn'								
4	×	....	0	....	....	....	d'	....	....	....	$d'n$							
5	×	0	....	....	....	....	0	....	....	....	....	$d'm$						
6	×	....	....	0	0	....	....	....	....	....	....	.....	dm'					
7	−	....	....	....	....	....	....	0	0	....	....	.....	.....	$(mn' - m'n)$				
8	−	....	....	....	....	....	....	....	....	0	0	.....	.....	....	$(dn' - d'n)$			
9	−	....	....	....	....	....	....	....	....	....	....	0	0	....	....	$(d'm - dm')$		
10	÷	....	....	....	....	....	....	....	....	....	....	.....	.....	$(mn' - m'n)$	0	....	$\frac{dn'-d'n}{mn'-m'n} = x$	
11	÷	....	....	....	....	....	....	....	....	....	....	.....	.....	0	....	0	....	$\frac{d'm-dm'}{mn'-m'n} = y$

World's first computer program (for calculating Bernoulli numbers), by Ada Lovelace, 1843:

Number of Operation	Nature of Operation	Variables acted upon	Variables receiving results	Indication of change in the value on any Variable	Statement of Results	Data			Working Variables										Result Variables			
						$^1V_1$	$^1V_2$	$^1V_3$	$^0V_4$	$^0V_5$	$^0V_6$	$^0V_7$	$^0V_8$	$^0V_9$	$^0V_{10}$	$^0V_{11}$	$^0V_{12}$	$^0V_{13} \cdots$	$^1V_{21}$	$^1V_{22}$	$^1V_{23}$	$^0V_{24} \cdots$
						○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
						0	0	0	0	0	0	0	0	0	0	0	0	0	B in a dec. fract.	B in a dec. fract.	B in a dec. fract.	0
						0	0	0	0	0	0	0	0	0	0	0	0	0				0
						1	2	4	0	0	0	0	0	0	0	0	0	0				0
						$\boxed{1}$	$\boxed{2}$	$\boxed{n}$											$\boxed{B_1}$	$\boxed{B_3}$	$\boxed{B_5}$	$\boxed{B_7}$
1	×	$^1V_2 \times {}^1V_3$	$^1V_4, {}^1V_5, {}^1V_6$	$\begin{cases} ^1V_2 = {}^1V_2 \\ ^1V_3 = {}^1V_3 \end{cases}$	$= 2n$	......	2	n	$2n$	$2n$	$2n$											
2	−	$^1V_4 - {}^1V_1$	$^2V_4$	$\begin{cases} ^1V_4 = {}^2V_4 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= 2n - 1$	1	......	......	$2n - 1$													
3	+	$^1V_5 + {}^1V_1$	$^2V_5$	$\begin{cases} ^1V_5 = {}^2V_5 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= 2n + 1$	1	......	......	......	$2n + 1$												
4	÷	$^2V_5 \div {}^2V_4$	$^1V_{11}$	$\begin{cases} ^2V_5 = {}^0V_5 \\ ^2V_4 = {}^0V_4 \end{cases}$	$= \frac{2n-1}{2n+1}$	......	......	......	0	0	......	......	......	......	......	$\frac{2n-1}{2n+1}$						
5	÷	$^1V_{11} \div {}^1V_2$	$^2V_{11}$	$\begin{cases} ^1V_{11} = {}^2V_{11} \\ ^1V_2 = {}^1V_2 \end{cases}$	$= \frac{1}{2} \cdot \frac{2n-1}{2n+1}$	......	2	......	......	......	......	......	......	......	......	$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$						
6	−	$^0V_{13} - {}^2V_{11}$	$^1V_{13}$	$\begin{cases} ^2V_{11} = {}^0V_{11} \\ ^0V_{13} = {}^1V_{13} \end{cases}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$	......	......	......	......	......	......	......	......	......	......	0	......	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$				
7	−	$^1V_3 - {}^1V_1$	$^1V_{10}$	$\begin{cases} ^1V_3 = {}^1V_3 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= n - 1 (= 3)$	1	......	n	......	......	......	......	......	......	$n - 1$							
8	+	$^1V_2 + {}^0V_7$	$^1V_7$	$\begin{cases} ^1V_2 = {}^1V_2 \\ ^0V_7 = {}^1V_7 \end{cases}$	$= 2 + 0 = 2$	......	2	......	......	......	......	2										
9	÷	$^1V_6 \div {}^1V_7$	$^3V_{11}$	$\begin{cases} ^1V_6 = {}^1V_6 \\ ^0V_{11} = {}^3V_{11} \end{cases}$	$= \frac{2n}{2} = A_1$	......	......	......	......	......	$2n$	2	......	......	......	$\frac{2n}{2} = A_1$						
10	×	$^1V_{21} \times {}^3V_{11}$	$^1V_{12}$	$\begin{cases} ^1V_{21} = {}^1V_{21} \\ ^3V_{11} = {}^3V_{11} \end{cases}$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$	......	......	......	......	......	......	......	......	......	......	$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1 A_1$	......	B_1			
11	+	$^1V_{12} + {}^1V_{13}$	$^2V_{13}$	$\begin{cases} ^1V_{12} = {}^0V_{12} \\ ^1V_{13} = {}^2V_{13} \end{cases}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$	......	......	......	......	......	......	......	......	......	......	......	0	$\left\{-\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}\right\}$				
12	−	$^1V_{10} - {}^1V_1$	$^2V_{10}$	$\begin{cases} ^1V_{10} = {}^2V_{10} \\ ^1V_1 = {}^1V_1 \end{cases}$	$= n - 2 (= 2)$	1	......	......	......	......	......	......	......	......	$n - 2$							
13	−	$^1V_6 - {}^1V_1$	$^2V_6$	$\begin{cases} ^1V_6 = {}^2V_6 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= 2n - 1$	1	......	......	......	......	$2n - 1$											
14	+	$^1V_1 + {}^1V_7$	$^2V_7$	$\begin{cases} ^1V_1 = {}^1V_1 \\ ^1V_7 = {}^2V_7 \end{cases}$	$= 2 + 1 = 3$	1	......	......	......	......	......	3										
15	÷	$^2V_6 \div {}^2V_7$	$^1V_8$	$\begin{cases} ^2V_6 = {}^2V_6 \\ ^2V_7 = {}^2V_7 \end{cases}$	$= \frac{2n-1}{3}$	......	......	......	......	......	$2n - 1$	3	$\frac{2n-1}{3}$									
16	×	$^1V_8 \times {}^3V_{11}$	$^4V_{11}$	$\begin{cases} ^1V_8 = {}^0V_8 \\ ^3V_{11} = {}^4V_{11} \end{cases}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3}$	......	......	......	......	......	......	......	0	......	......	$\frac{2n}{2} \cdot \frac{2n-1}{3}$						
17	−	$^2V_6 - {}^1V_1$	$^3V_6$	$\begin{cases} ^2V_6 = {}^3V_6 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= 2n - 2$	1	......	......	......	......	$2n - 2$											
18	+	$^1V_1 + {}^2V_7$	$^3V_7$	$\begin{cases} ^2V_7 = {}^3V_7 \\ ^1V_1 = {}^1V_1 \end{cases}$	$= 3 + 1 = 4$	1	......	......	......	......	......	4										
19	÷	$^3V_6 \div {}^3V_7$	$^1V_9$	$\begin{cases} ^3V_6 = {}^3V_6 \\ ^3V_7 = {}^3V_7 \end{cases}$	$= \frac{2n-2}{4}$	......	......	......	......	......	$2n - 2$	4	......	$\frac{2n-2}{4}$	......							
20	×	$^1V_9 \times {}^4V_{11}$	$^5V_{11}$	$\begin{cases} ^1V_9 = {}^0V_9 \\ ^4V_{11} = {}^5V_{11} \end{cases}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$	......	......	......	......	......	......	......	......	0		$\left\{\frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4}\right\} = A_3$						
21	×	$^1V_{22} \times {}^5V_{11}$	$^0V_{12}$	$\begin{cases} ^1V_{22} = {}^1V_{22} \\ ^0V_{12} = {}^2V_{12} \end{cases}$	$= B_3 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = B_3 A_3$	......	......	......	......	......	......	......	......	......	......	0	$B_3 A_3$	......	......	B_3		
22	+	$^2V_{12} + {}^2V_{13}$	$^3V_{13}$	$\begin{cases} ^2V_{12} = {}^0V_{12} \\ ^2V_{13} = {}^3V_{13} \end{cases}$	$= A_0 + B_1 A_1 + B_3 A_3$	......	......	......	......	......	......	......	......	......	......	......	0	$\{A_0 + B_1 A_1 + B_3 A_3\}$				
23	−	$^2V_{10} - {}^1V_1$	$^3V_{10}$	$\begin{cases} ^2V_{10} = {}^3V_{10} \\ ^1V_1 = {}^1V_1 \end{cases}$	$= n - 3 (= 1)$	1	......	......	......	......	......	......	......	......	$n - 3$							
Here follows a repetition of Operations thirteen to twenty-three																						
24	+	$^4V_{13} + {}^0V_{24}$	$^1V_{24}$	$\begin{cases} ^4V_{13} = {}^0V_{13} \\ ^0V_{24} = {}^1V_{24} \end{cases}$	$= B_7$	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	B_7
25	+	$^1V_1 + {}^1V_3$	$^1V_3$	$\begin{cases} ^1V_1 = {}^1V_1 \\ ^1V_3 = {}^1V_3 \\ ^5V_6 = {}^0V_6 \\ ^5V_7 = {}^0V_7 \end{cases}$	$= n + 1 = 4 + 1 = 5$ by a Variable-card. by a Variable-card.	1	......	$n + 1$	......	......	0	0										

Quotes from the Ada Lovelace notes on
"Sketch of the Analytical Engine Invented by Charles Babbage", 1843

"We may say most aptly, that the Analytical Engine *weaves algebraical patterns* just as the Jacquard-loom weaves flowers and leaves."



"Again, it might act upon other things besides *number*, were objects found whose mutual fundamental relations could be expressed by those of the abstract science of operations, and which should be also susceptible of adaptations to the action of the operating notation and mechanism of the engine. Supposing, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent."

Quotes from the Ada Lovelace notes on

“Sketch of the Analytical Engine Invented by Charles Babbage”, 1843

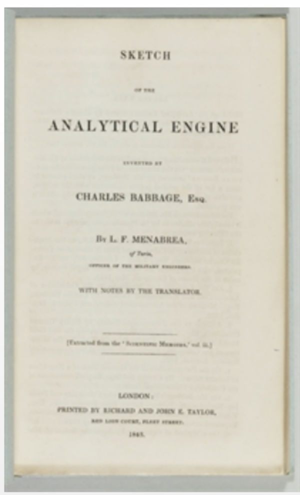
“Many persons who are not conversant with mathematical studies, imagine that because the business of the engine is to give its results in *numerical notation*, the *nature of its processes* must consequently be *arithmetical* and *numerical*, rather than *algebraical* and *analytical*. This is an error. The engine can arrange and combine its numerical quantities exactly as if they were *letters* or any other *general* symbols; and in fact it might bring out its results in algebraical *notation*, were provisions made accordingly.”

“But it would be a mistake to suppose that because its *results* are given in the *notation* of a more restricted science, its *processes* are therefore restricted to those of that science. The object of the engine is in fact to give the *utmost practical efficiency* to the resources of *numerical interpretations* of the higher science of analysis, while it uses the processes and combinations of this latter.”

LOT 21 / SALE 2013

EMAIL | PRINT

[BABBAGE]. -- MENABREA, Luigi Federico (1809-1896). *Sketch of the Analytical Engine invented by Charles Babbage... with notes by the translator*. Offprint from: *Scientific Memoirs*. Translated by Augusta Ada King, Countess of Lovelace (1809-1896). Volume 3. London: Richard and John E. Taylor, 1843.



ENLARGE & ZOOM

Price Realized (Set Currency)
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Price includes buyer's premium

Estimate
$10,000 - $15,000

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LOTS IN THIS SALE

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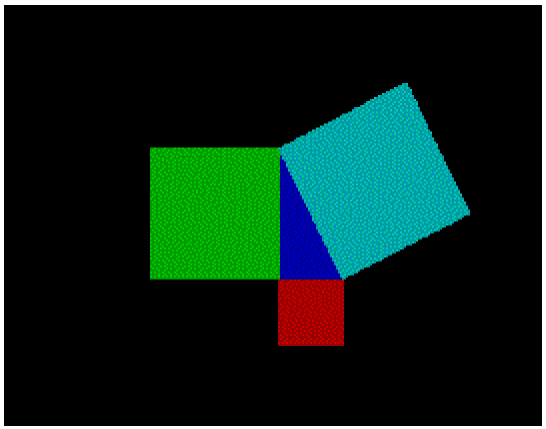
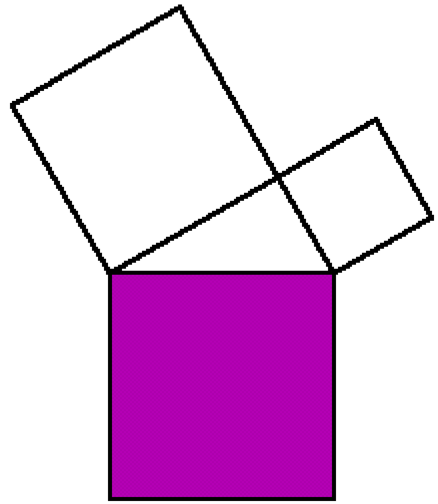
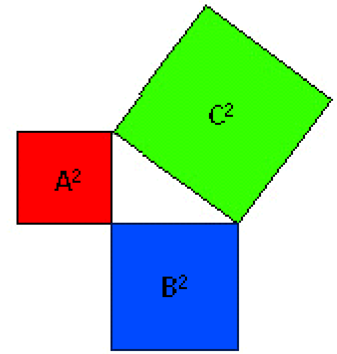
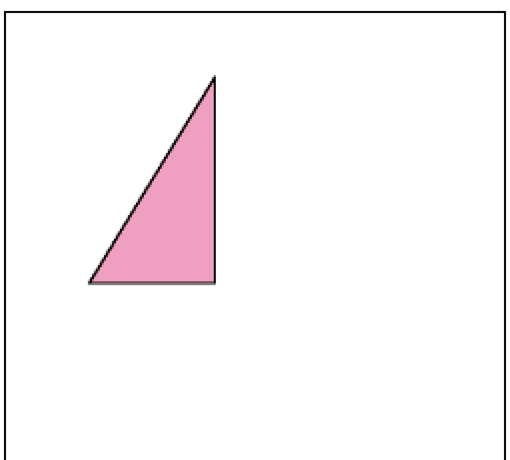
LOT #21 [BABBAGE]. -- MENABREA, Luigi Federico...
LOT #22 BABBAGE, Charles. *Passages from the...*
LOT #23 BABBAGE, Charles. *The Ninth...*
LOT #24 [BALLISTICS]. *Une merveille du génie...*
LOT #25 [BALLISTICS]. BRITISH INFORMATION...
LOT #26 [BALLISTICS]. *The United States...*
LOT #27 BAYER, Johann (1572-1625)....
LOT #28 BEAUMONT, William (1785-1853)....
LOT #30

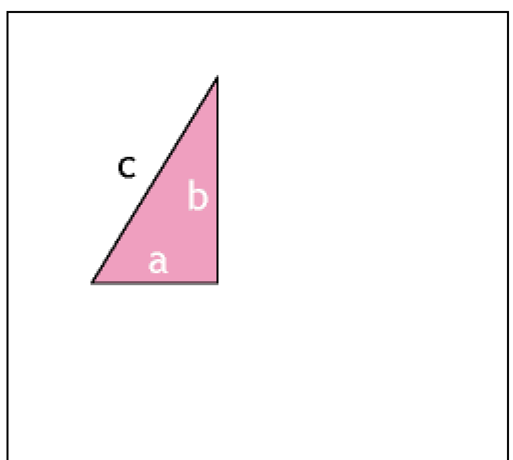
Problem: Give as many proofs as you can for the Pythagorean Theorem. i.e., $a^2 + b^2 = c^2$ holds for any right triangle with sides a & b and hypotenuse c.



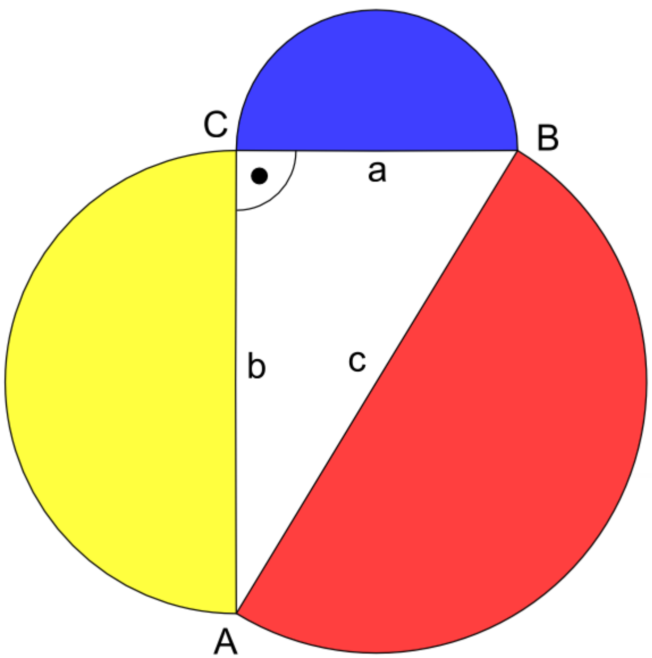


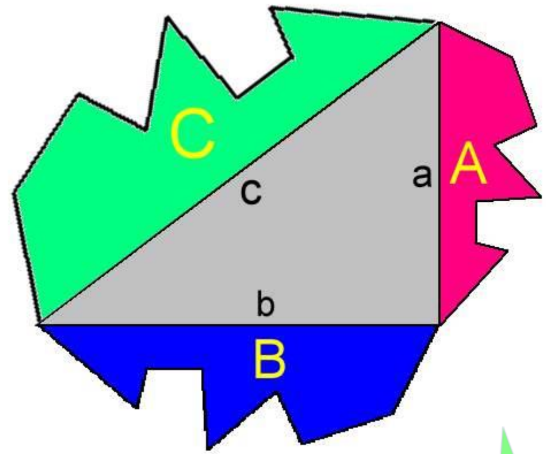


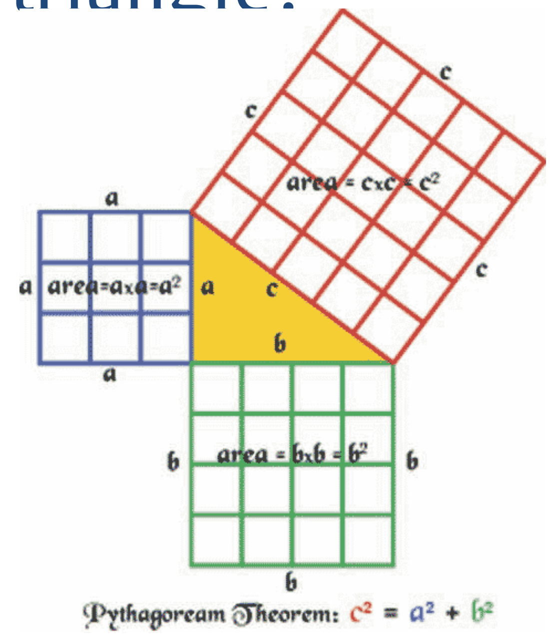


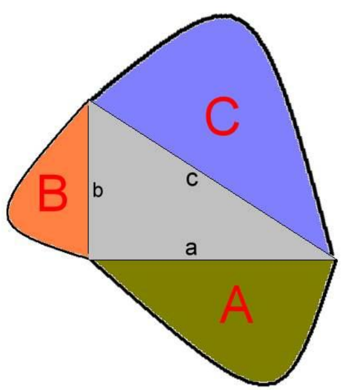


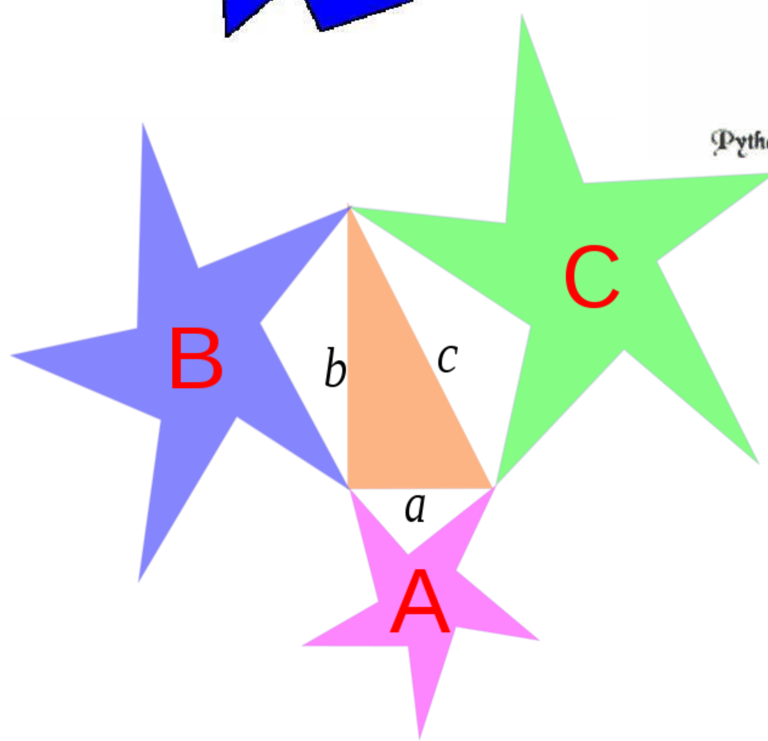
Problem: Does the Pythagorean theorem generalize to arbitrary figures on the sides of a right triangle?









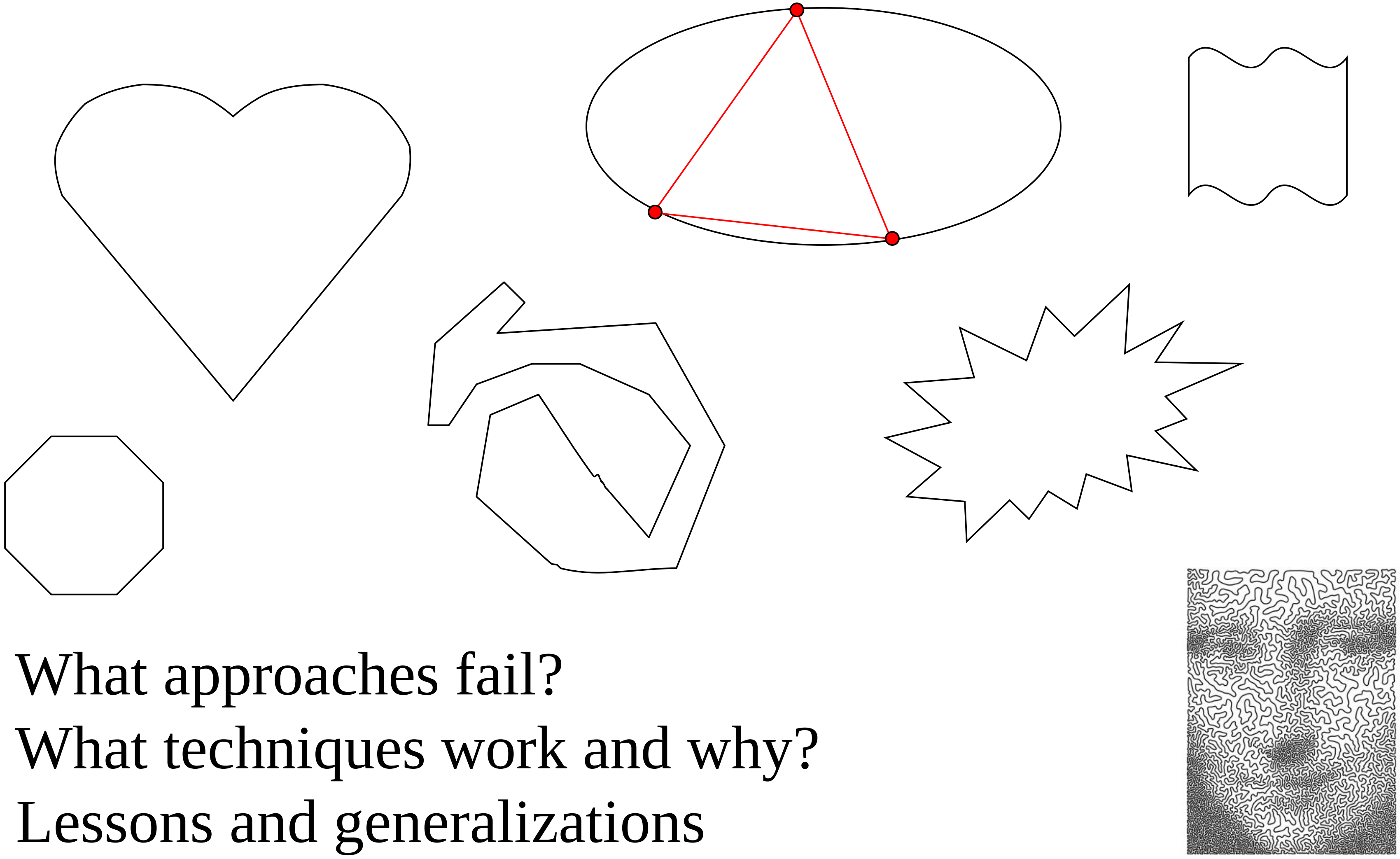


Problem: compute 111111111^2 in your head.

