Arrays and Vectors

Mechanism for representing lists

Lists

- Problem solving often requires information be viewed as a list
  - List may be one-dimensional or multidimensional
- C++ provides two list mechanisms
  - Arrays
    - Traditional and important because of legacy libraries
    - Restrictions on its use
  - Container classes
    - First-class list representation
    - Common containers provided by STL
      - Vector, queue, stack, map, …
    - Preferred long-term programming practice

Lists

- Analogies
  - Egg carton
  - Apartments
  - Cassette carrier

Array Terminology

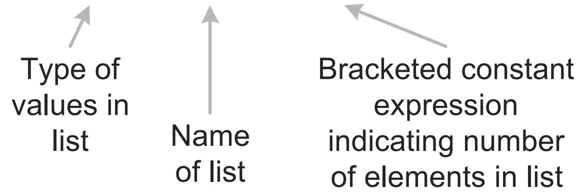
- List is composed of *elements*
- Elements in a list have a *common name*
  - The list as a whole is referenced through the common name
- List elements are of the same type — the *base* type
- Elements of a list are referenced by *subscripting* or *indexing* the common name

C++ Restrictions

- Subscripts are denoted as expressions within brackets: [ ]
- Base type can be any fundamental, library-defined, or programmer-defined type
- The index type is integer and the index range must be 0 ... `n`-1
  - where `n` is a programmer-defined constant expression.
- Parameter passing style
  - Always call by reference (no indication necessary)

Basic Array Definition

```
BaseType Id [ SizeExp ] ;
```



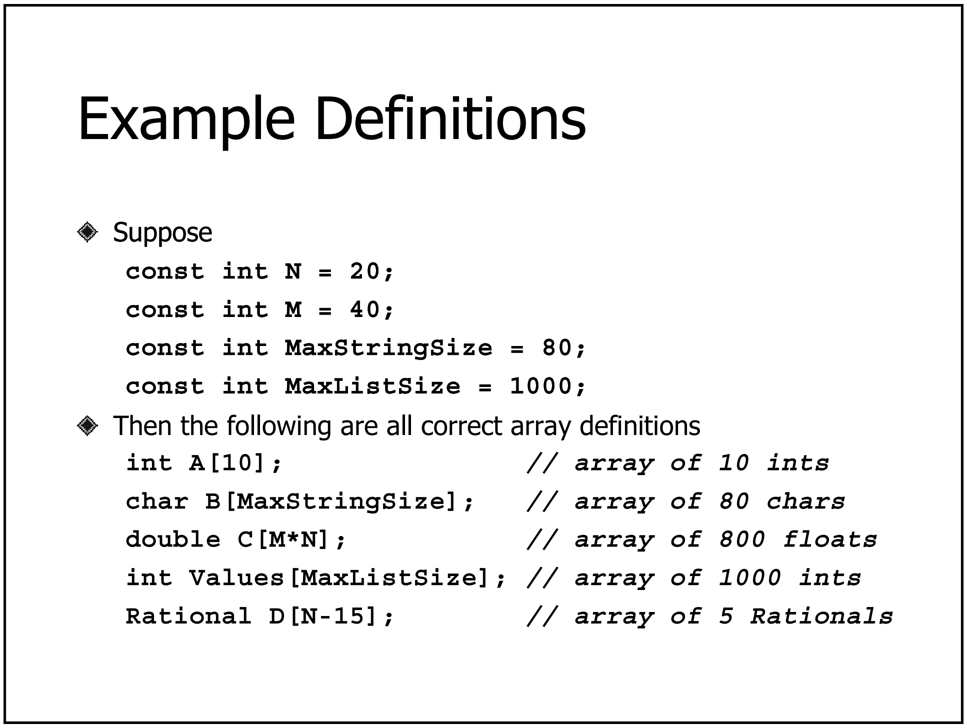
Type of values in list

Name of list

Bracketed constant expression indicating number of elements in list

```
double X [ 100 ] ;

                // Subscripts are 0 through 99
```