Problem: What is the approximate value of:

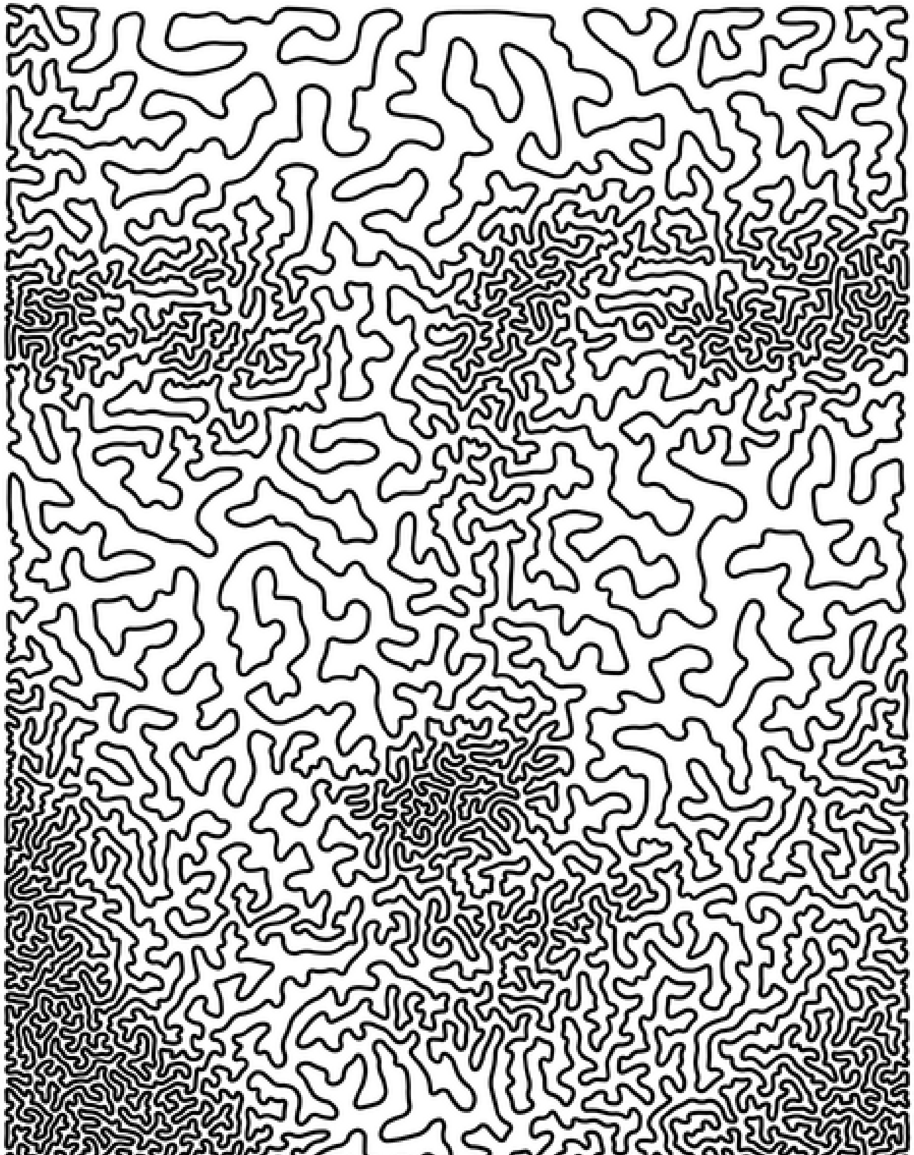
$$(1+9\text{^}(-(4\text{^}(7*6))))\text{^}(3\text{^}(2\text{^}85)) \approx ?$$

Problem: Does every closed simple curve contain the vertices of an equilateral triangle?

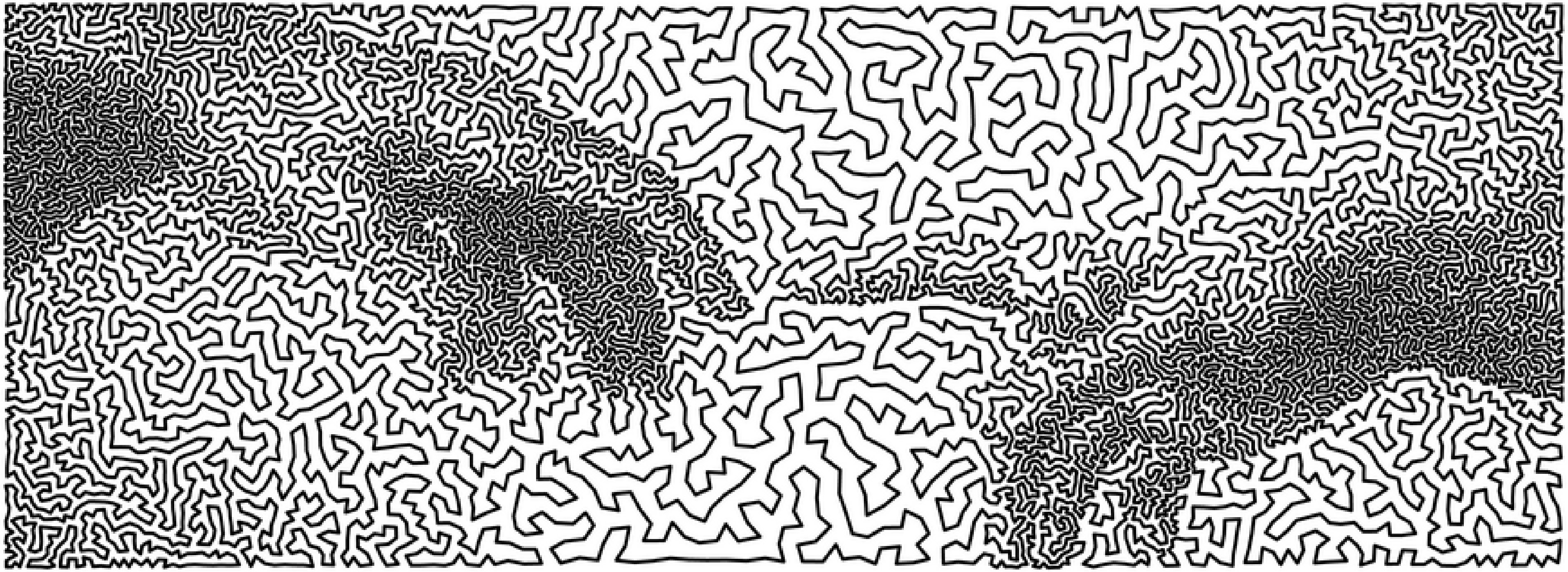


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

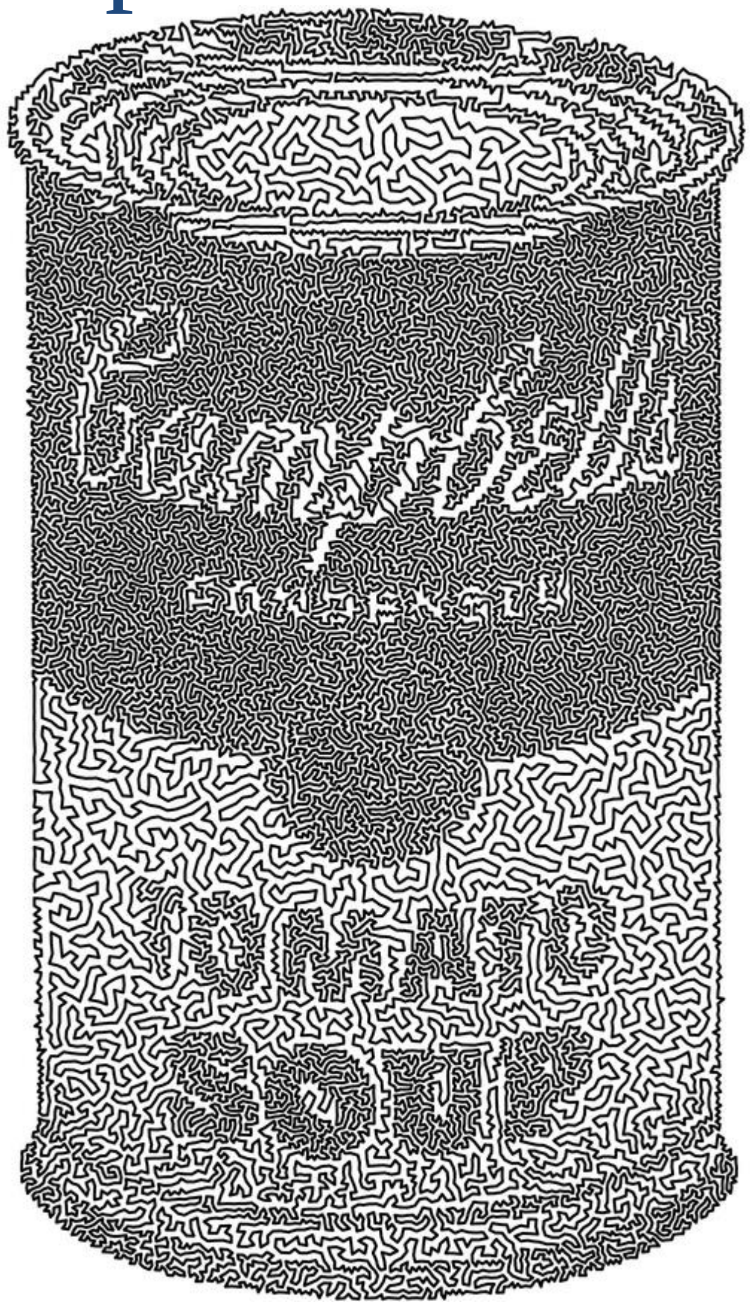
A Simple Closed Curve!



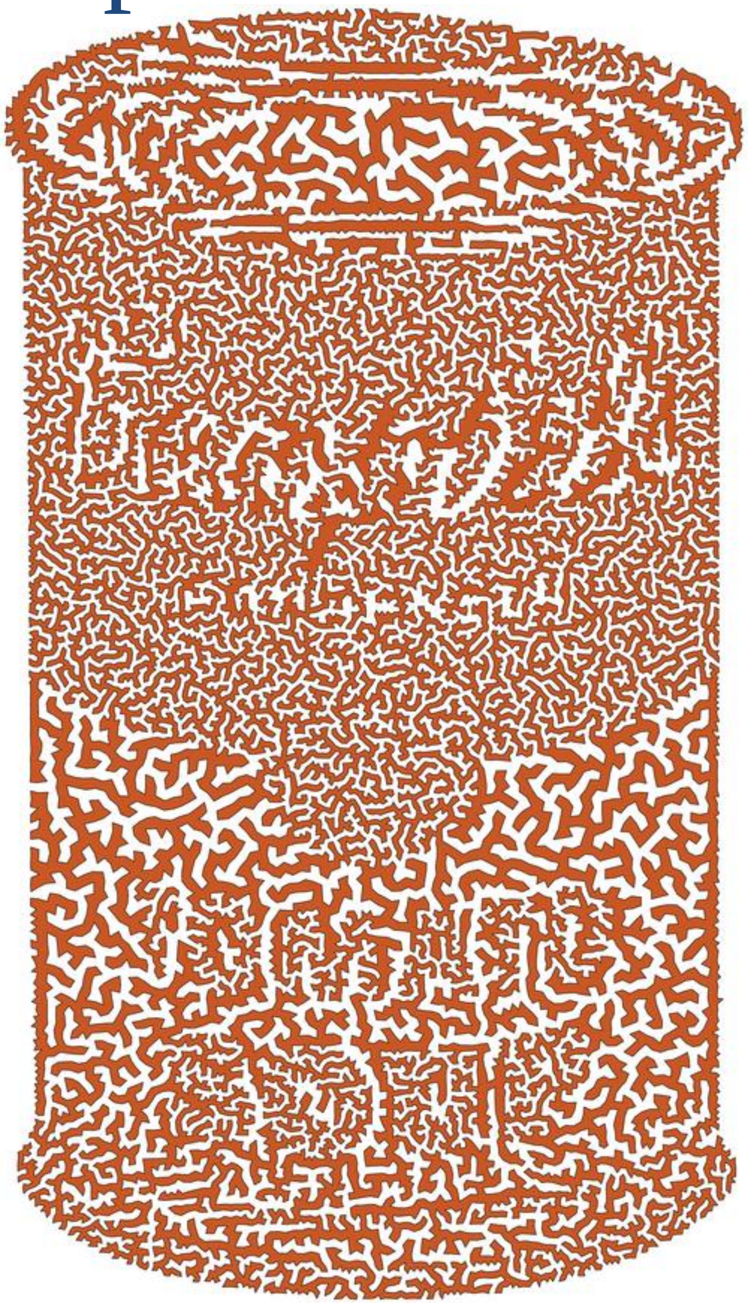
A Simple Closed Curve!



A Simple Closed Curve!

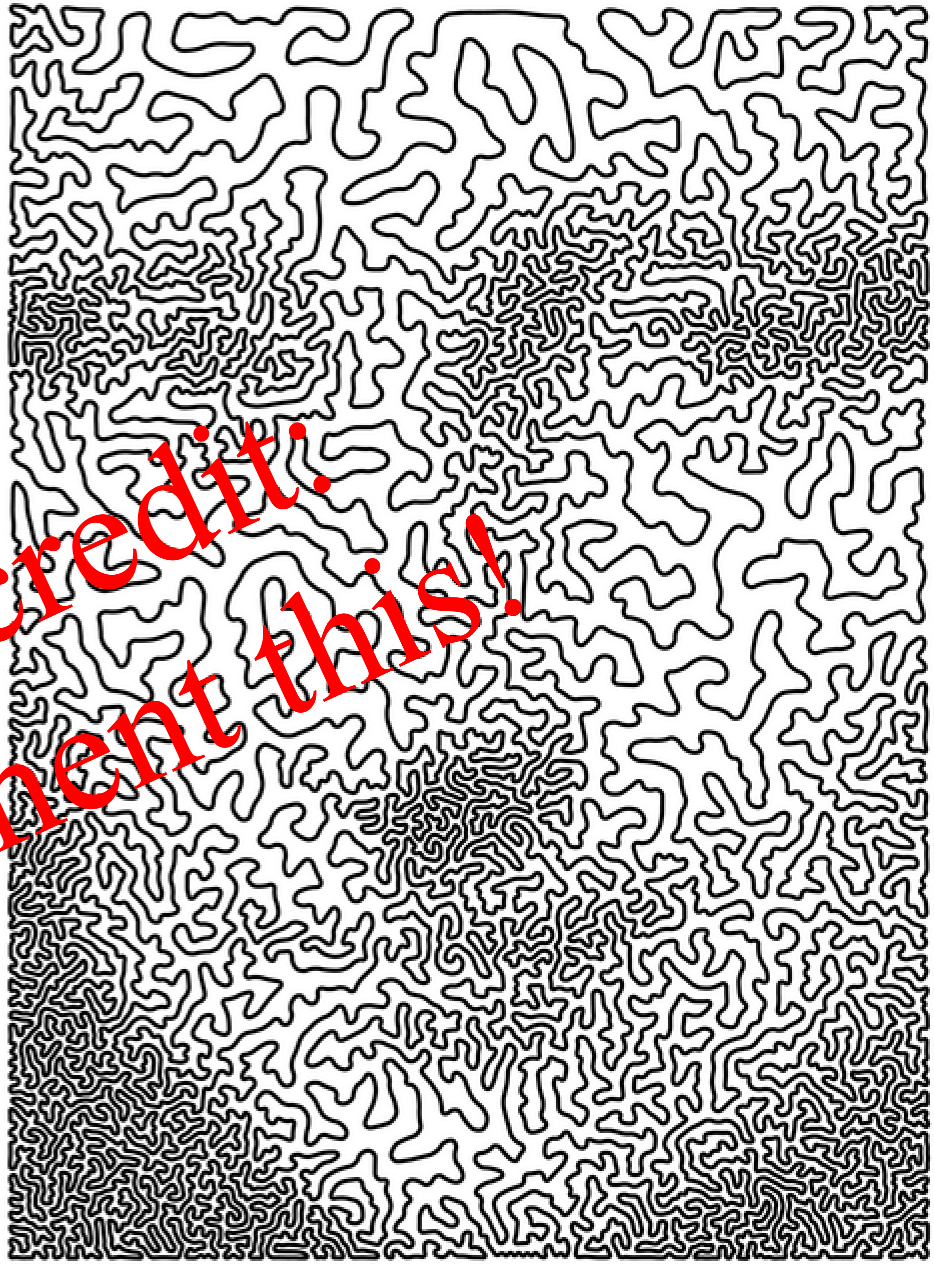


A Simple Closed Curve!



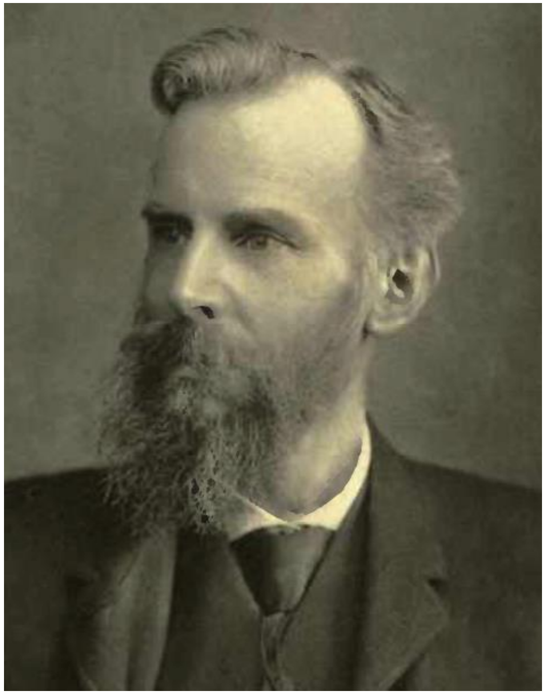
Traveling Salesperson Art

- Compute TSP Tour
- Optimal is NP-complete
  So use heuristics
- Convert image to B&W
- Sample image density
  to obtain a pointset
- Run TSP heuristics
- Can use minimum spanning
  trees (easy to compute)
- Can also use minimum
  matchings (easy to compute)
- What about colors?



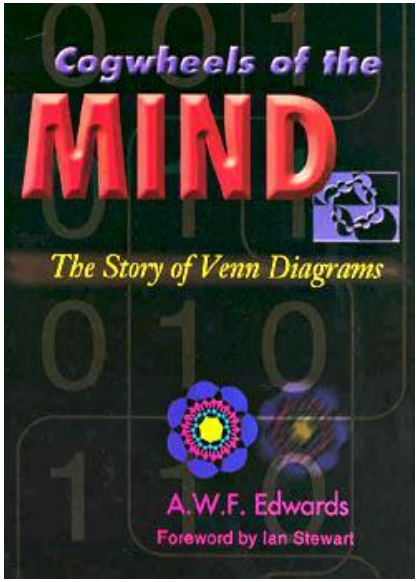
Extra credit:
implement this!

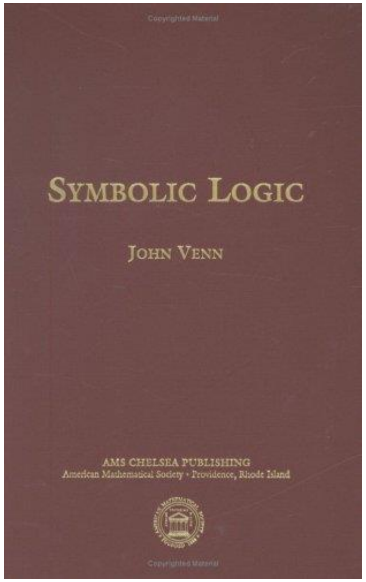
Historical Perspectives

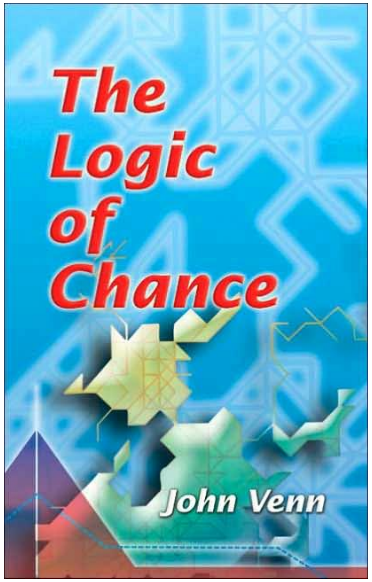


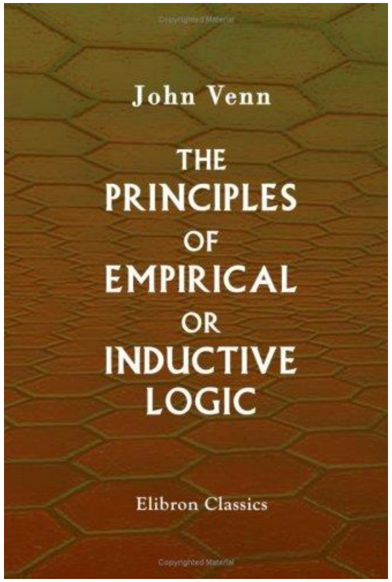
John Venn (1834-1923)

- Logician and philosopher
- Worked in logic, probability, set theory
- Introduced the "Venn diagram" (1880)
  - Very widely used, many applications
  - Ties together fundamental concepts from logic, geometry, combinatorics, knot theory

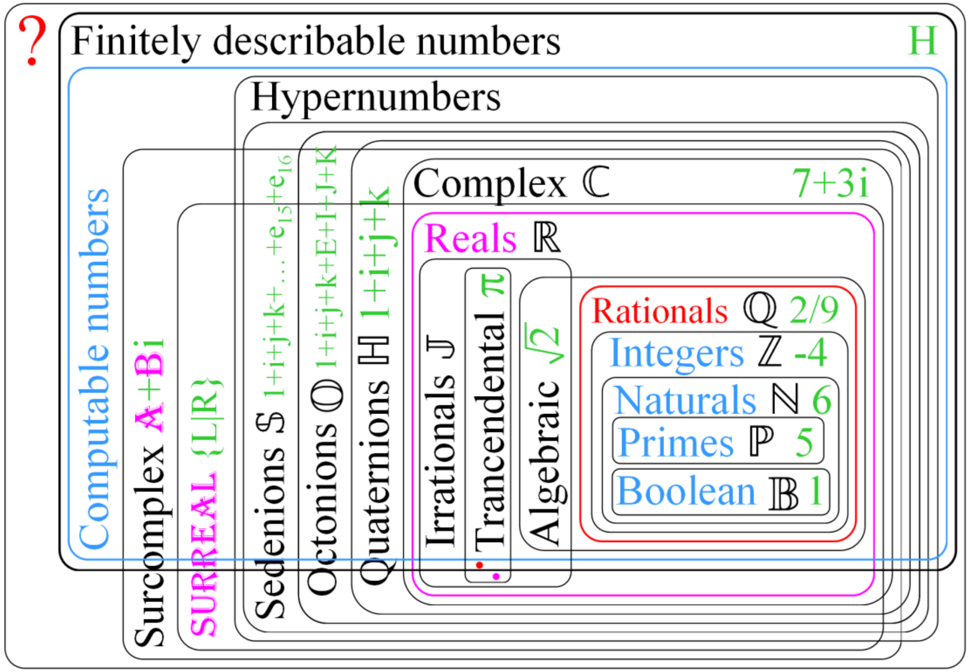




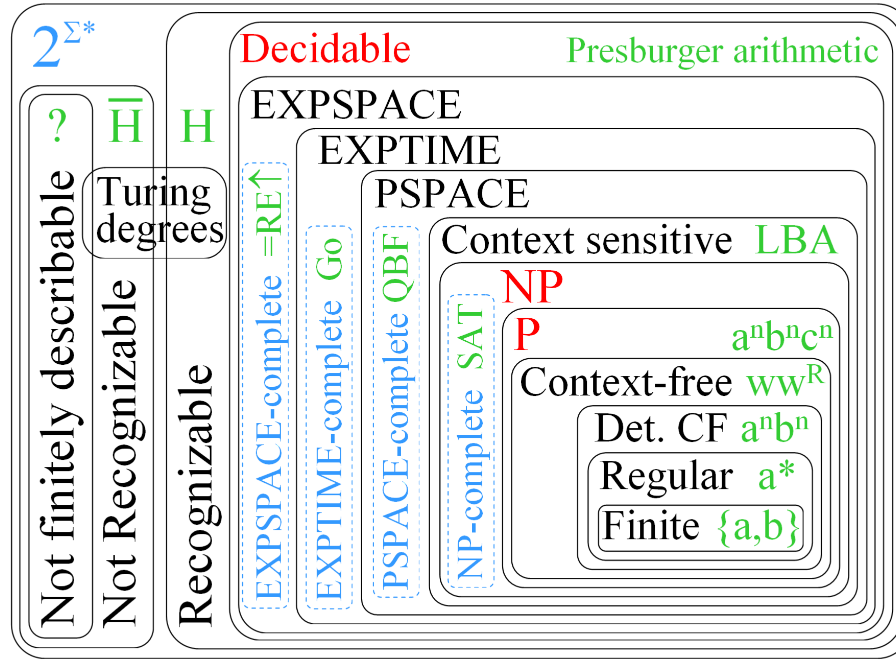


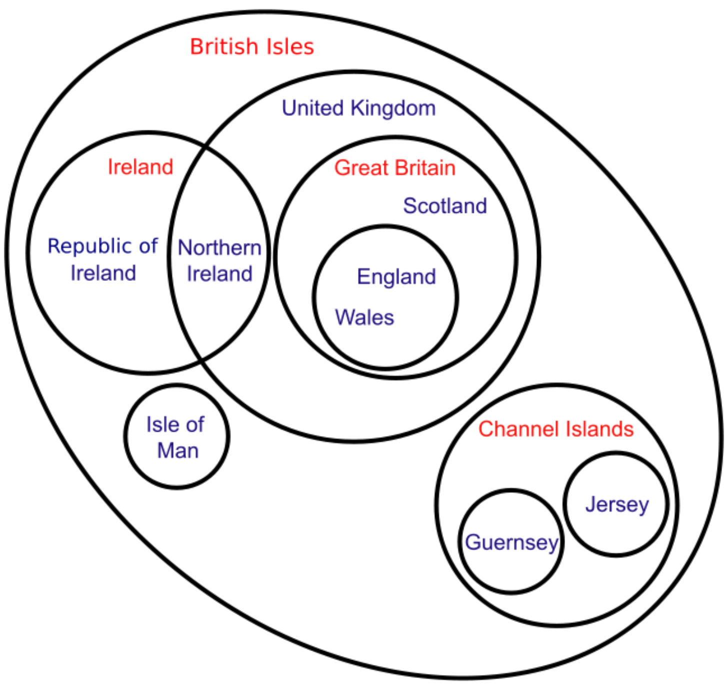


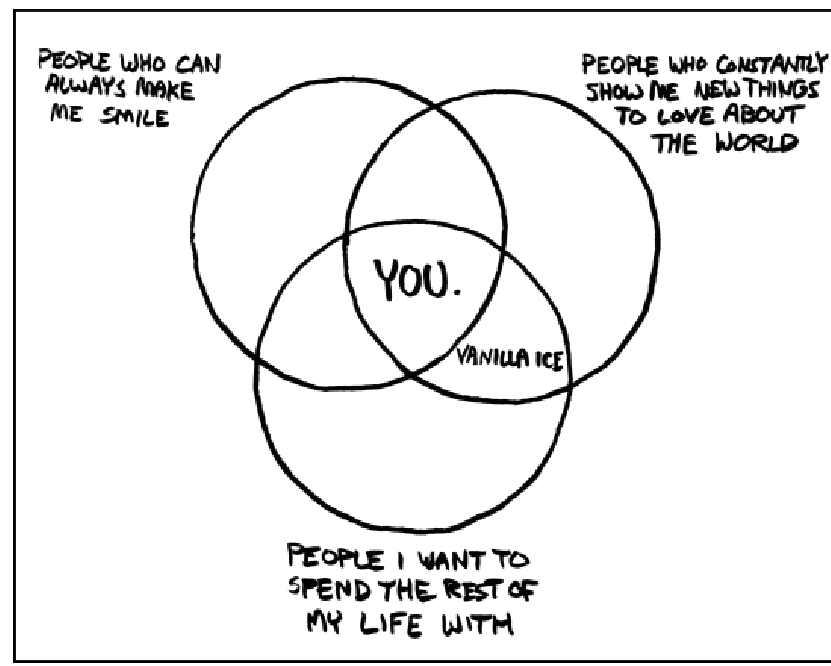
Generalized Numbers

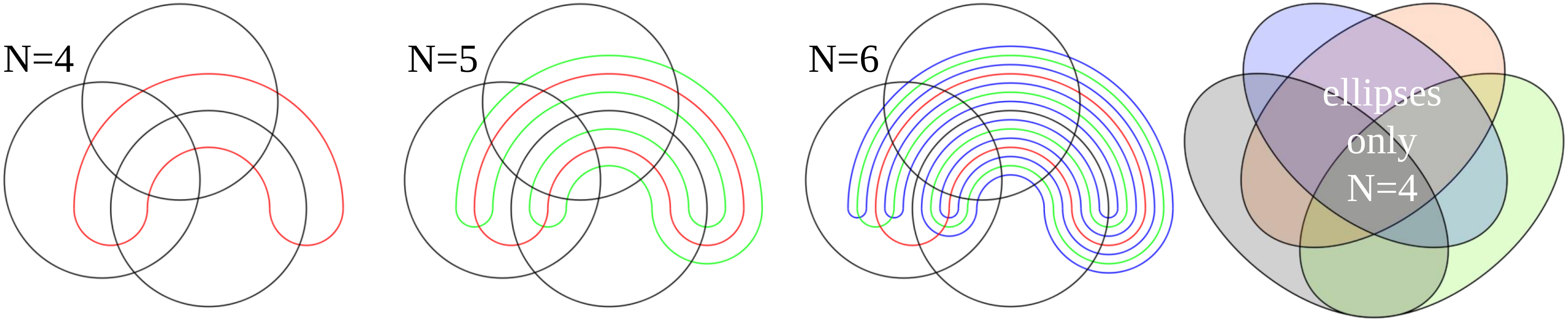


The Extended Chomsky Hierarchy

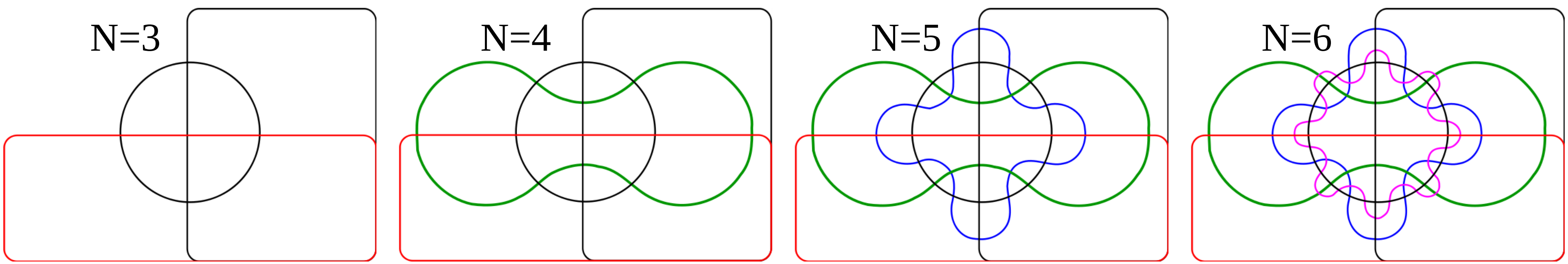


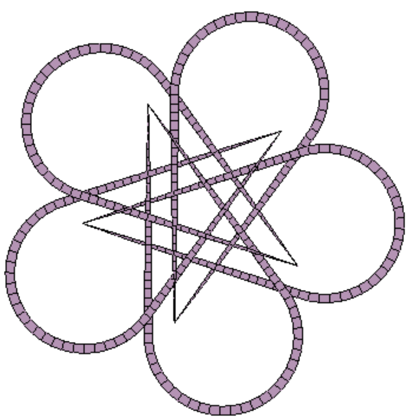


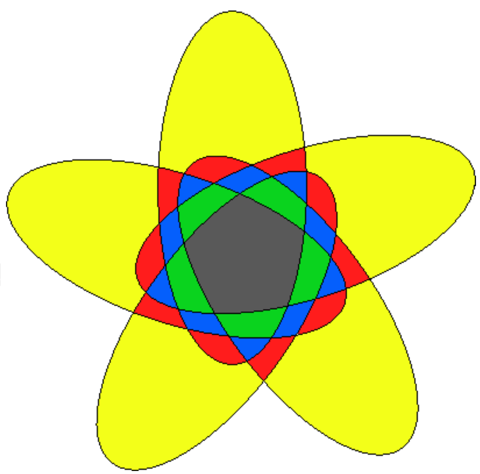




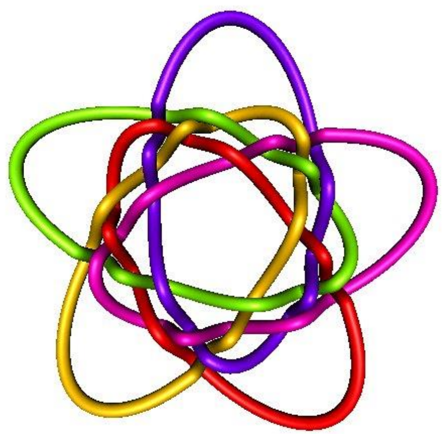
Generalized Venn diagrams [John Venn, 1880]



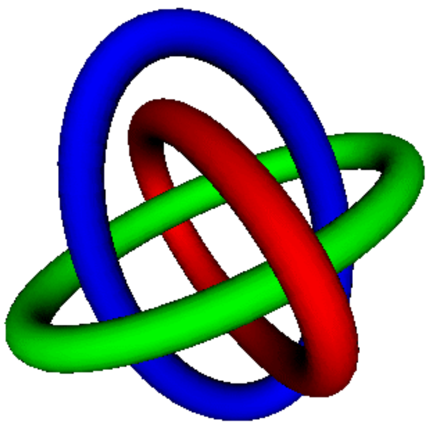




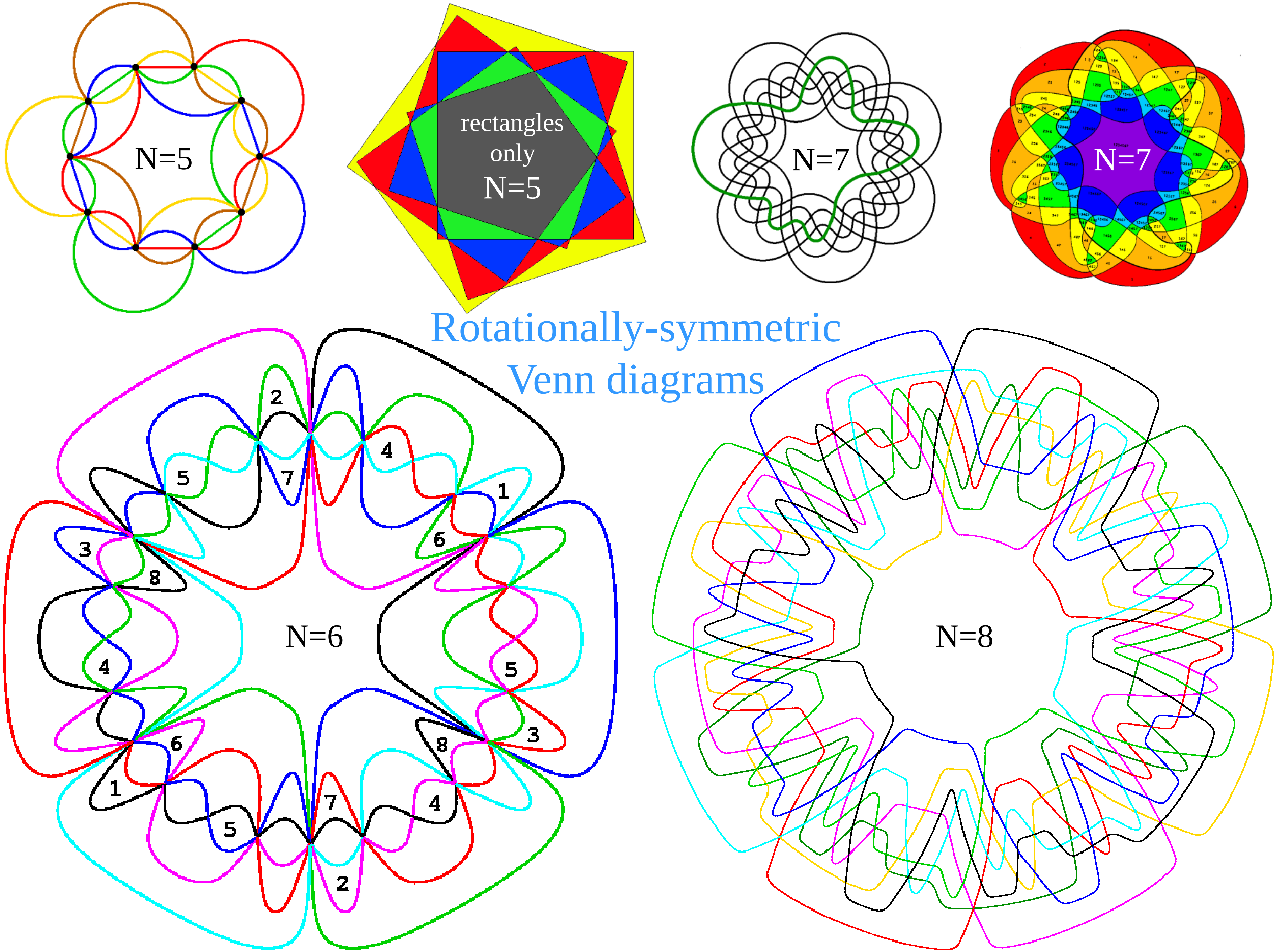
Ellipses only N=5



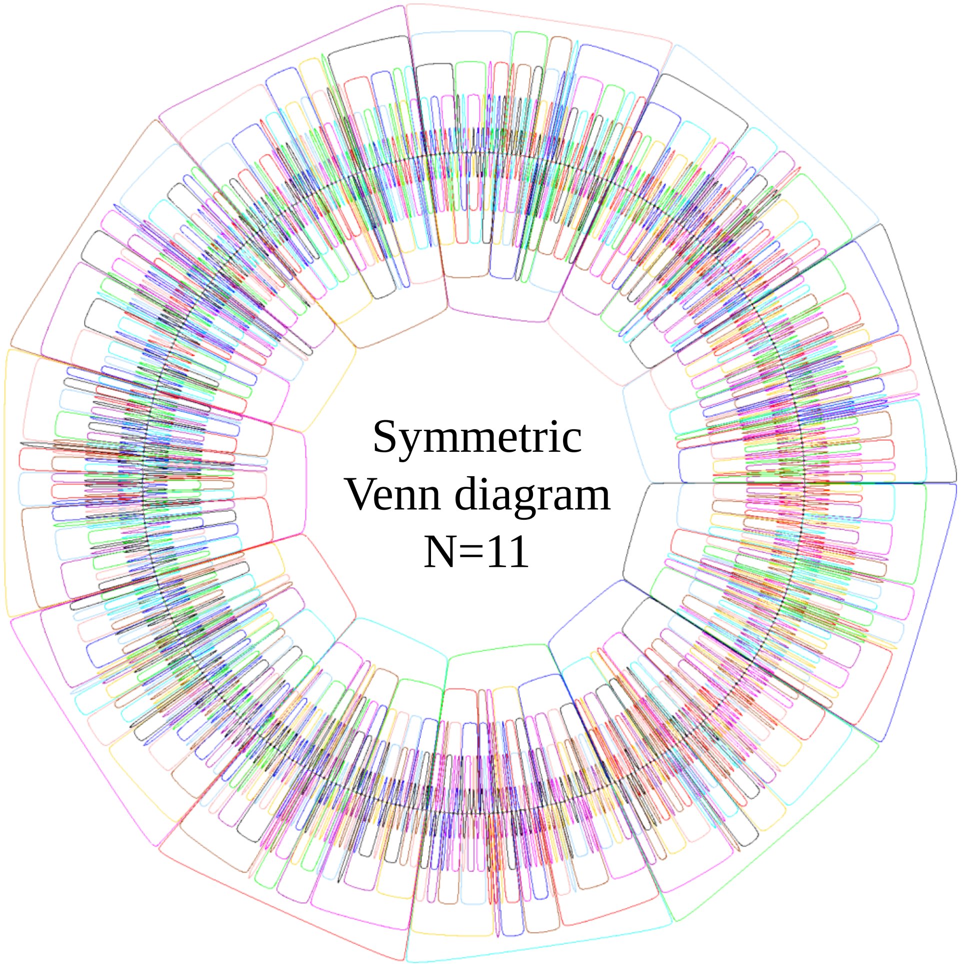
Borromean rings analogue N=5

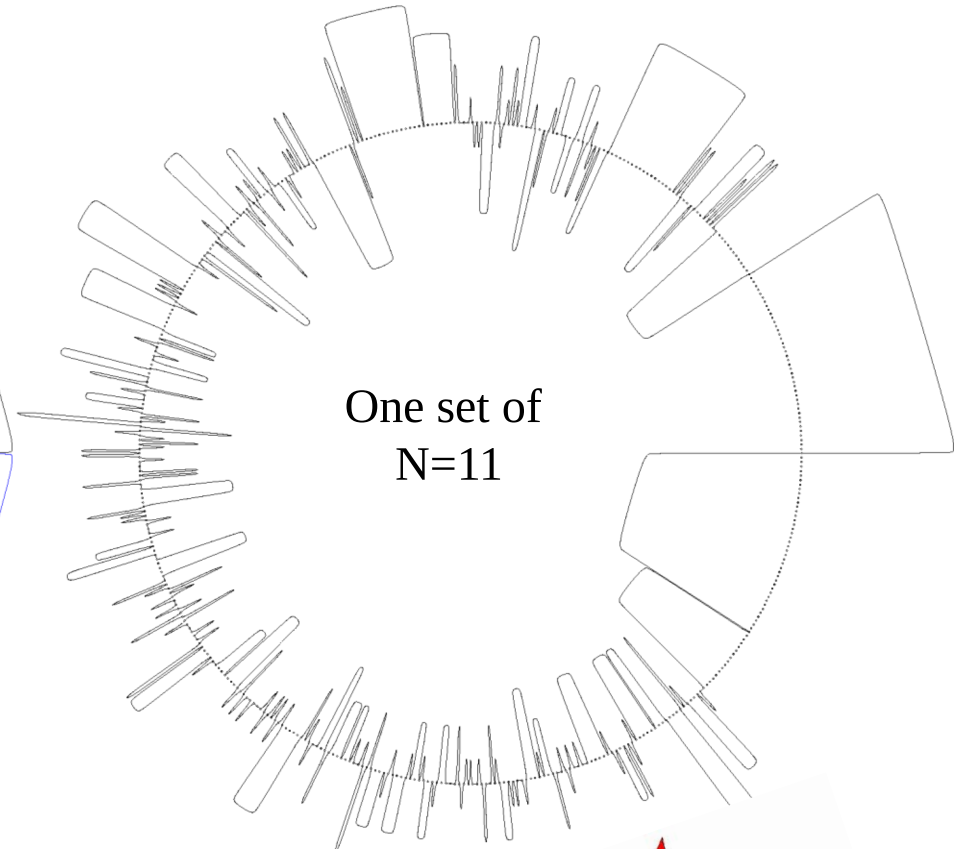


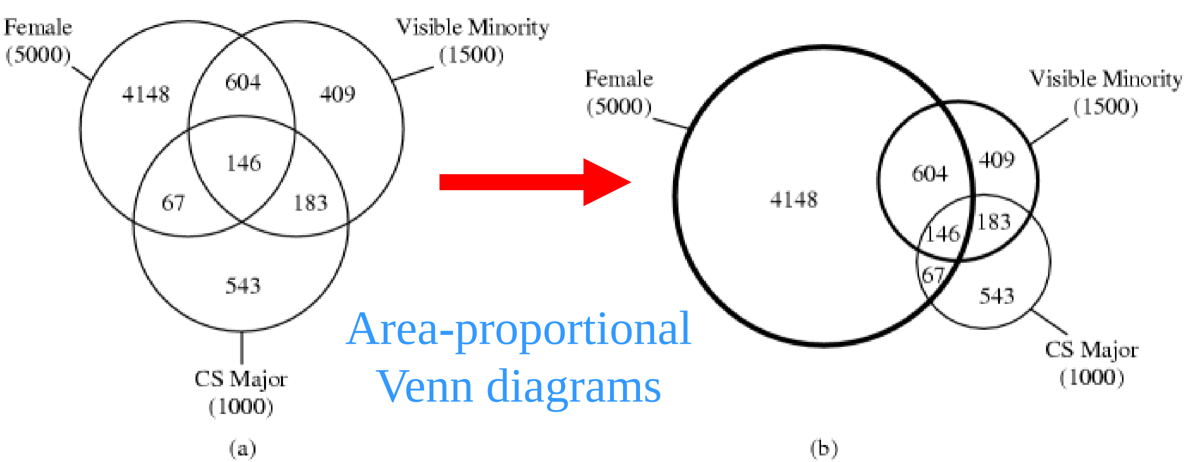
Borromean rings N=3



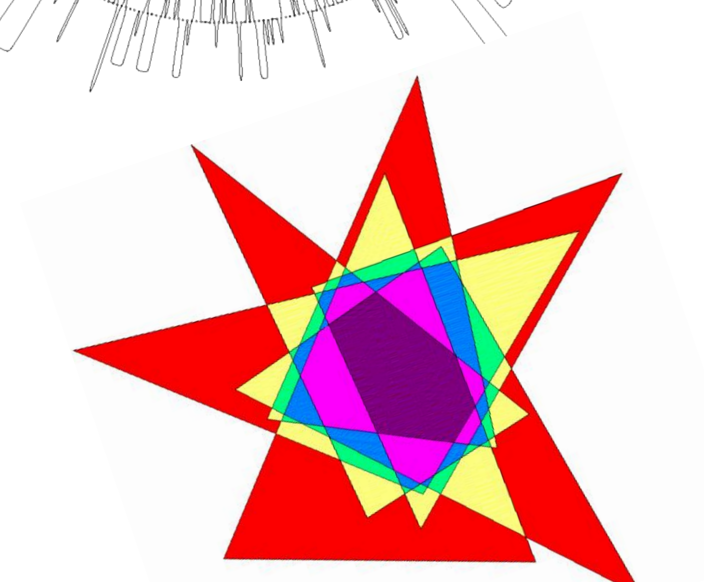
Rotationally-symmetric Venn diagrams
N=5
rectangles only
N=5
N=7
N=7
N=6
N=8



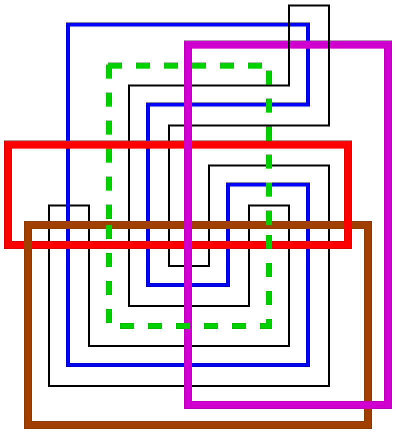




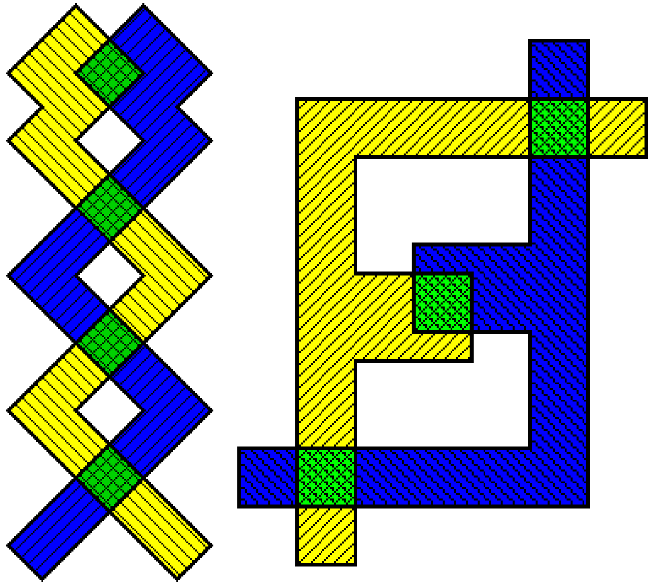
Area-proportional
Venn diagrams



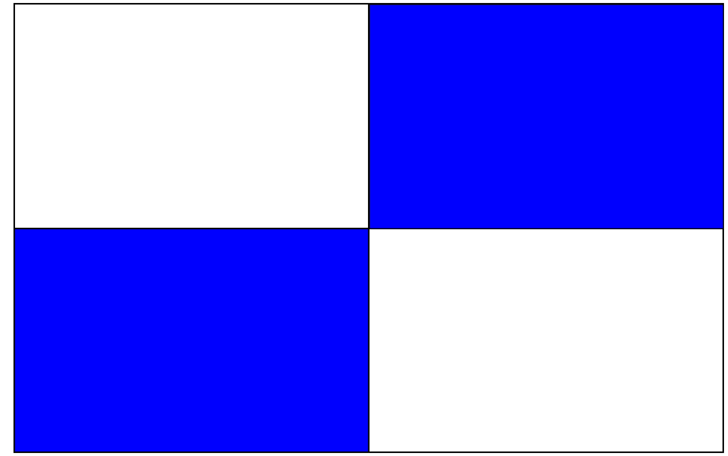
Triangles only Venn diagram (N=6)

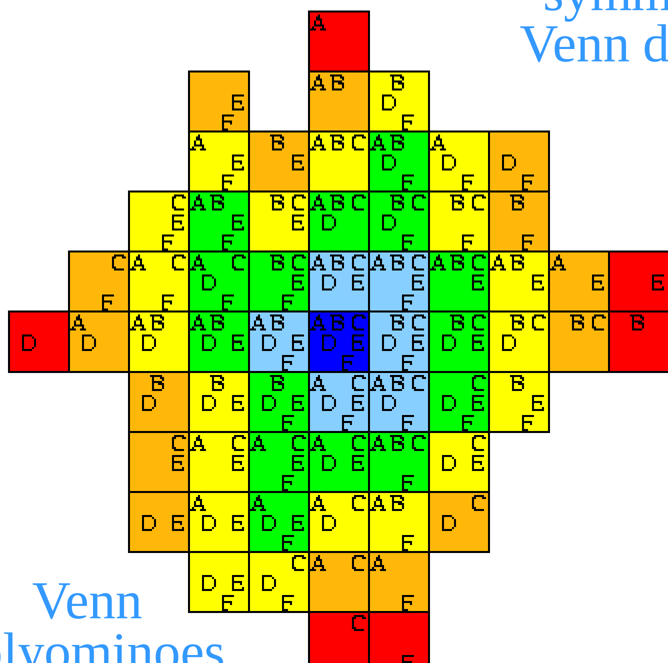


exposed Venn diagrams n=5

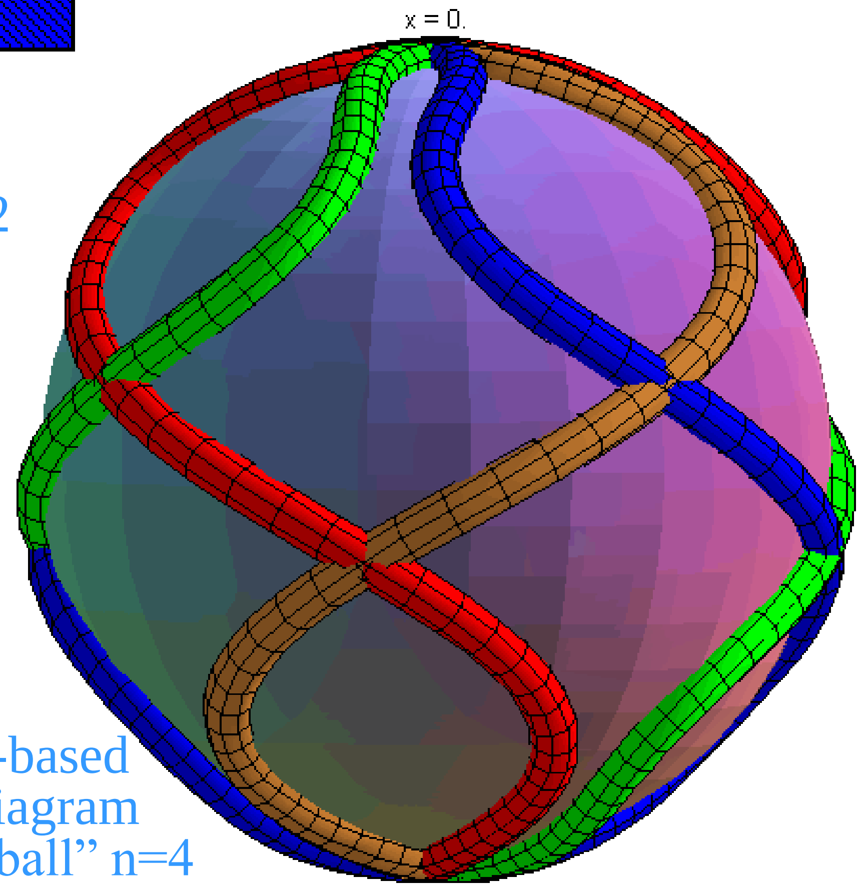


symmetric k-fold Venn diagrams n=2





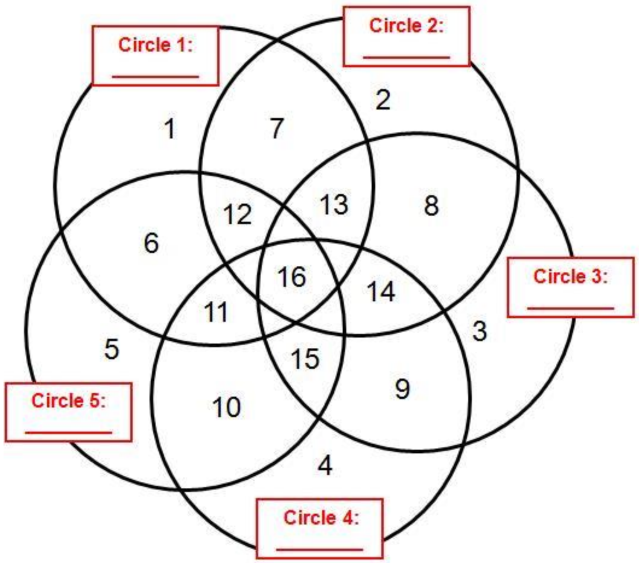
Venn polyominoes



sphere-based Venn diagram “Vennice ball” n=4

http://www.combinatorics.org/Surveys/ds5/VennEJC.html

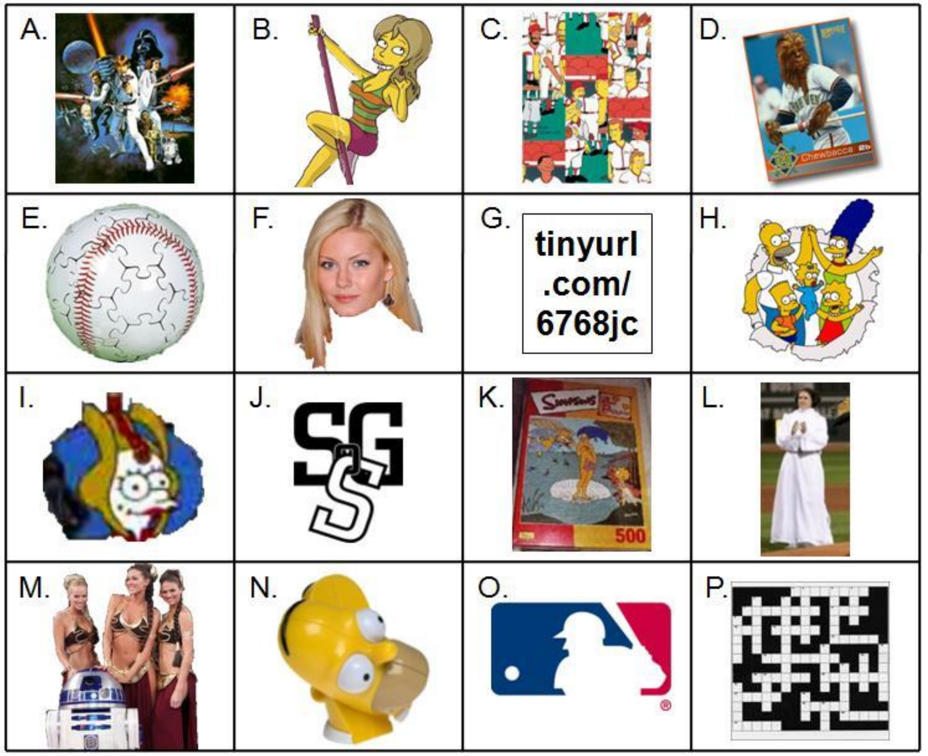
Venn diagram puzzles:



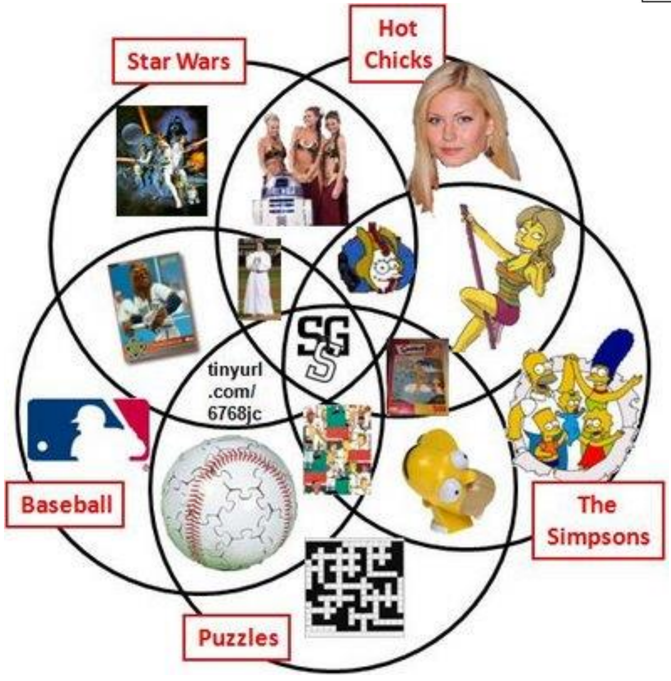
Answer Panel:

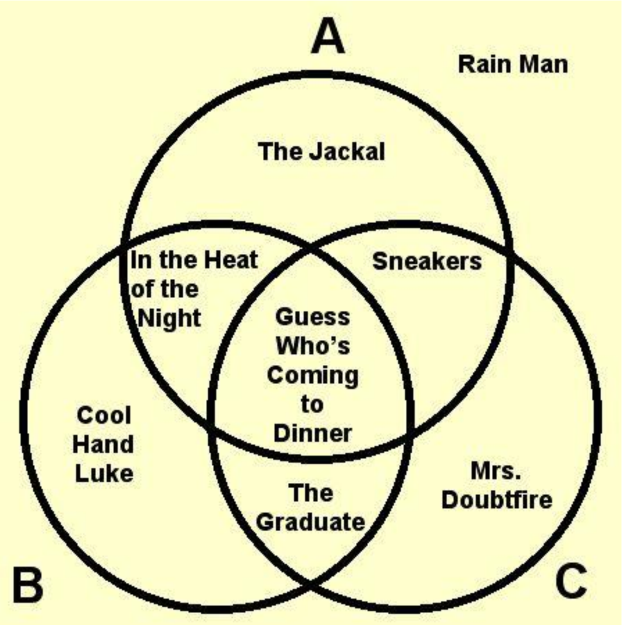
1. A
2. ?
3. ?
4. ?
5. ?
6. ?
7. ?
8. ?
9. ?
10. ?
11. ?
12. ?
13. ?
14. ?
15. ?
16. ?

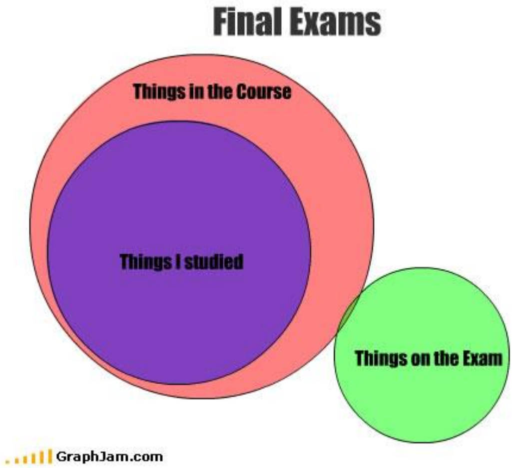
Circle 1: ?
Circle 2: ?
Circle 3: ?
Circle 4: ?
Circle 5: ?

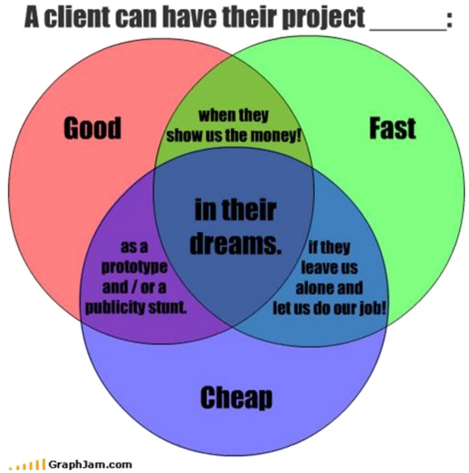


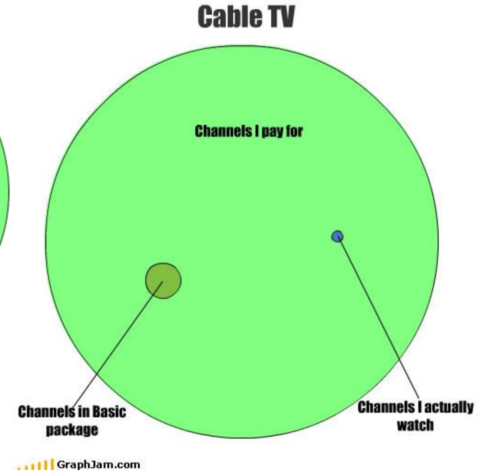
Puzzle solution:

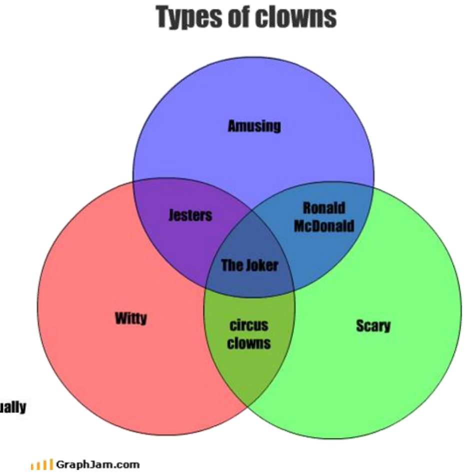


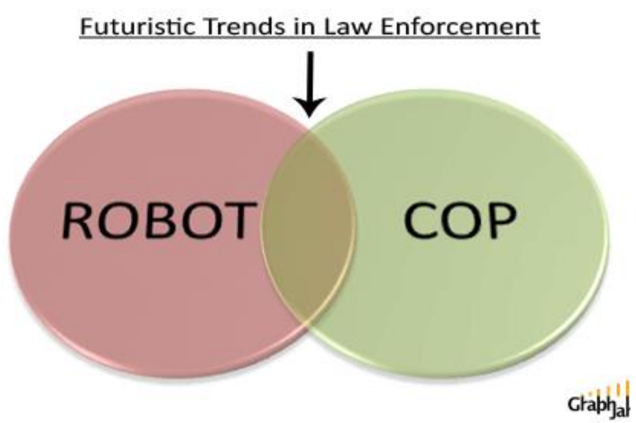


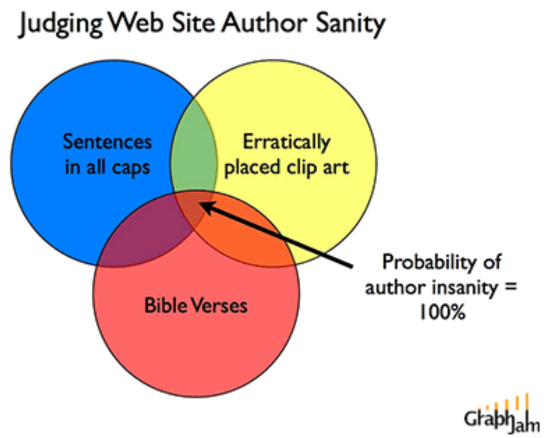

Final Exams
Things in the Course
Things I studied
Things on the Exam
GraphJam.com

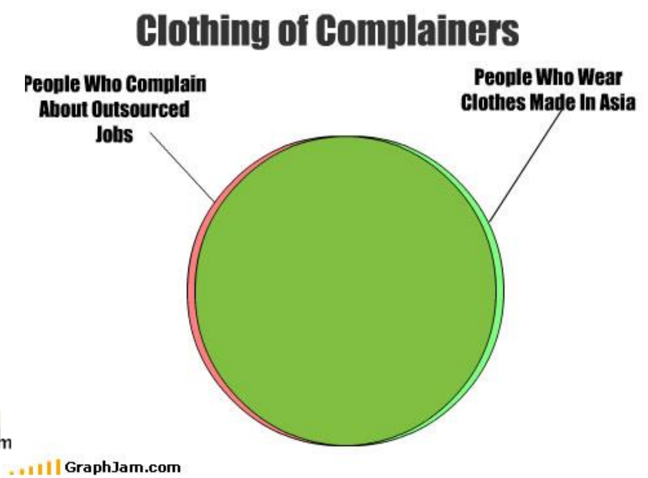

A client can have their project ______:
Good
Fast
Cheap
when they show us the money!
in their dreams.
as a prototype and / or a publicity stunt.
if they leave us alone and let us do our job!
GraphJam.com

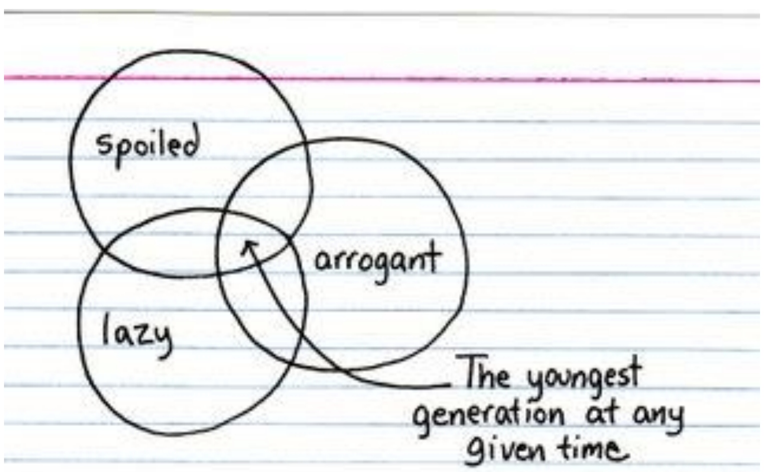

Cable TV
Channels I pay for
Channels in Basic package
Channels I actually watch
GraphJam.com


Types of clowns
Amusing
Jesters
Ronald McDonald
The Joker
Witty
circus clowns
Scary
GraphJam.com

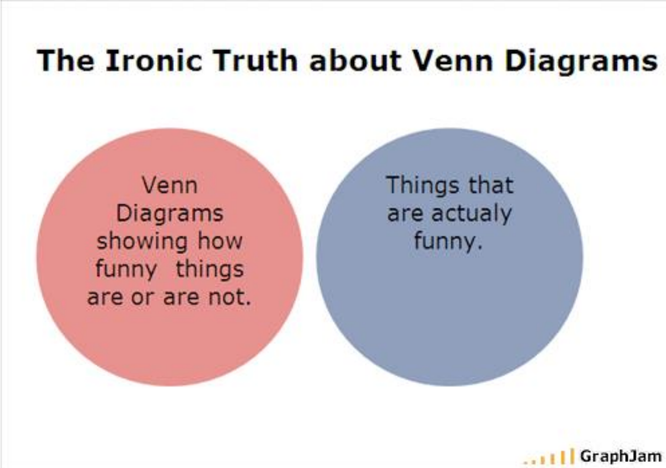

Futuristic Trends in Law Enforcement
ROBOT
COP
GraphJam

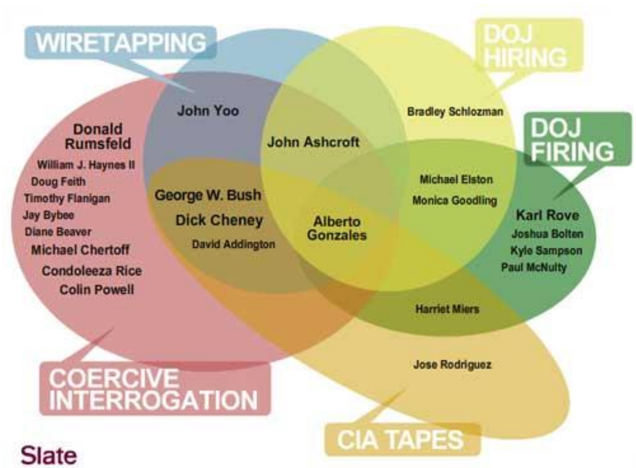

Judging Web Site Author Sanity
Sentences in all caps
Erratically placed clip art
Bible Verses
Probability of author insanity = 100%
GraphJam

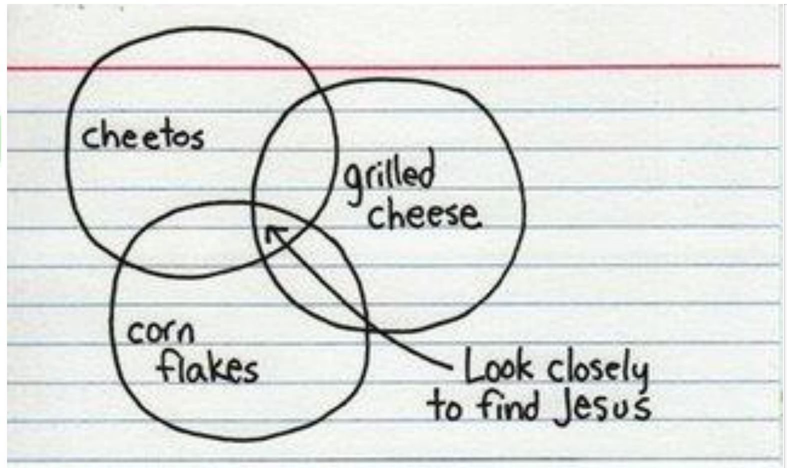

Clothing of Complainers
People Who Complain About Outsourced Jobs
People Who Wear Clothes Made In Asia
GraphJam.com

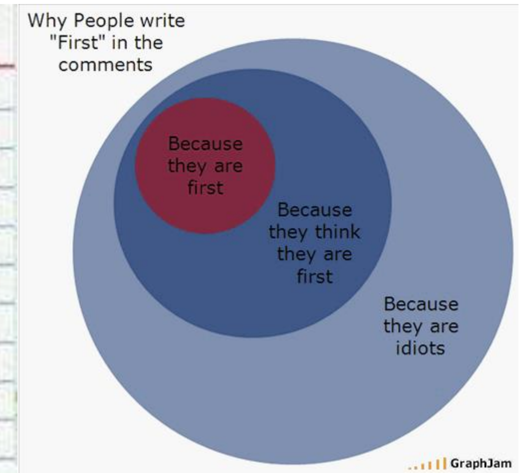

spoiled
arrogant
lazy
The youngest generation at any given time


The Marx Brothers
Funny Accent
Karl
Chico
Facial Hair
Musician
Groucho
Harpo
Chases Girls
GraphJam.com

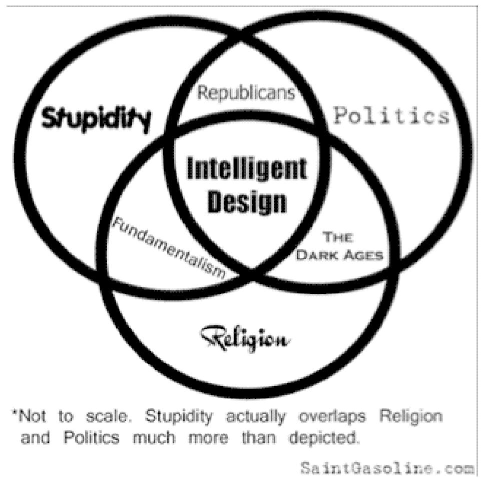

The Ironic Truth about Venn Diagrams
Venn Diagrams showing how funny things are or are not.
Things that are actualy funny.
GraphJam

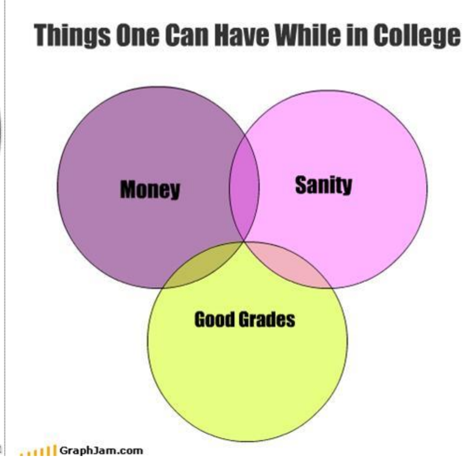

WIRETAPPING
DOJ HIRING
DOJ FIRING
John Yoo
Donald Rumsfeld
William J. Haynes II
Doug Feith
Timothy Flanigan
Jay Bybee
Diane Beaver
Michael Chertoff
Condoleeza Rice
Colin Powell
John Ashcroft
Bradley Schlozman
George W. Bush
Dick Cheney
David Addington
Alberto Gonzales
Michael Elston
Monica Goodling
Karl Rove
Joshua Bolten
Kyle Sampson
Paul McNulty
Harriet Miers
Jose Rodriguez
COERCIVE INTERROGATION
CIA TAPES
Slate

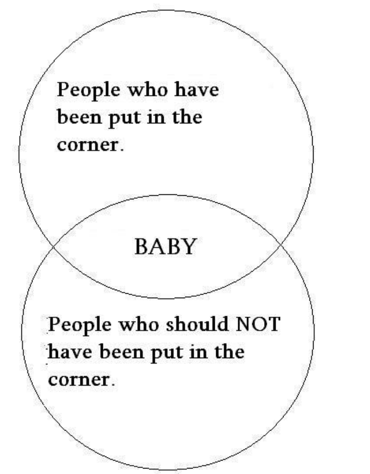

cheetos
grilled cheese
corn flakes
Look closely to find Jesus

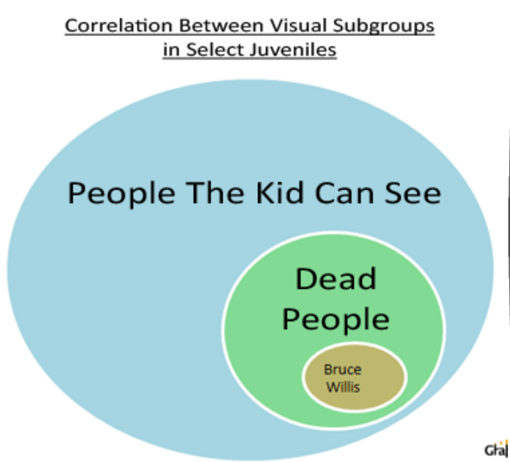

Why People write "First" in the comments
Because they are first
Because they think they are first
Because they are idiots
GraphJam

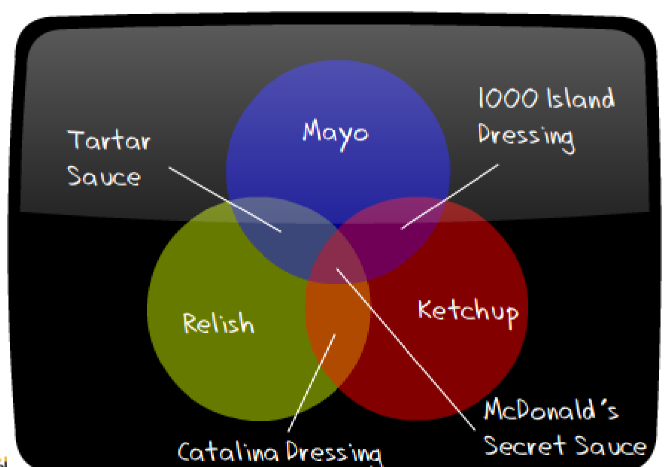

please, no more mascots
the real winner: the worldwide spandex industry
participation
excellence
hope
competition
unity
there should be at least one incredible wipeout where the person's body goes limp and equipment goes flying all over the place
any country can fake it for two weeks!


Stupidity
Republicans
Politics
Intelligent Design
Fundamentalism
The Dark Ages
Religion
*Not to scale. Stupidity actually overlaps Religion and Politics much more than depicted.
SaintGasoline.com

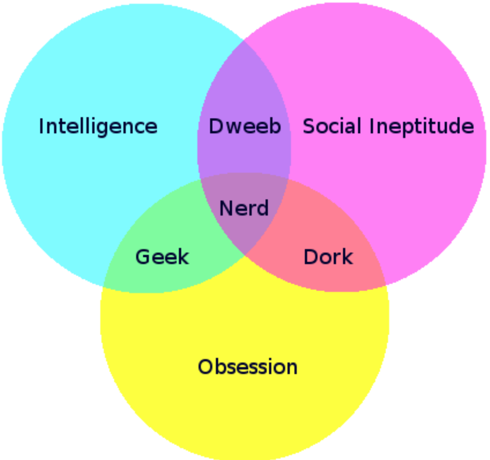

Things One Can Have While in College
Money
Sanity
Good Grades
GraphJam.com

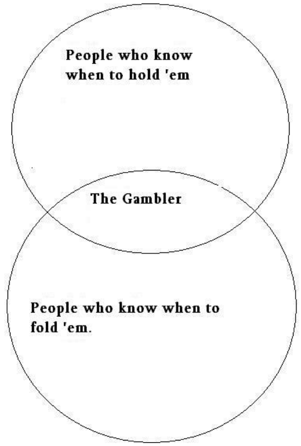

People who have been put in the corner.
BABY
People who should NOT have been put in the corner.

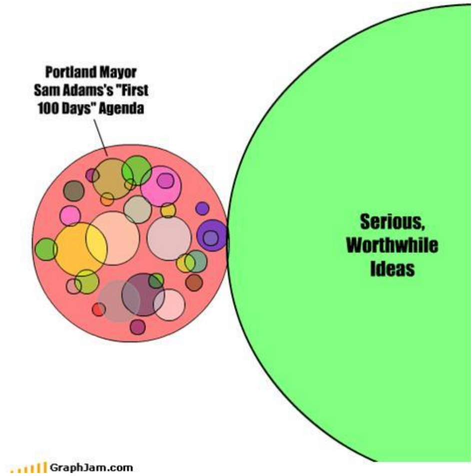

Correlation Between Visual Subgroups in Select Juveniles
People The Kid Can See
Dead People
Bruce Willis

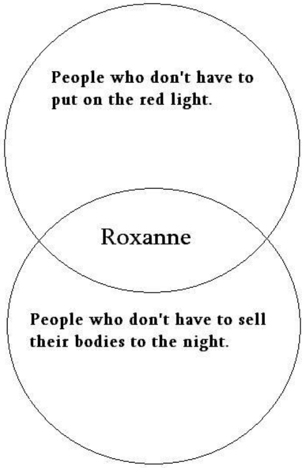

Tartar Sauce
Mayo
1000 Island Dressing
Relish
Ketchup
Catalina Dressing
McDonald's Secret Sauce


PONZI SCHEMES
SITUATIONS WHERE IT STINKS BEING THE BOTTOM
TWO PERSON HORSE COSTUMES

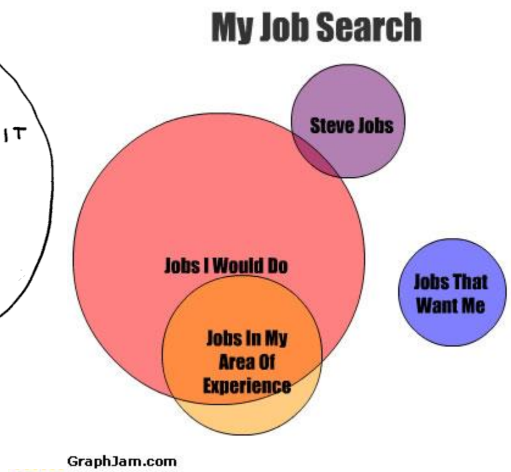

Intelligence
Dweeb
Social Ineptitude
Nerd
Geek
Dork
Obsession

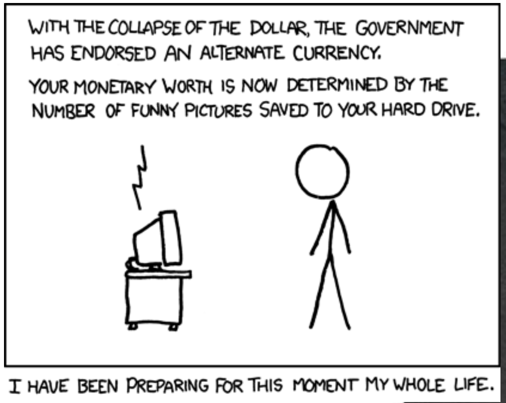

People who know when to hold 'em
The Gambler
People who know when to fold 'em.


Portland Mayor Sam Adams's "First 100 Days" Agenda
Serious, Worthwhile Ideas
GraphJam.com

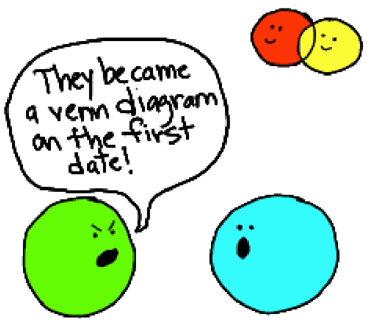

People who don't have to put on the red light.
Roxanne
People who don't have to sell their bodies to the night.

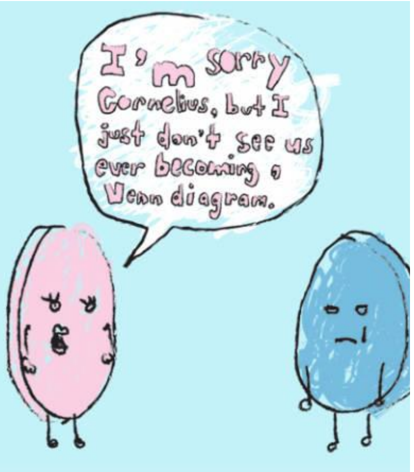

THOSE WHO GET IT
THOSE WHO GET IT AND DO NOT GET IT
THOSE WHO DO NOT GET IT
THOSE WHO ARE VERY CONFUSED
THOSE WHO GET IT BUT REALISE THERE IS NOTHING TO GET
THOSE WHO DO NOT GET IT AND REALISE THERE IS NOTHING TO GET
THOSE WHO REALISE THERE IS NOTHING TO GET

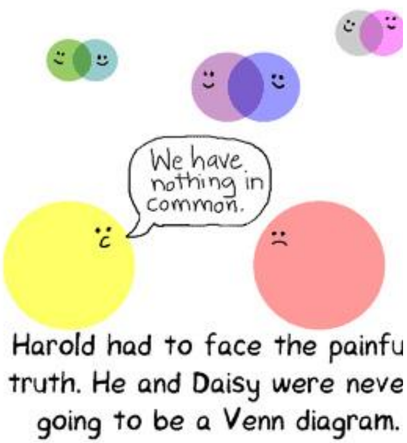

My Job Search
Steve Jobs
Jobs I Would Do
Jobs In My Area Of Experience
Jobs That Want Me
GraphJam.com


WITH THE COLLAPSE OF THE DOLLAR, THE GOVERNMENT HAS ENDORSED AN ALTERNATE CURRENCY.
YOUR MONETARY WORTH IS NOW DETERMINED BY THE NUMBER OF FUNNY PICTURES SAVED TO YOUR HARD DRIVE.
I HAVE BEEN PREPARING FOR THIS MOMENT MY WHOLE LIFE.


You don't need to see his identification.
MOVE ALONG
These aren't the droids you're looking for.
He can go about his business.
These aren't the droids you're looking for.


They became a venn diagram on the first date!

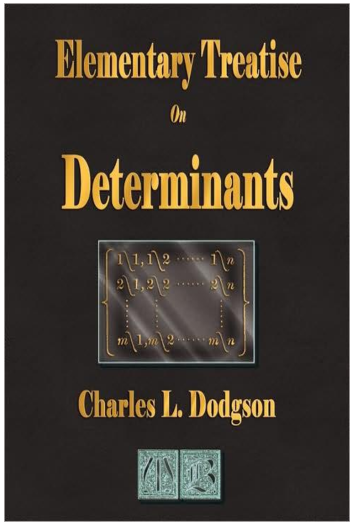

I'm sorry Cornelius, but I just don't see us ever becoming a Venn diagram.

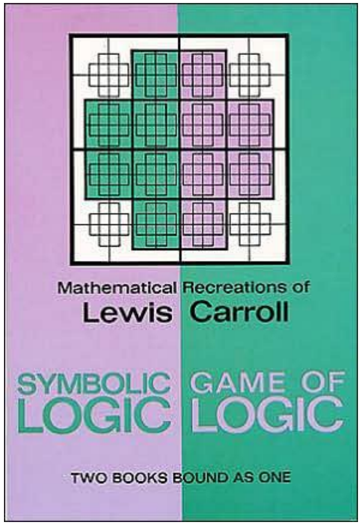

We have nothing in common.
Harold had to face the painfu
truth. He and Daisy were neve
going to be a Venn diagram.

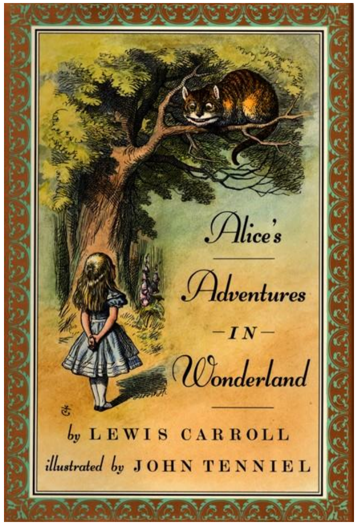
Historical Perspectives

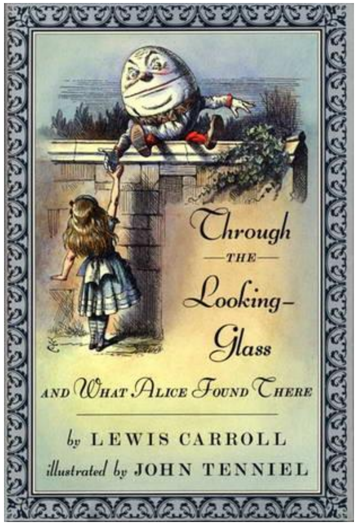
Charles Dodgson (1832-1898)

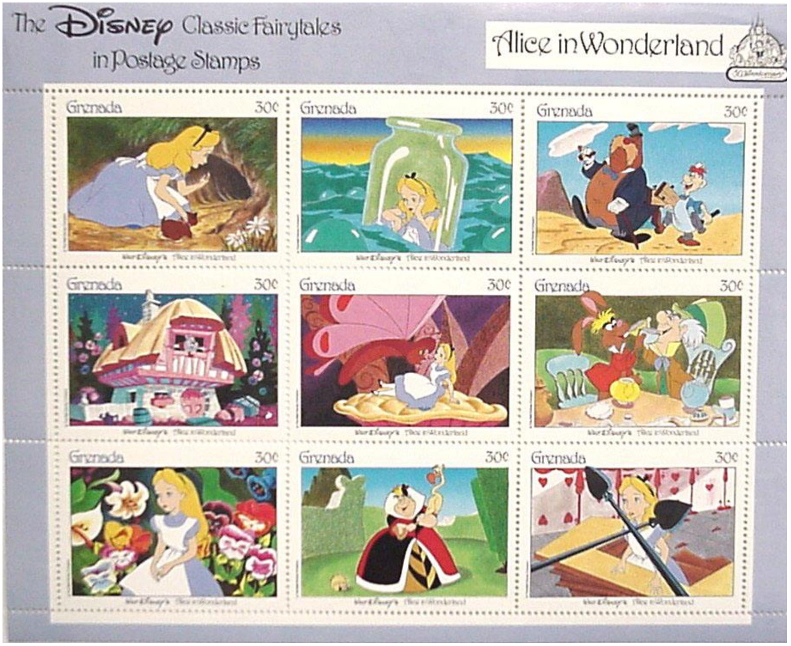
- AKA "Lewis Carroll"
- Mathematician, logician, author, photographer
- Wrote "Alice in Wonderland", "Jabberwocky", and "Through the Looking Glass"
- Popularized logic & syllogisms and made it fun!
- Invented "Scrabble" and "word ladder" games
- Profoundly influenced literature, art, and culture










The Disney Classic Fairytales in Postage Stamps
Alice in Wonderland
Grenada 30¢

OFFICIALLY SEALED
WONDERLAND
DEAD LETTER
OFFICE

13p
Alice's Adventures in Wonderland
The Year of the Child

Through the Looking-Glass
LEWIS CARROLL
63

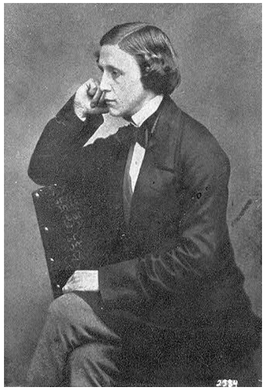
Lewis Carroll

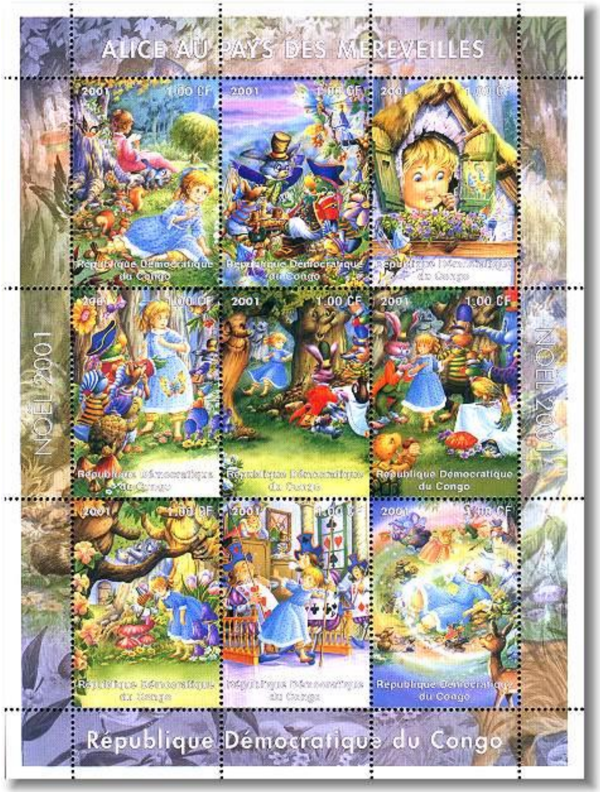
Charles Lutwidge Dodgson
(Lewis Carroll)



REPUBLIQUE du MALI
150e Anniversaire de la naissance de LEWIS CARROLL
110F

37
usa

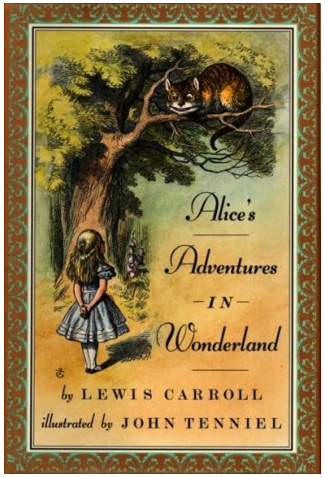


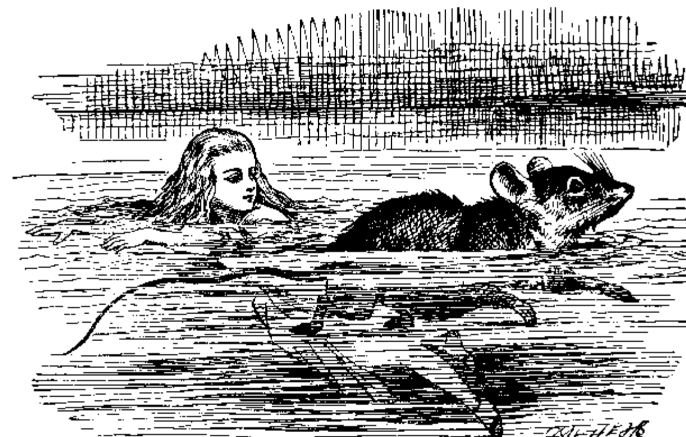

ALICE AU PAYS DES MERVEILLES
NOEL 2001
République Démocratique du Congo

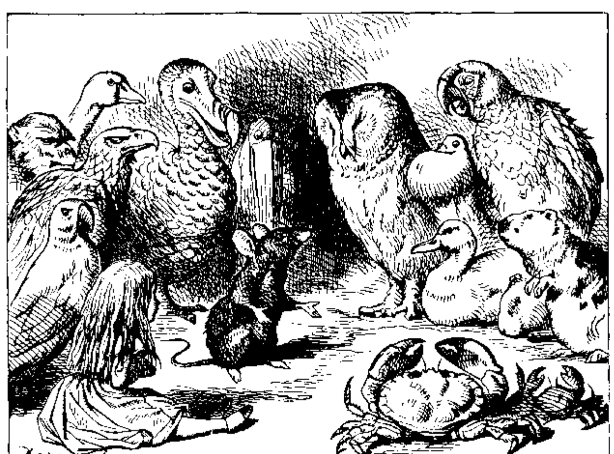
MALDIVES

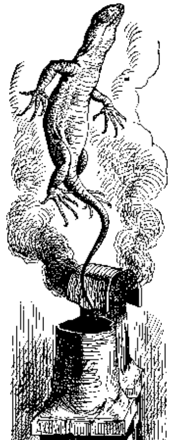
TIME
The Weekly Newsmagazine

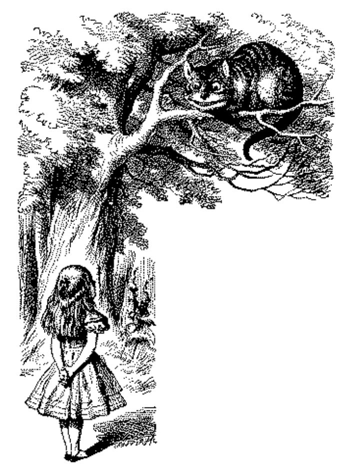
WONDERLAND
JABBERWOCKY CENTENNIAL
10
WINKS
JABBERWOCKY CENTENNIAL
WONDERLAND



Alice's
Adventures
IN
Wonderland
by LEWIS CARROLL
illustrated by JOHN TENNIEL







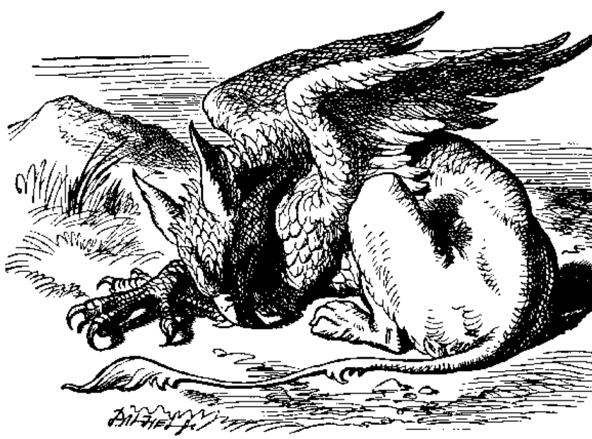












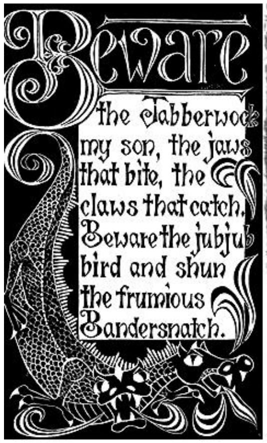

In this Style 10/6

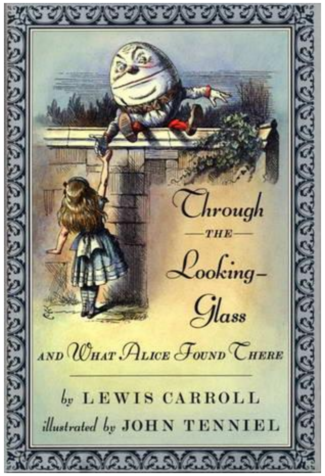




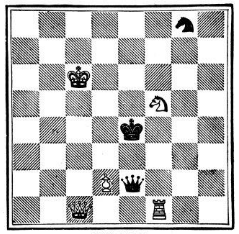





Beware
the Jabberwock
my son, the jaws
that bite, the
claws that catch.
Beware the jubjub
bird and shun
the frumious
Bandersnatch.



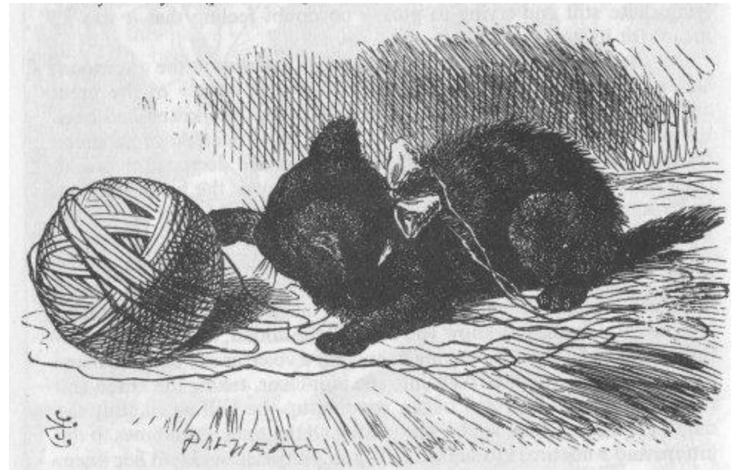
RED

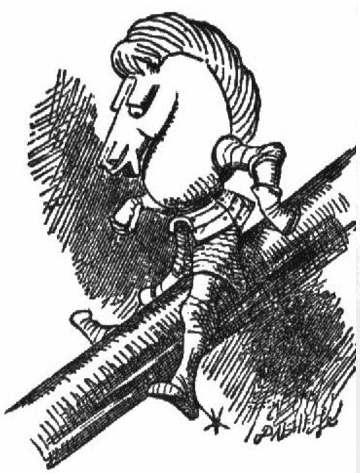


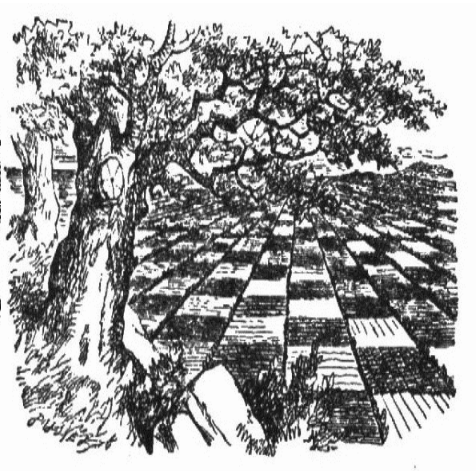
WHITE.

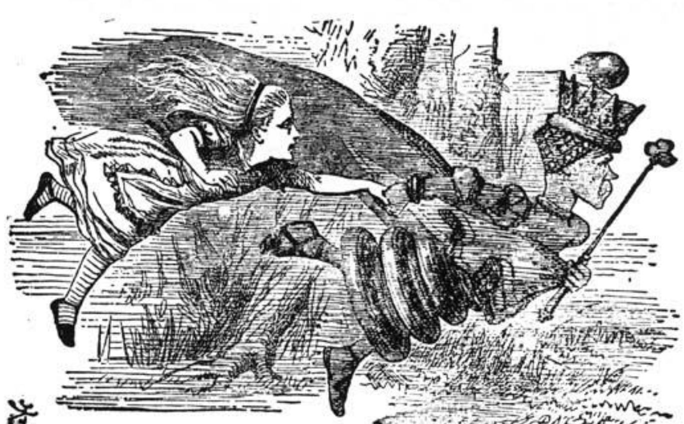
White Pawn (Alice) to play, and win in eleven moves.

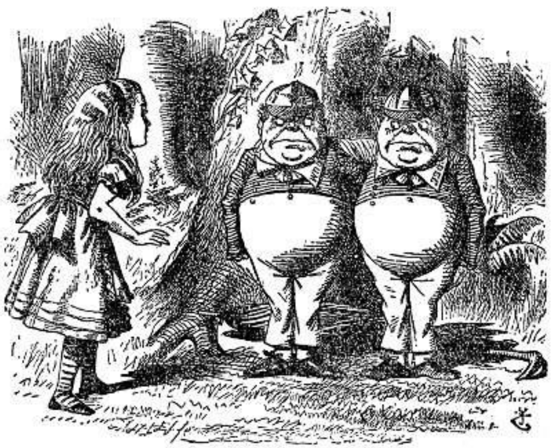
	PAGE	
1. Alice meets R. Q.	140	1. R. Q. to K. R.'s 4th
2. Alice through Q.'s 3rd *(by railway)* to Q.'s 4th *(Tweedledum and Tweedledee)*	147 / 149	2. W. Q. to Q. B.'s 4th *(after shawl)*
3. Alice meets W. Q. *(with shawl)*	168	3. W. Q. to Q. B.'s 5th *(becomes sheep)*
4. Alice to Q.'s 5th *(shop, river, shop)*	173	4. W. Q. to K. B.'s 8th *(leaves egg on shelf)*
5. Alice to Q.'s 6th *(Humpty Dumpty)*	179	5. W. Q. to Q. B.'s 8th *(flying from R. Kt.)*
6. Alice to Q.'s 7th *(forest)*	200	6. R. Kt. to K.'s 2nd (ch.)
7. W. Kt. takes R. Kt	202	7. W. Kt. to K. B.'s 5th
8. Alice to Q.'s 8th *(coronation)*	213	8. R. Q. to K.'s sq. *(examination)*
9. Alice becomes Queen	220	9. Queens castle
10. Alice castles *(feast)*	223	10. W. Q. to Q. R.'s 6th *(soup)*
11. Alice takes R.Q. & wins	230	

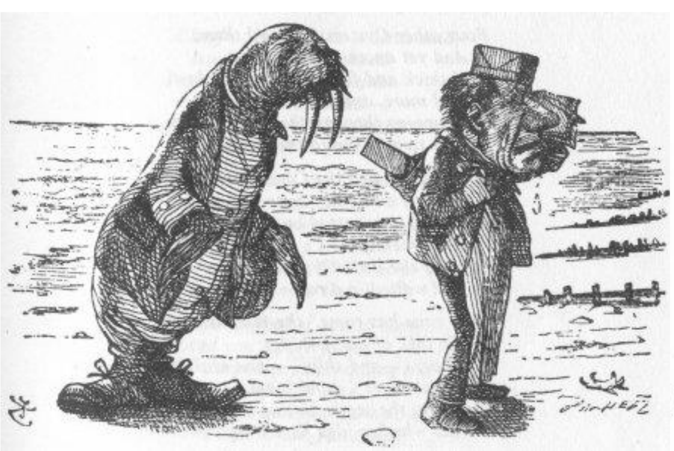










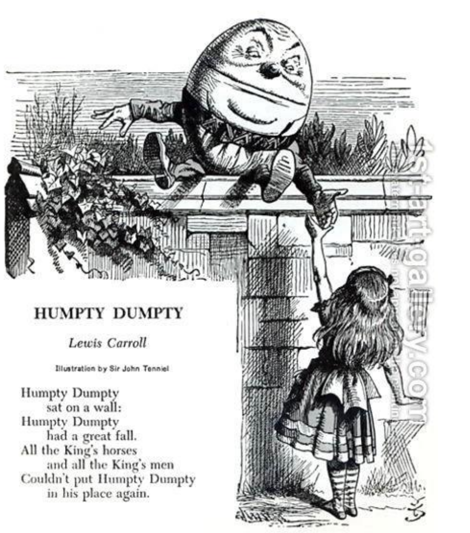











HUMPTY DUMPTY
Lewis Carroll
Illustration by Sir John Tenniel
Humpty Dumpty
sat on a wall:
Humpty Dumpty
had a great fall.
All the King's horses
and all the King's men
Couldn't put Humpty Dumpty
in his place again.
1st-art-gallery.com














QUEEN ALICE

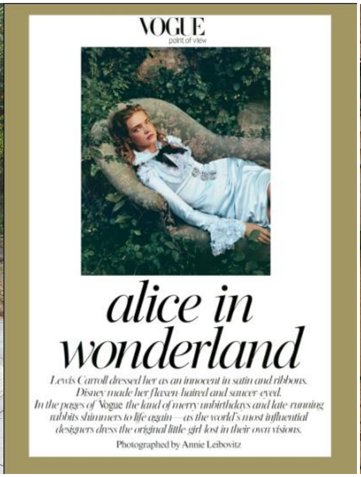








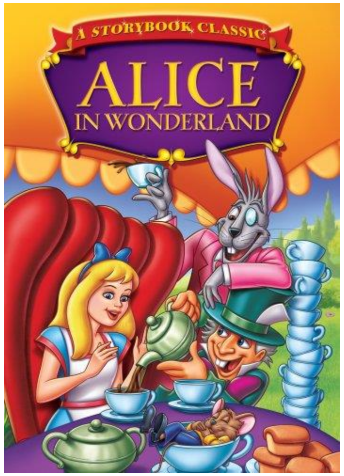
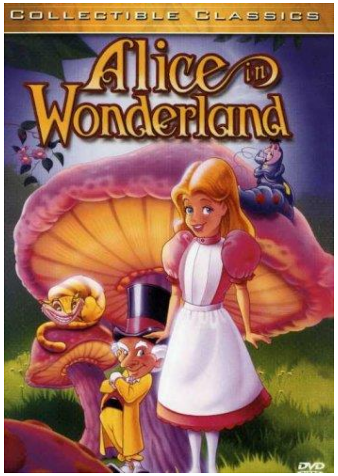
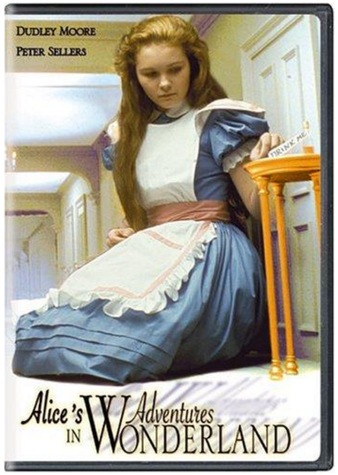


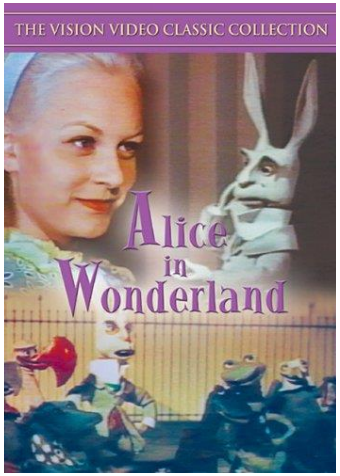
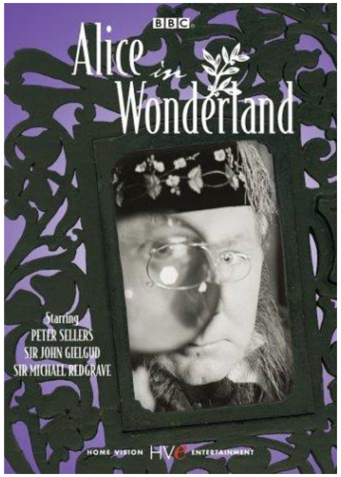
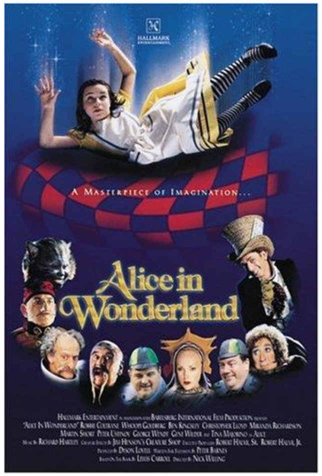


VOGUE
point of view

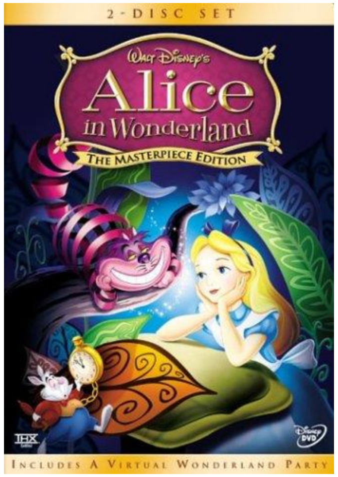
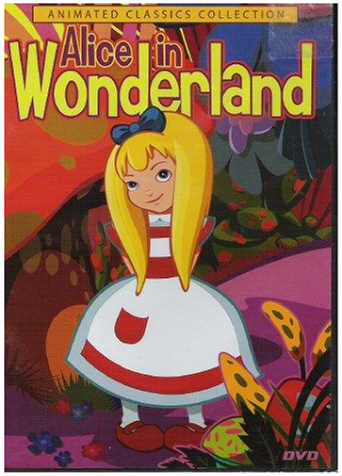
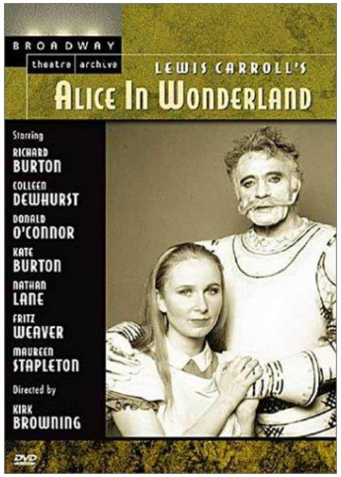
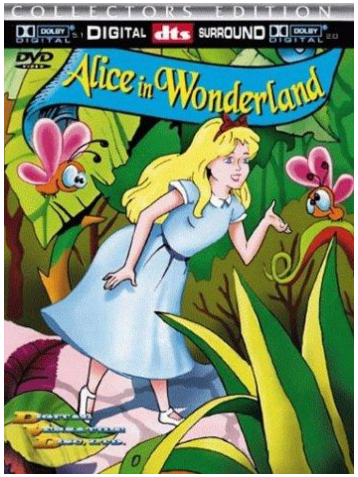
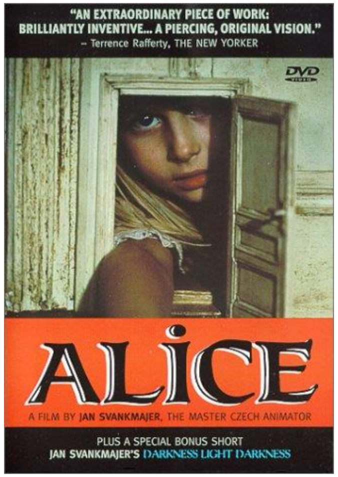
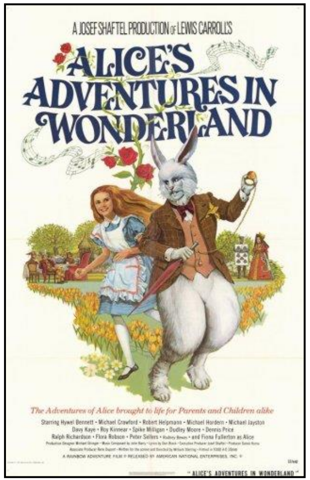
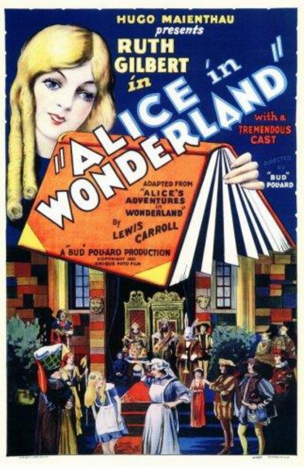
alice in wonderland

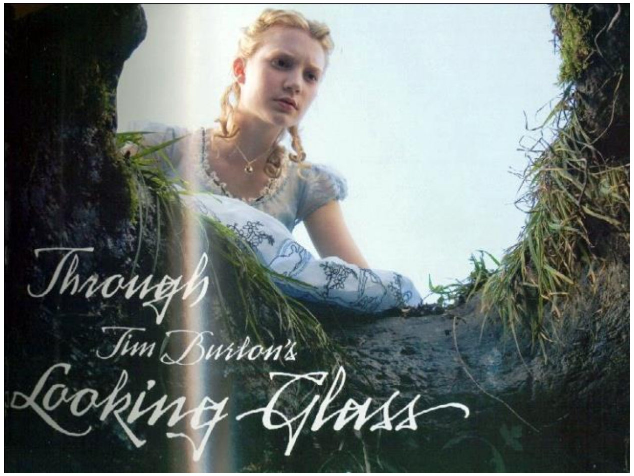
Lewis Carroll dressed her as an innocent in satin and ribbons. Disney made her flaxen-haired and saucer-eyed. In the pages of Vogue the land of merry unbirthdays and late-running rabbits shimmers to life again—as the world's most influential designers dress the original little girl lost in their own visions.

Photographed by Annie Leibovitz

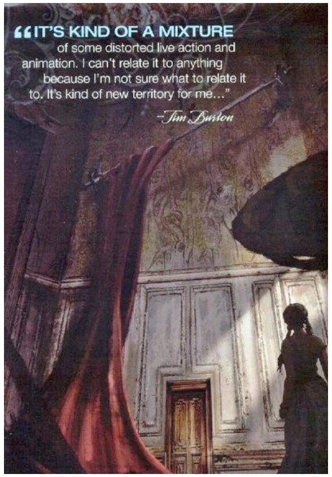

A STORYBOOK CLASSIC
ALICE
IN WONDERLAND

COLLECTIBLE CLASSICS
Alice in Wonderland
DVD

DUDLEY MOORE
PETER SELLERS
Alice's Adventures IN WONDERLAND

Alice IN Wonderland
THE STUNNING ADAPTATION OF A FAMILY CLASSIC!
DVD

THE VISION VIDEO CLASSIC COLLECTION
Alice in Wonderland

BBC
Alice in Wonderland
Starring
PETER SELLERS
SIR JOHN GIELGUD
SIR MICHAEL REDGRAVE
HOME VISION ENTERTAINMENT

A MASTERPIECE OF IMAGINATION...
Alice in Wonderland

PETER SELLERS
DUDLEY MOORE
MICHAEL CRAWFORD
FIONA FULLERTON
ALICE'S ADVENTURES in WONDERLAND
JOURNEY DOWN THE RABBIT HOLE IN THIS TUNEFUL FAMILY CLASSIC!

2-DISC SET
Walt Disney's
Alice in Wonderland
THE MASTERPIECE EDITION
INCLUDES A VIRTUAL WONDERLAND PARTY

ANIMATED CLASSICS COLLECTION
Alice in Wonderland
DVD

BROADWAY
theatre archive
LEWIS CARROLL'S
ALICE IN WONDERLAND
Starring
RICHARD BURTON
COLLEEN DEWHURST
DONALD O'CONNOR
KATE BURTON
NATHAN LANE
FRITZ WEAVER
MAUREEN STAPLETON
Directed by
KIRK BROWNING
DVD

COLLECTORS EDITION
DIGITAL dts SURROUND
DVD
Alice in Wonderland

"AN EXTRAORDINARY PIECE OF WORK: BRILLIANTLY INVENTIVE... A PIERCING, ORIGINAL VISION."
– Terrence Rafferty, THE NEW YORKER
DVD
ALICE
A FILM BY JAN SVANKMAJER, THE MASTER CZECH ANIMATOR
PLUS A SPECIAL BONUS SHORT
JAN SVANKMAJER'S DARKNESS LIGHT DARKNESS

A JOSEF SHAFTEL PRODUCTION of LEWIS CARROLL'S
ALICE'S ADVENTURES IN WONDERLAND

HUGO MAIENTHAU
presents
RUTH GILBERT
in
"ALICE IN WONDERLAND"
with a TREMENDOUS CAST
ADAPTED FROM "ALICE'S ADVENTURES IN WONDERLAND"
By LEWIS CARROLL
A BUD POLLARD PRODUCTION


Through
Tim Burton's
Looking Glass


© Disney


"IT'S KIND OF A MIXTURE
of some distorted live action and
animation. I can't relate it to anything
because I'm not sure what to relate it
to. It's kind of new territory for me..."
–Tim Burton

Alice and the White Knight:

A Lesson in Logic, Semantics, and Pointers

`You are sad,' the Knight said in an anxious tone: `let me sing you a song to comfort you.'

`Is it very long?' Alice asked, for she had heard a good deal of poetry that day.

`It's long,' said the Knight, `but it's very, *very* beautiful. Everybody that hears me sing it -- either it brings the *tears* into their eyes, or else --'

logical disjunction!

`Or else what?' said Alice, for the Knight had made a sudden pause.

law of the excluded middle!

`Or else it doesn't, you know. The name of the song is called "***Haddocks' Eyes***".'

pointer to a pointer!

`Oh, that's the name of the song, is it?' Alice said, trying to feel interested.

`No, you don't understand,' the Knight said, looking a little vexed. `That's what the name is *called*. The name really *is* "***The Aged Aged Man***".'

pointer dereferencing: meta-pointer resolved!

`Then I ought to have said "That's what the *song* is called"?' Alice corrected herself.

separation of abstractions: variable vs. pointer!

`No, you oughtn't: that's quite another thing! The *song* is called "***Ways and Means***": but that's only what it's *called*, you know!'

call-by-name vs. call-by-value!

`Well, what *is* the song, then?' said Alice, who was by this time completely bewildered.

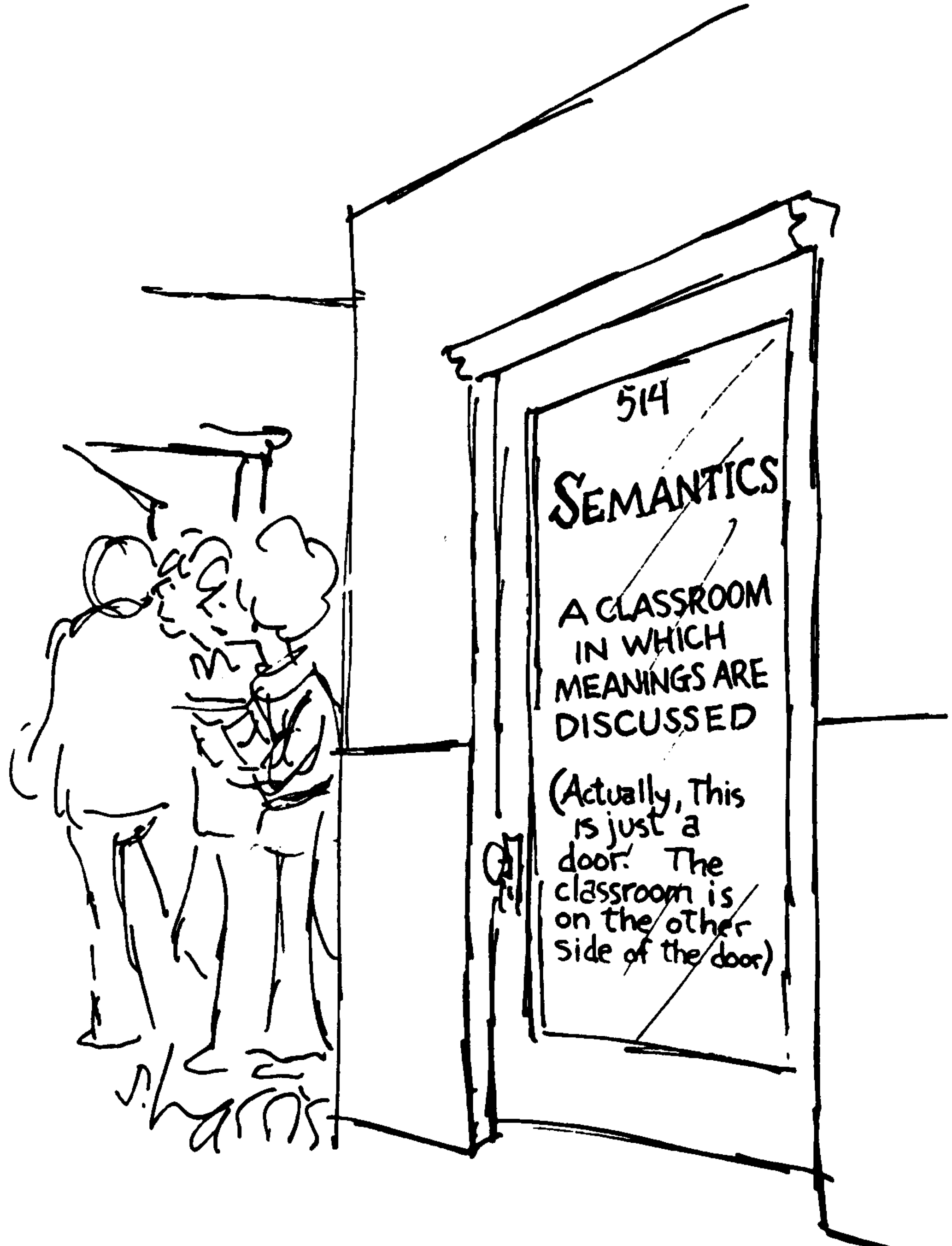
`I was coming to that,' the Knight said. `The song really *is* "***A-sitting On a Gate***": and the tune's my own invention.'

the name is called
"Haddocks' Eyes"

the name of the song is
"The Aged Aged Man"

the song is
"A-sitting On a Gate"

the song is called
"Ways and Means"


514
SEMANTICS
A CLASSROOM IN WHICH MEANINGS ARE DISCUSSED
(Actually, This is just a door. The classroom is on the other side of the door)
S. Harris

Lewis Carroll Society of North America

WELCOME

Welcome to The Lewis Carroll Society of North America (LCSNA) homepage. The LCSNA is a non-profit organization dedicated to furthering Carroll studies, increasing accessibility of research material, and maintaining public awareness of Carroll's contributions to society and culture. This website is one way we share information with Carroll enthusiasts around the World. If you are a Carrollian and would like to help in these endeavors, or if you simply enjoy Carroll and want to be among other people with a like interest, please consider joining the LCSNA.

For detailed information about C.L.Dodgson ("Lewis Carroll") and his creations, please access the Lewis Carroll Homepage.

Spring Meeting

The 2009 Spring meeting will be held in beautiful Sante Fe, New Mexico, on May 9. Please consult the **newly updated (as of April 24th)** meeting agenda for all of the details. See you there.

The Lewis Carroll Society

The Lewis Carroll Society

founded 1969 ~ registered charity no. 266239

Navigation
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Events, People, Places
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Welcome to the Lewis Carroll Society Website

The Lewis Carroll Society was formed in 1969 with the aim of encouraging research into the life and works of Lewis Carroll (Charles Lutwidge Dodgson). The Society has members around the world, including many leading libraries and institutions, authors, researchers and many who simply enjoy Carroll's books and want to find out more about the man and his work.

Why not join the LCS - for your own interest and entertainment or to make a contribution to Carroll scholarship? Our subscription are remarkably low for a society of this nature. **Click Here for membership details**.

Events at Lyndhurst: from 15 May 2009

This wonderful season of Alice-related events has something for everyone! The village of Lyndhurst, in the beautiful New Forest, celebrates its Alice connections with walks, talks, tea-parties, musicals, and many other activities. Vist the **Alice Adventure** website for more details.

Events at Oxford: 4 July 2009

The city of Oxford plays host to the second Alice's Day this year, with a busy programme of events on 4 July. There are live performances, reading, drama workshops, exhibitions, talks and other activities for all the family.

The Lewis Carroll Society is hosting a series of lectures at the Natural History Museum from 10:15. Edward Wakeling talks about the real Alice and the original telling of her adventures, Anne Varty investigates the child-actresses who played Alice and were friends of Lewis Carroll and Mark Richards explores the connections between Carroll and Charles Darwin. All are welcome to attend - come and go as you please.

Alice.org - Mozilla Firefox

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Alice.org

Alice

An Educational Software that teaches students computer programming in a 3D environment

FREE!!

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The Alice Project announces a unique collaboration with Sun Microsystems

read more...

Alice 3 News

Alice 3 wins Duke's Choice Award at JavaOne 2009!

Read more...

All about Alice

Alice is an innovative 3D programming environment that makes it easy to create an animation for telling a story, playing an interactive game, or a video to share on the web. Alice is a teaching tool for introductory computing. It uses 3D graphics and a drag-and-drop interface to facilitate a more engaging, less frustrating first programming experience.

Read more...

Teaching Materials

Alice is a teaching tool designed as a revolutionary approach to teaching and learning introductory programming concepts. The Alice team has developed instructional materials to support students and teachers in using this new approach. Resources include textbooks, lessons, sample syllabuses, test banks, and more. Other authors have generously joined our efforts, creating additional textbooks.

Read more...

Downloads

Alice 2.2, Alice 2.0
Designed for High School and College

Storytelling Alice
Designed for Middle School

Alice 3 beta
Get a sneak peek at the future of Alice

3D Models Gallery
Additional free 3D models

Alice Blog

Check out the Alice blog! The Alice team discusses the latest in Alice development. View screencasts demonstrating new features, tips and techniques!

Visit blog...

Community Forums

Share and gather knowledge about Alice through our community forums. Students, teachers and enthusiasts are all welcome! If you have a question or comment about Alice, post it here!

View forums...

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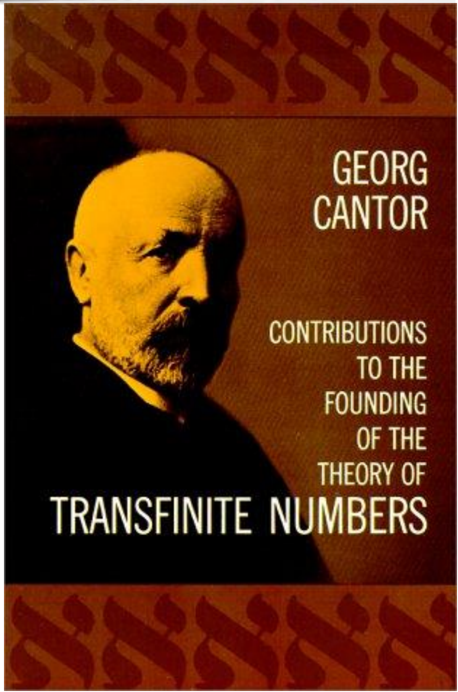
Done

Historical Perspectives

Georg Cantor (1845-1918)

- Created modern set theory
- Invented trans-finite arithmetic (highly controvertial at the time)
- Invented diagonalization argument
- First to use 1-to-1 correspondences with sets
- Proved some infinities "bigger" than others
- Showed an infinite hierarchy of infinities
- Formulated continuum hypothesis
- Cantor's theorem, "Cantor set", Cantor dust, Cantor cube, Cantor space, Cantor's paradox
- Laid foundation for computer science theory
- Influenced Hilbert, Godel, Church, Turing




GEORG CANTOR
His Mathematics and Philosophy of the Infinite
Joseph Warren Dauben
David Foster Wallace
GEORG CANTOR
Der Jahrhundertmathematiker und die Entdeckung des Unendlichen
PIPER
MIDHAT J. GAZALÉ
NUMBER
From Ahmes to Cantor
princeton science library
Infinity and the Mind
the science and philosophy of the infinite
RUDY RUCKER
On the reading list!
THE NATIONAL BESTSELLER
The Mystery of the Aleph
Mathematics, the Kabbalah, and the Search for Infinity
"Excellent."
Amir D. Aczel
Philosophical Perspectives on Infinity
Graham Oppy
CAMBRIDGE
GREAT DISCOVERIES
EVERYTHING AND MORE
A Compact History of ∞
DAVID FOSTER WALLACE
david foster wallace
everything and more
cantor & zeno
& mathematics & abstraction & ∞
USES OF INFINITY
LEO ZIPPIN
ELI MAOR
To Infinity and Beyond
A Cultural History of the Infinite
THE ART of THE INFINITE
THE PLEASURES OF MATHEMATICS
ROBERT KAPLAN
+
ELLEN KAPLAN
THE MATHEMATICS OF INFINITY
A Guide to Great Ideas
THEODORE G. FATICONI
ONE
TWO
THREE...
INFINITY
Facts and Speculations of Science
George Gamow
KEYS TO Infinity
CLIFFORD A. PICKOVER
PLAYING WITH INFINITY
MATHEMATICAL EXPLORATIONS AND EXCURSIONS
Rózsa Péter
INFINITY
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Satan, Cantor & Infinity
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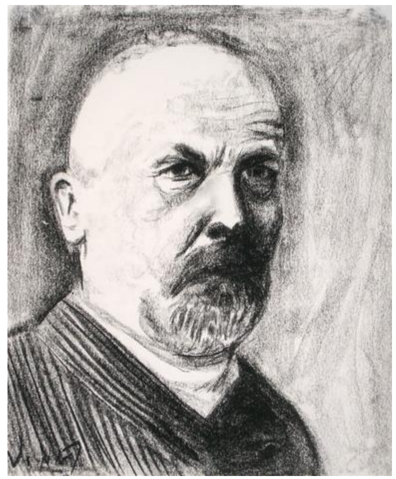
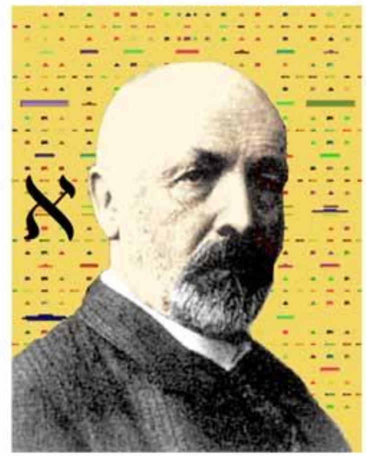
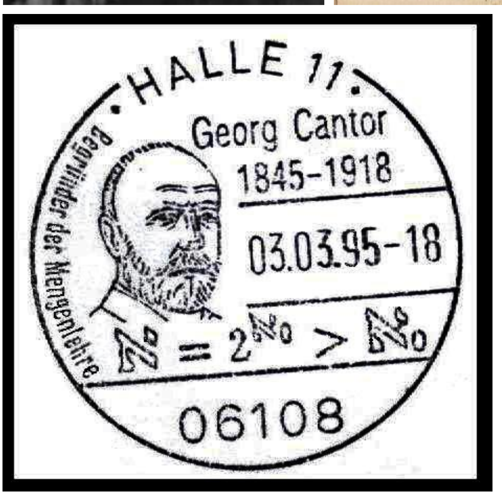
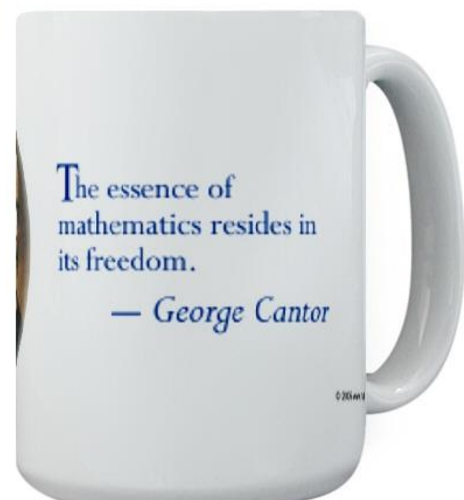



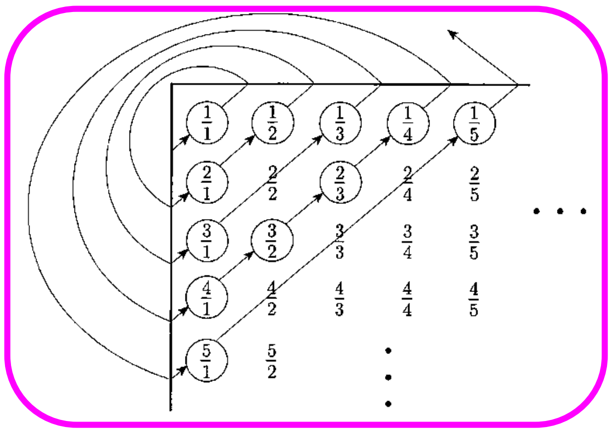
Georg Cantor
1845 - 1918





To get this list we make an infinite matrix containing all the positive rational numbers, as shown in Figure 4.16. The ith row contains all numbers with numerator i and the jth column has all numbers with denominator j. So the number $\frac{i}{j}$ occurs in the ith row and jth column.

Now we turn this matrix into a list. One (bad) way to attempt it would be to begin the list with all the elements in the first row. That isn't a good approach because the first row is infinite, so the list would never get to the second row. Instead we list the elements on the diagonals, starting from the corner, which are superimposed on the diagram. The first diagonal contains the single element $\frac{1}{1}$, and the second diagonal contains the two elements $\frac{2}{1}$ and $\frac{1}{2}$. So the first three elements on the list are $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{1}{2}$. In the third diagonal a complication arises. It contains $\frac{3}{1}$, $\frac{2}{2}$, and $\frac{1}{3}$. If we simply added these to the list, we would repeat $\frac{1}{1} = \frac{2}{2}$. We avoid doing so by skipping an element when it would cause a repetition. So we add only the two new elements $\frac{3}{1}$ and $\frac{1}{3}$. Continuing in this way we obtain a list of all the elements of $\mathcal{Q}$.



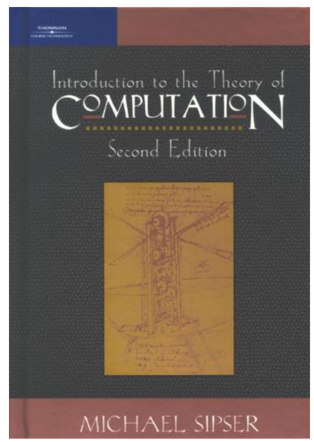


FIGURE 4.16
A correspondence of $\mathcal{N}$ and $\mathcal{Q}$

After seeing the correspondence of $\mathcal{N}$ and $\mathcal{Q}$, you might think that any two infinite sets can be shown to have the same size. After all, you need only demonstrate a correspondence, and this example shows that surprising correspondences do exist. However, for some infinite sets no correspondence with $\mathcal{N}$ exists. These sets are simply too big. Such sets are called ***uncountable***.

The set of real numbers is an example of an uncountable set. A ***real number*** is one that has a decimal representation. The numbers $\pi = 3.1415926\ldots$ and $\sqrt{2} = 1.4142135\ldots$ are examples of real numbers. Let $\mathcal{R}$ be the set of real numbers. Cantor proved that $\mathcal{R}$ is uncountable. In doing so he introduced the diagonalization method.

A_{DFA} and A_{CFG} were decidable, A_{TM} is not. Let

$$A_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}.$$

THEOREM 4.11

A_{TM} is undecidable.

Before we get to the proof, let's first observe that A_{TM} is Turing-recognizable. Thus this theorem shows that recognizers *are* more powerful than deciders. Requiring a TM to halt on all inputs restricts the kinds of languages that it can recognize. The following Turing machine U recognizes A_{TM}.

U = "On input $\langle M, w\rangle$, where M is a TM and w is a string:
1. Simulate M on input w.
2. If M ever enters its accept state, *accept*; if M ever enters its reject state, *reject*."

Note that this machine loops on input $\langle M, w\rangle$ if M loops on w, which is why this machine does not decide A_{TM}. If the algorithm had some way to determine that M was not halting on w, it could *reject*. Hence A_{TM} is sometimes called the ***halting problem***. As we demonstrate, an algorithm has no way to make this determination.

The Turing machine U is interesting in its own right. It is an example of the ***universal Turing machine*** first proposed by Turing. This machine is called universal because it is capable of simulating any other Turing machine from the description of that machine. The universal Turing machine played an important early role in stimulating the development of stored-program computers.

THE DIAGONALIZATION METHOD

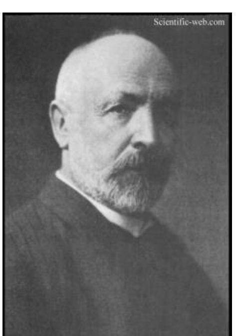
The proof of the undecidability of the halting problem uses a technique called ***diagonalization***, discovered by mathematician Georg Cantor in 1873. Cantor was concerned with the problem of measuring the sizes of infinite sets. If we have two infinite sets, how can we tell whether one is larger than the other or whether they are of the same size? For finite sets, of course, answering these questions is easy. We simply count the elements in a finite set, and the resulting number is its size. But, if we try to count the elements of an infinite set, we will never finish! So we can't use the counting method to determine the relative sizes of infinite sets.

For example, take the set of even integers and the set of all strings over $\{0,1\}$. Both sets are infinite and thus larger than any finite set, but is one of the two larger than the other? How can we compare their relative size?

Cantor proposed a rather nice solution to this problem. He observed that two finite sets have the same size if the elements of one set can be paired with the elements of the other set. This method compares the sizes without resorting to counting. We can extend this idea to infinite sets. Let's see what it means more precisely.

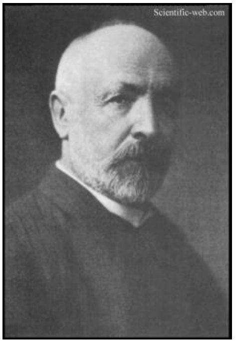
Problem: How can a new guest be accommodated in a full infinite hotel?

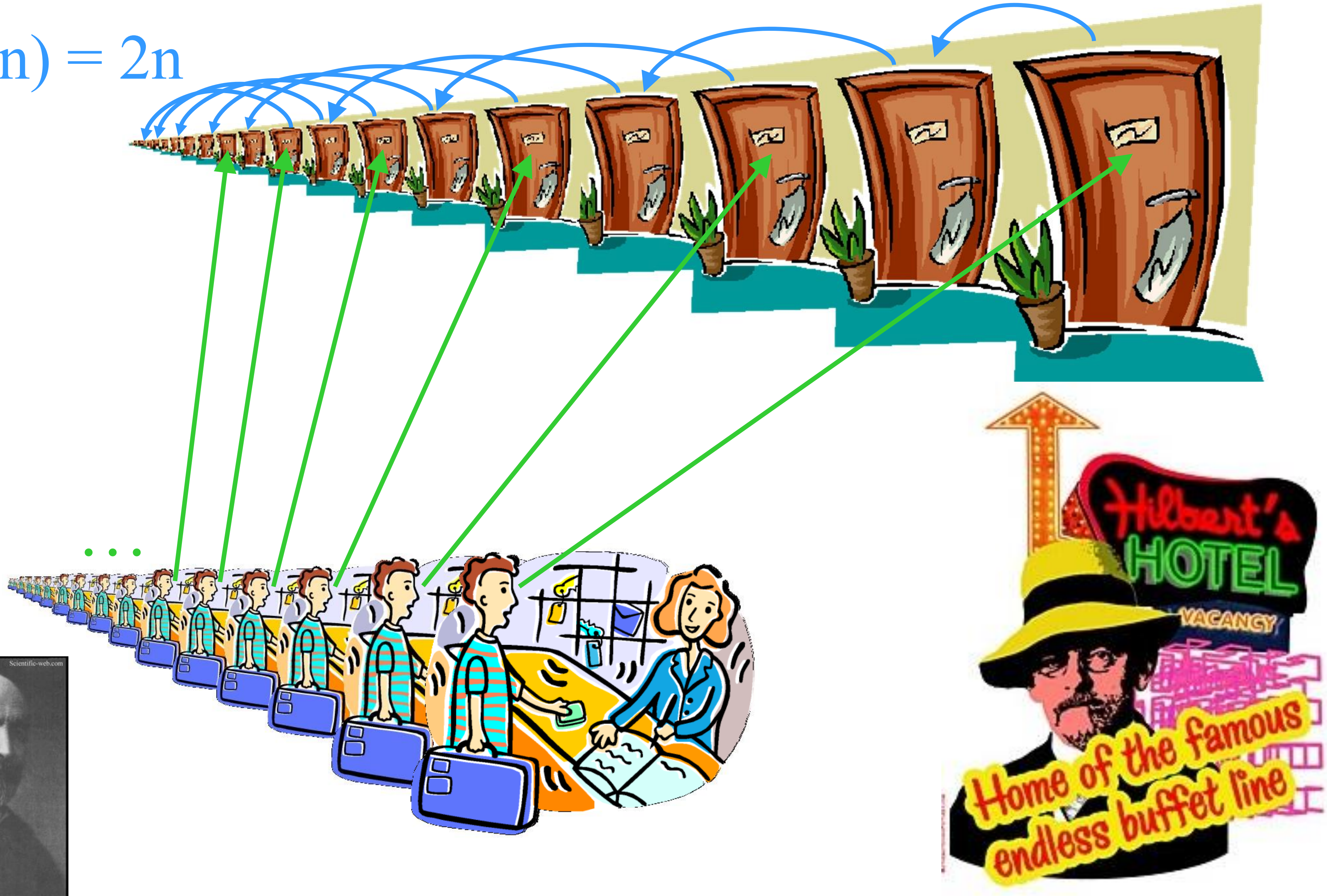
$f(n) = n+1$



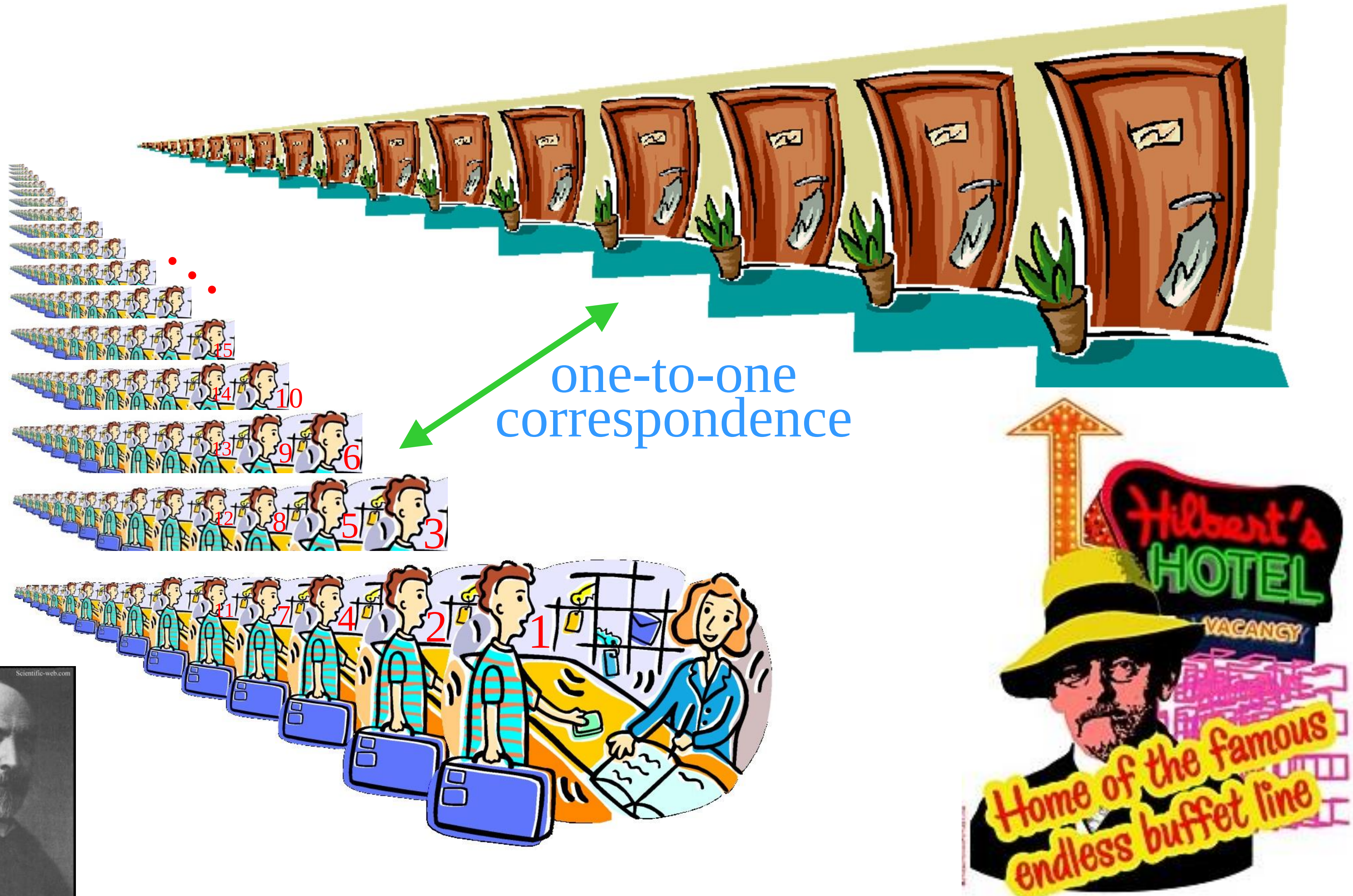
Problem: How can an infinity of new guests be accommodated in a full infinite hotel?

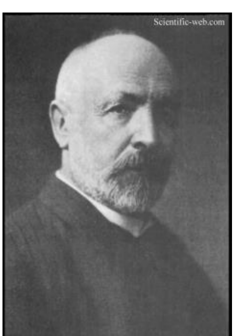
$f(\mathrm{n}) = 2\mathrm{n}$





Problem: How can an infinity of infinities of new guests be accommodated in a full infinite hotel?





"HILBERT'S HOTEL"
©2002 CAROL LAY
THE INFINITE HOTEL WAS ALWAYS FILLED TO CAPACITY.
HILBERT'S HOTEL ∞
NO VACANCY
WWW.WAYLAY.COM
YET, IF A NEW GUEST ARRIVED, SHE WAS ALWAYS GIVEN A ROOM.
AFTER ALL, THERE WERE AN INFINITE NUMBER OF ROOMS.
ROOM 843,722,948,6714,392,645, 913,653,898,652,732 NEEDS FRESH TOWELS.
THIS PARADOX ASSUMED THAT MANAGEMENT COULD ALWAYS ADD ONE OR MORE TO INFINITY.
WE'LL SIMPLY PUT YOU IN ROOM 1, THEN ASK EVERYONE TO SHIFT OVER ONE ROOM.
THE BRAIN-BRUISING HOTEL ATTRACTED A LOT OF MATHEMATICIANS AND PHILOSOPHERS.
JUST A MOMENT, PLEASE.
THEY LIKED TO ARGUE INTO THE WEE HOURS ABOUT THE NATURE OF INFINITY.
UNFORTUNATELY, THEY WERE A BUNCH OF SLOBS.
OH, DEAR.
TSK
MANAGEMENT HAD TO HIRE A NEW MAID TO KEEP UP WITH THE MESS.
RING IF YOU GET LOST...
DAUNTED BY THE NUMBER OF ROOMS TO CLEAN...
RM. 632946851 7432
NOK NOK
THE MAID SET FIRE TO THE JOINT.
SCREW IT.
ETHER FLAMMABLE
THE PHILOSOPHERS ESCAPED...
BUT THE HOTEL BURNED FOREVER.

Ode to Aleph Null
$\aleph_0$
Aleph-null bottles of beer on the wall,
Aleph-null bottles of beer,
if one of those bottles should happen to fall,
Aleph-null bottles of beer on the wall.

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HOTEL INFINITY
Amanda Boyle

average rating from 27 members

drama | 2004 | London | Switzerland/ Mandarin | 10 min

Published 28 Feb 07

What happens when an hotel of infinite rooms suddenly becomes full?

send to a friend

PLAY NOW
Requires windows media player or real player.

synopsis
There was once a hotel in the mountains that was so popular with guests, the Manager decided to extend it. Yet still it remained full. The Manager continued to extend it, until eventually it became infinitely large. One day much to his surprise, no spare rooms could be found in his now infinite hotel. All the mathematical calculations, which he normally relied on to find the rooms, just wouldn't work...

rate this film
5
4
3
2
1

http://www.bbc.co.uk/mediaselector/check/filmnetwork/media/shorts/A20082089?size=16x9&bgc=C0C0C0&nbwm=1&bbwm=1&nbram=1&bbram=1

Problem: Are there more integers than natural #'s?

$\mathbb{N} \subset \mathbb{Z}$

$\mathbb{N} \neq \mathbb{Z}$

So $|\mathbb{N}| < |\mathbb{Z}|$?

$\mathbb{Z}$

-4	-3	-2	-1	0	1	2	3	4

Rearrangement:

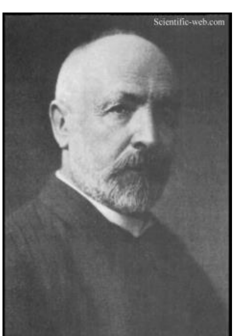
Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Z}$

$\mathbb{Z}$

$\mathbb{N}$ 1 2 3 4 5 6 7 8 9

$\Rightarrow |\mathbb{N}| = |\mathbb{Z}|$



Problem: Are there more rationals than natural #'s?

$\mathbb{N} \subset \mathbb{Q}$

$\mathbb{N} \neq \mathbb{Q}$

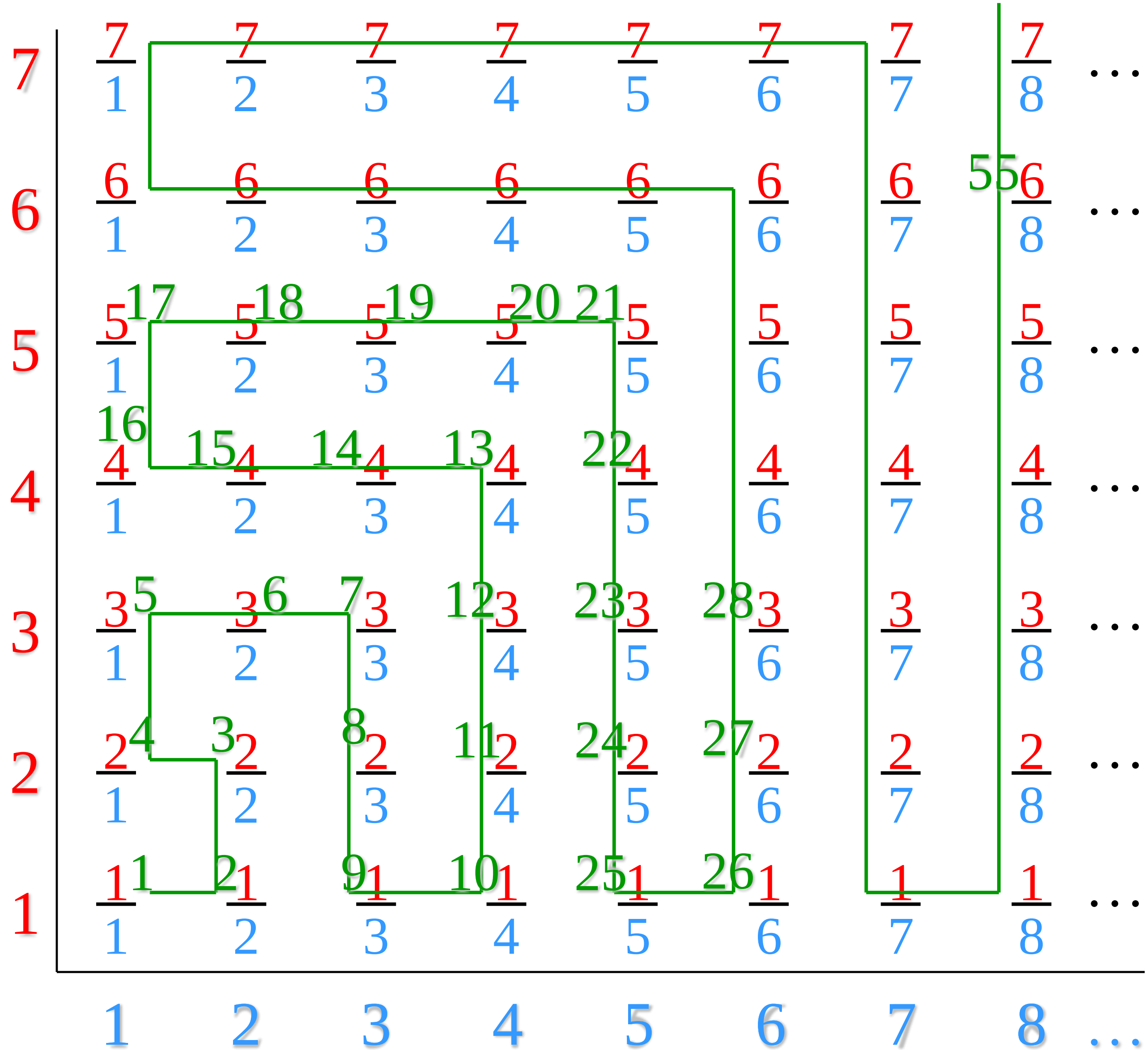
So $|\mathbb{N}|<|\mathbb{Q}|$?

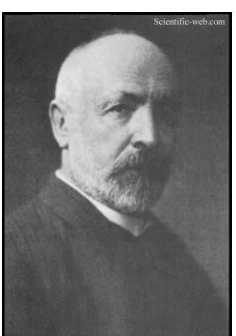
Dovetailing:

Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Q}$

$\Rightarrow |\mathbb{N}|=|\mathbb{Q}|$





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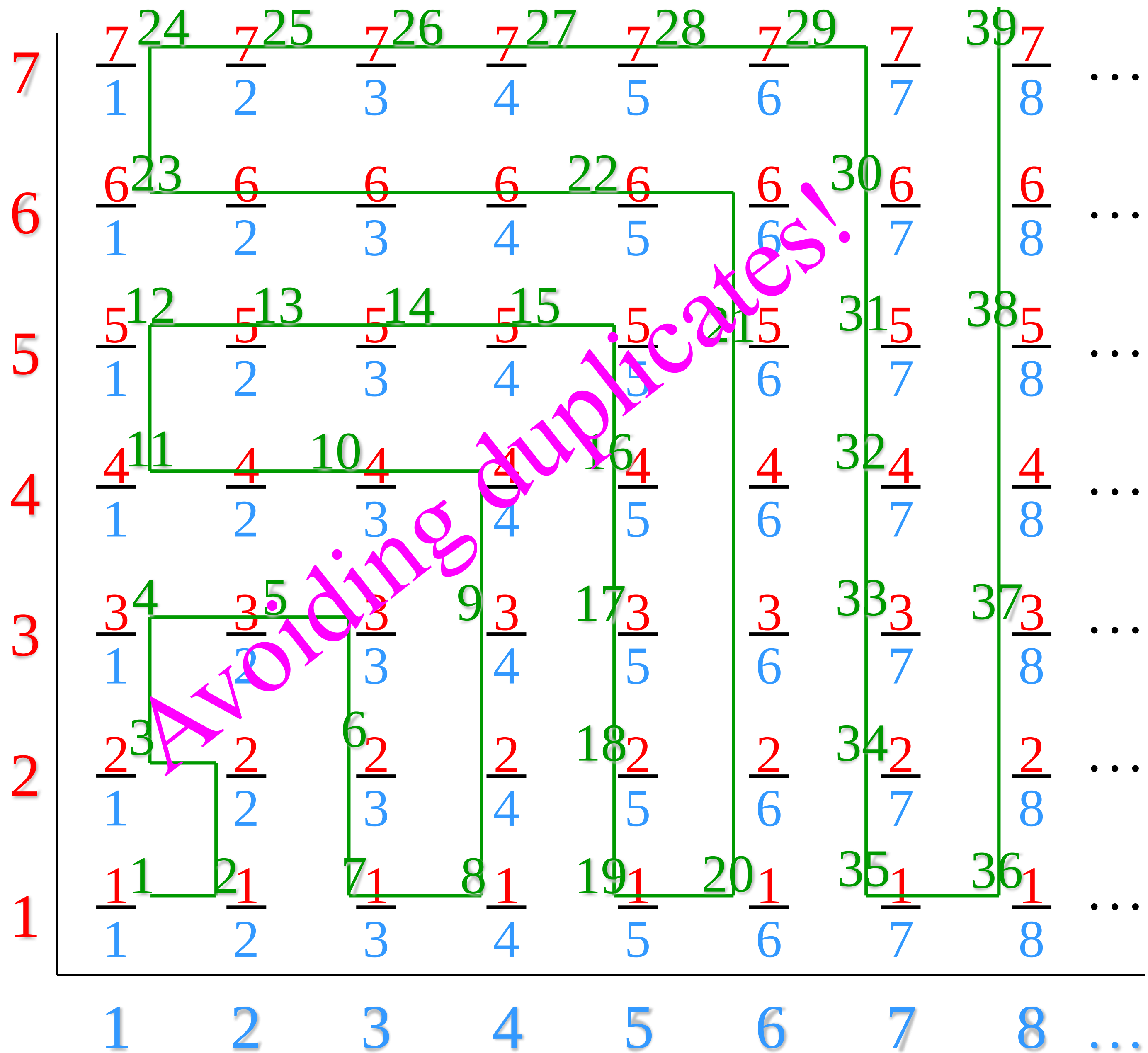
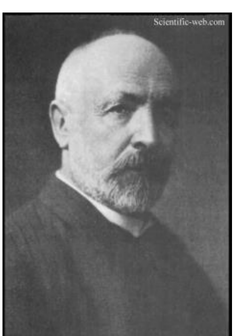
Dovetailing:

Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Q}$

$\Rightarrow |\mathbb{N}|=|\mathbb{Q}|$





Problem: Are there more rationals than natural #'s?

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$\mathbb{N} \neq \mathbb{Q}$

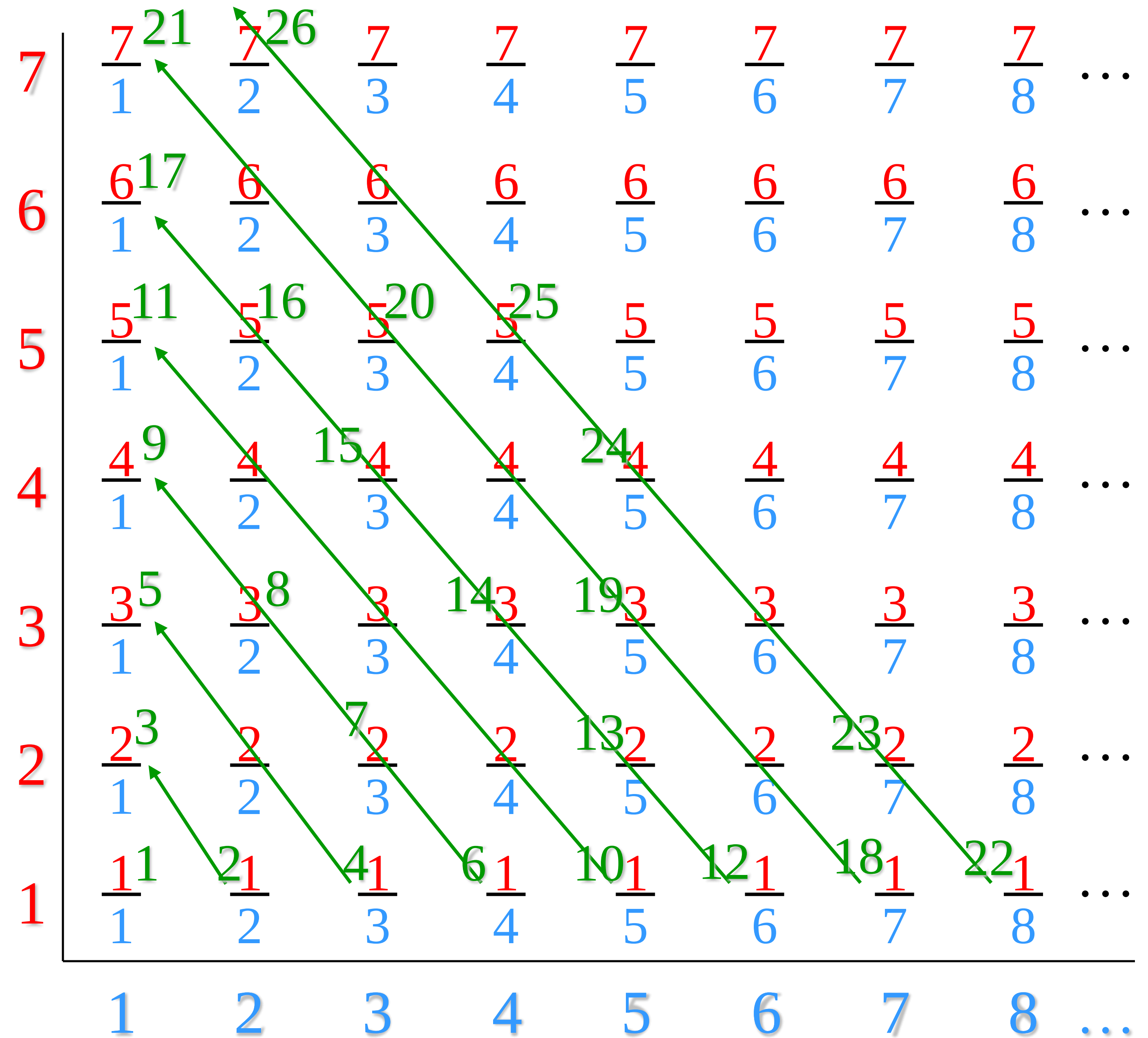
So $|\mathbb{N}|<|\mathbb{Q}|$?

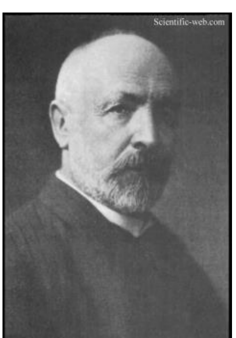
Dovetailing:

Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Q}$

$\Rightarrow |\mathbb{N}|=|\mathbb{Q}|$





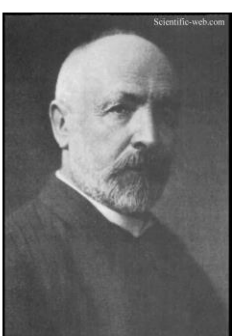
Problem: Why doesn't this "dovetailing" work?

There's no "last" element on the first line!

So the 2nd line is never reached!

$\Rightarrow$ 1-1 function is not defined!

7	$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	…
6	$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	…
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	…
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	…
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	…
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	…
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	…
	1	2	3	4	5	6	7	8	…



Dovetailing Reloaded

Dovetailing: $f:\mathbb{N} \leftrightarrow \mathbb{Z}$

-4	-3	-2	-1	0	1	2	3	4

0	1	2	3	4	5	6	7	8	. . .
-1	-2	-3	-4	-5	-6	-7	-8	-9	. . .

$\mathbb{Z}$									
$\mathbb{N}$	1	2	3	4	5	6	7	8	9

To show $|\mathbb{N}|=|\mathbb{Q}|$ we can construct $f:\mathbb{N}\leftrightarrow\mathbb{Q}$ by sorting x/y by increasing key max(|x|,|y|), while avoiding duplicates:

Dovetailing!

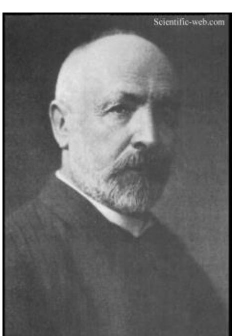
max(|x|,|y|) = 0 : {}

max(|x|,|y|) = 1 : $\overset{1}{0/1}$, $\overset{2}{1/1}$

max(|x|,|y|) = 2 : $\overset{3}{1/2}$, $\overset{4}{2/1}$

max(|x|,|y|) = 3 : $\overset{5}{1/3}$, $\overset{6}{2/3}$, $\overset{7}{3/1}$, $\overset{8}{3/2}$

. . . {finite new set at each step}



- Dovetailing can have many disguises!
- So can diagonalization!

Theorem: There are more reals than rationals / integers.

Proof [Cantor]: Assume a 1-1 correspondence $f: \mathbb{N} \leftrightarrow \mathbb{R}$

i.e., there exists a table containing all of $\mathbb{N}$ and all of $\mathbb{R}$:

$\mathbb{N}$	$\mathbb{R}$
$f(1) =$	3 . 1 4 1 5 9 2 6 5 3 ...
$f(2) =$	1 . 0 0 0 0 0 0 0 0 0 ...
$f(3) =$	2 . 7 1 8 2 8 1 8 2 8 ...
$f(4) =$	1 . 4 1 4 2 1 3 5 6 2 ...
$f(5) =$	0 . 3 3 3 3 3 3 3 3 3 ...
...	...

Diagonalization

Non-existence proof!

$X = 0 . 2\ 1\ 9\ 3\ 4\ \ldots \in \mathbb{R}$

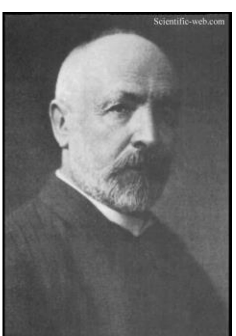
But X is missing from our table! $X \neq f(k)\ \forall\ k \in \mathbb{N}$

$\Rightarrow f$ not a 1-1 correspondence

$\Rightarrow$ contradiction

$\Rightarrow \mathbb{R}$ is not countable!

There are more reals than rationals / integers!



Problem 1: Why not just insert X into the table?

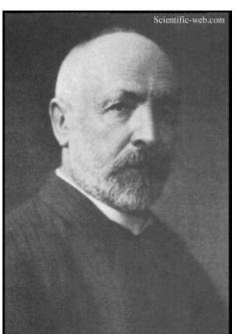
Problem 2: What if X=0.999… but 1.000… is already in table?

Diagonalization

Non-existence proof!

$\mathbb{N}$	$\mathbb{R}$
$f(1)$ =	3 . 1 4 1 5 9 2 6 5 3 …
$f(2)$ =	1 . 0 0 0 0 0 0 0 0 0 …
$f(3)$ =	2 . 7 1 8 2 8 1 8 2 8 …
$f(4)$ =	1 . 4 1 4 2 1 3 5 6 2 …
$f(5)$ =	0 . 3 3 3 3 3 3 3 3 3 …
…	…

X = 0 . 2 1 9 3 4 … $\in \mathbb{R}$



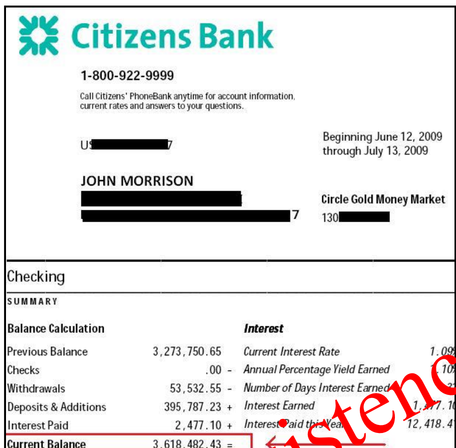
- Table with X inserted will have X' still missing! Inserting X (or any number of X's) will not help!
- To enforce unique table values, we can avoid using 9's and 0's in X.

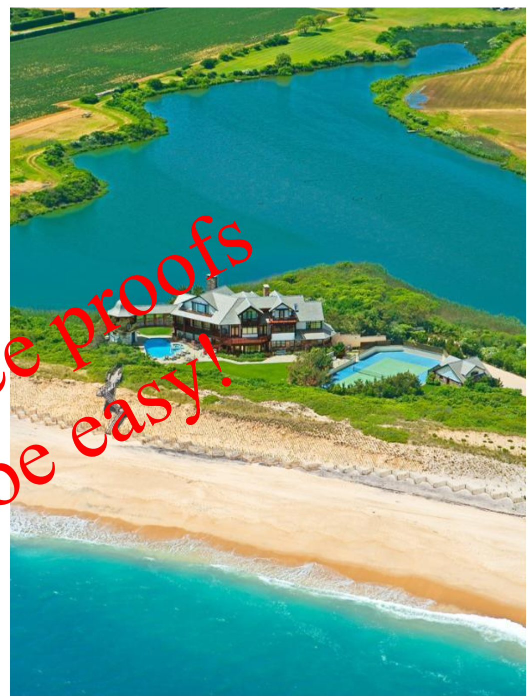

Celebrity Cruises
X
a true departure
WELCOME
TO
INFINITY
Infinity

Non-Existence Proofs

- Must cover all possible (usually infinite) scenarios!
- Examples / counter-examples are not convincing!
- Not "symmetric" to existence proofs!

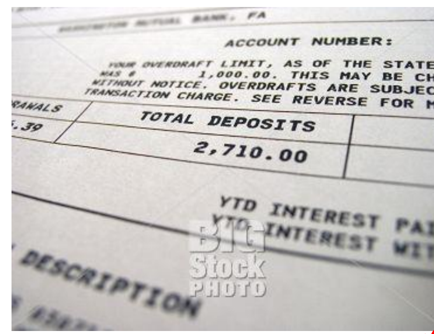
Ex: proofs that you are a millionaire:





Existence proofs can be easy!

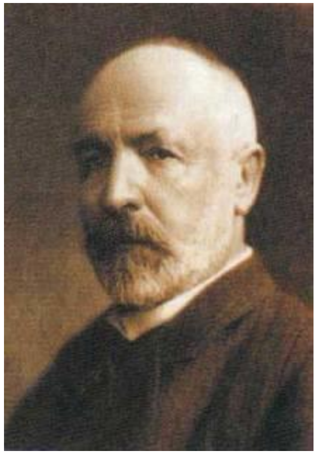
"Proof" that you are not a millionaire ?



Non-existence proofs are often hard!

P≠NP

Cantor set:



Start with unit segment

- Remove (open) middle third
- Repeat recursively on all remaining segments
- Cantor set is all the remaining points

Total length removed: $1/3 + 2/9 + 4/27 + 8/81 + \ldots = 1$

Cantor set does not contain any intervals

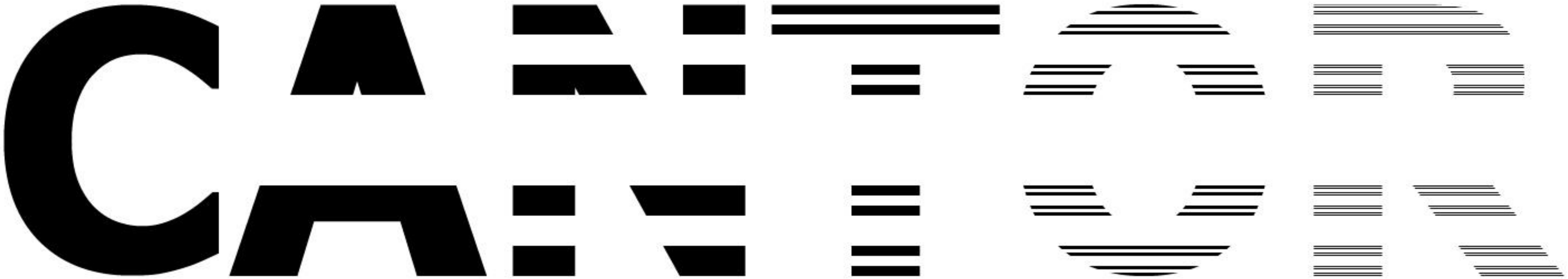
Cantor set is not empty (since, e.g. interval endpoints remain)

An uncountable number of non-endpoints remain as well (e.g., 1/4)

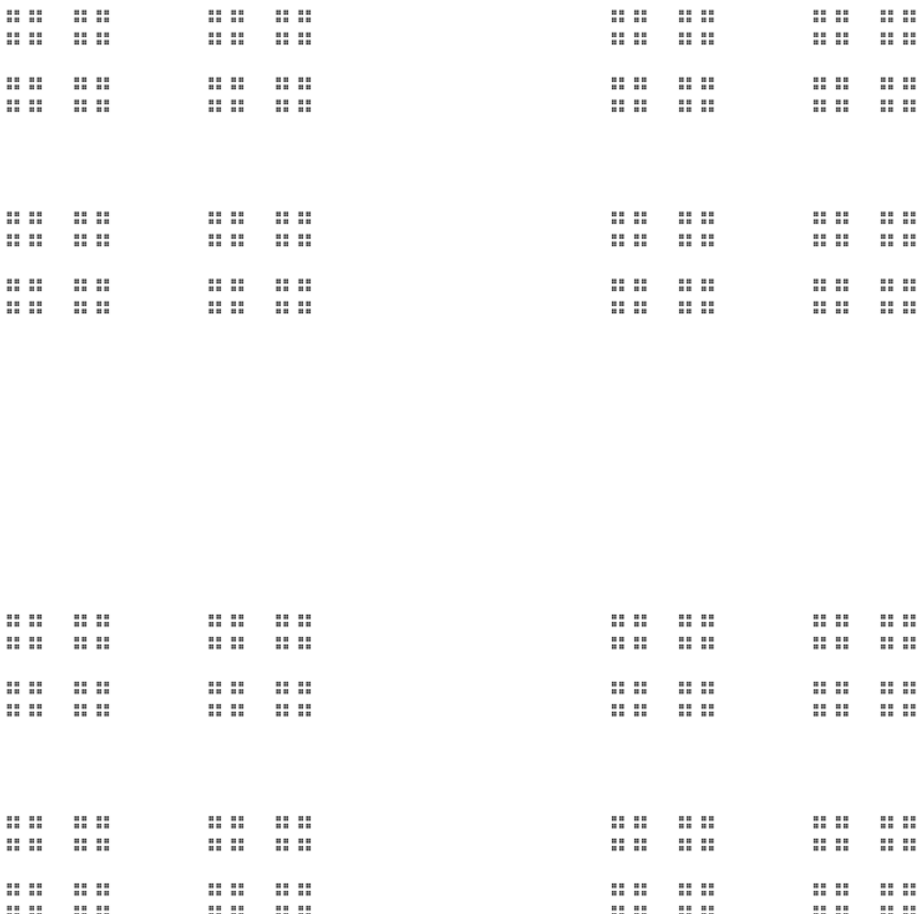
Cantor set is totally disconnected (no nontrivial connected subsets)

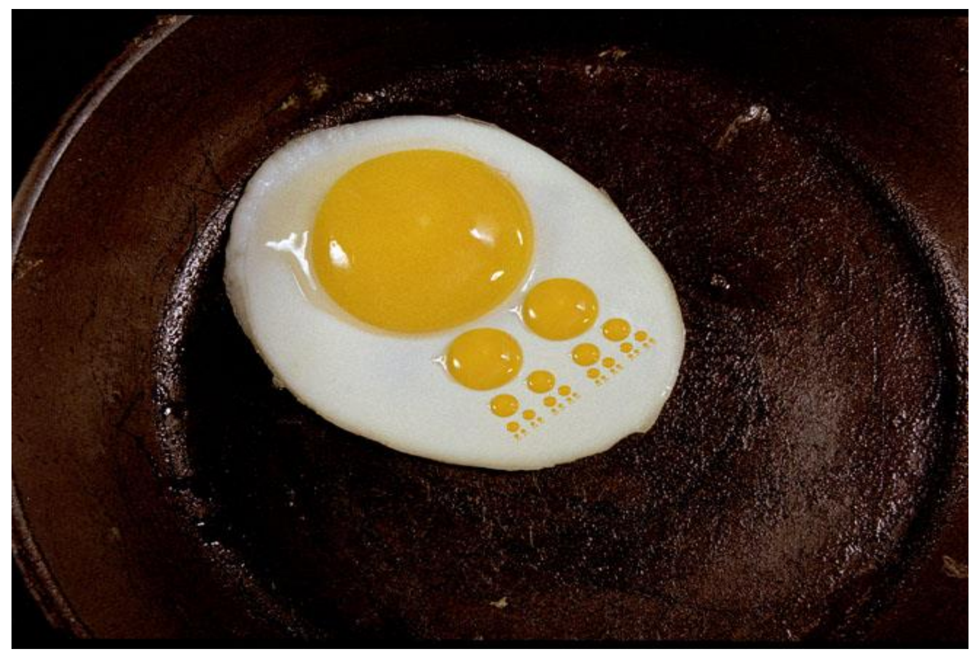
Cantor set is self-similar with Hausdorff dimension of $\log_3 2 = 1.585$

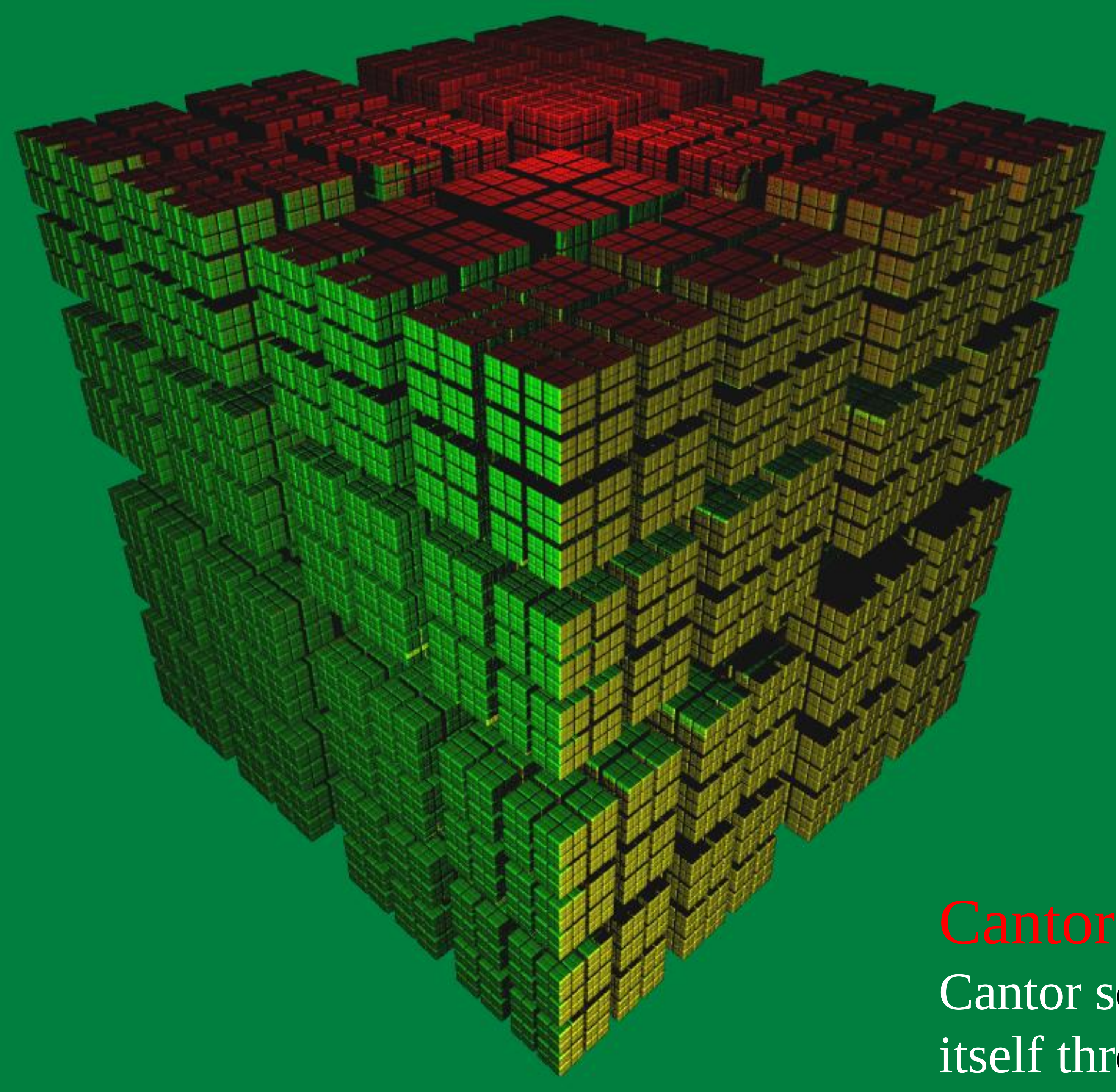
Cantor set is a closed, totally bounded, compact, complete metric space, with uncountable cardinality and lebesque measure zero



Cantor dust (2D generalization): Cantor set crossed with itself







Cantor cube (3D):

Cantor set crossed with itself three times