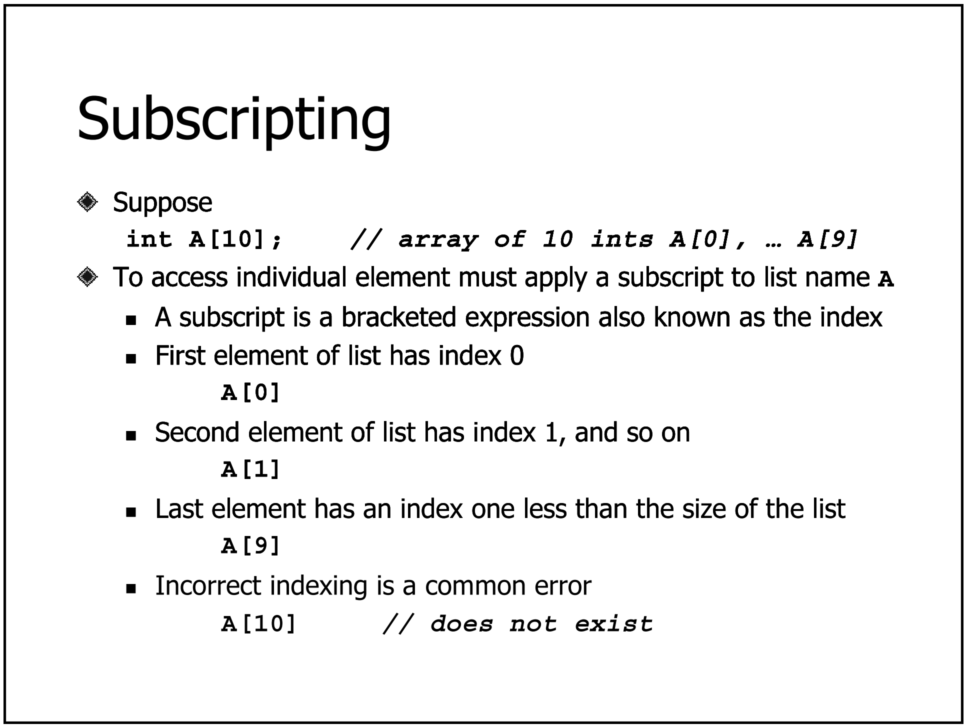
Example Definitions

- Suppose

```
const int N = 20;
const int M = 40;
const int MaxStringSize = 80;
const int MaxListSize = 1000;
```

- Then the following are all correct array definitions

```
int A[10];                // array of 10 ints
char B[MaxStringSize];    // array of 80 chars
double C[M*N];            // array of 800 floats
int Values[MaxListSize]; // array of 1000 ints
Rational D[N-15];         // array of 5 Rationals
```



Subscripting

- Suppose

```
int A[10];     // array of 10 ints A[0], … A[9]
```

- To access individual element must apply a subscript to list name **`A`**
  - A subscript is a bracketed expression also known as the index
  - First element of list has index 0

    ```
    A[0]
    ```

  - Second element of list has index 1, and so on

    ```
    A[1]
    ```

  - Last element has an index one less than the size of the list

    ```
    A[9]
    ```

  - Incorrect indexing is a common error

    ```
    A[10]     // does not exist
    ```

Array Elements

- Suppose

```
int A[10];    // array of 10 uninitialized ints
```

A	--	--	--	--	--	--	--	--	--	--
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]

- To access an individual element we must apply a subscript to list name `A`

Array Element Manipulation

- Consider

```
int i = 7, j = 2, k = 4;
A[0] = 1;
A[i] = 5;
A[j] = A[i] + 3;
A[j+1] = A[i] + A[0];
A[A[j]] = 12;
cin >> A[k]; // where next input value is 3
```

A	--	--	--	--	--	--	--	--	--	--
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]

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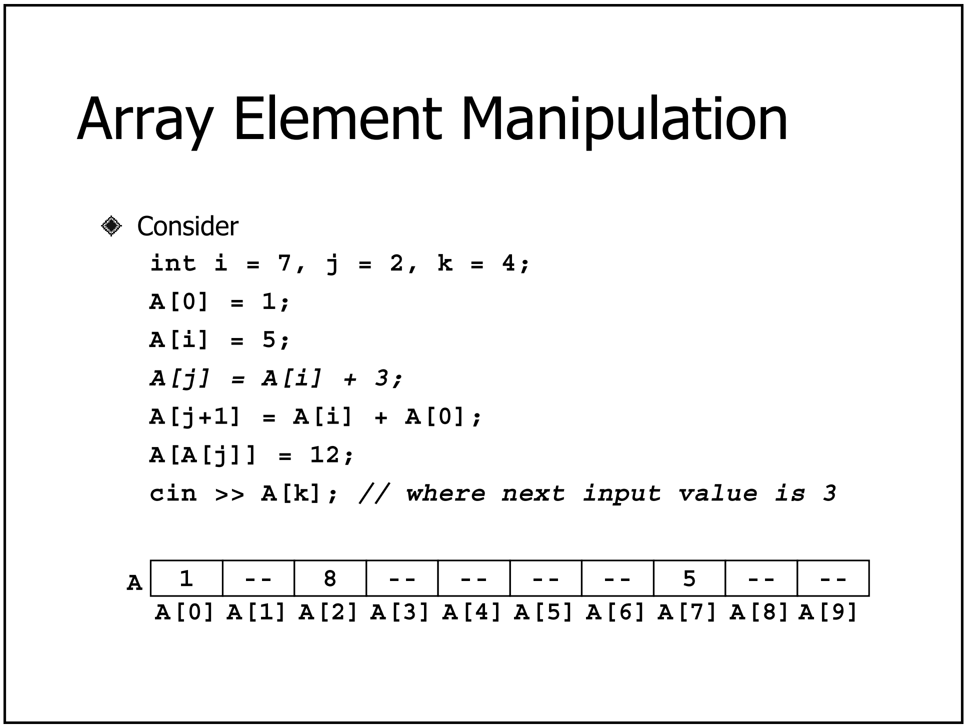
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]



Array Element Manipulation

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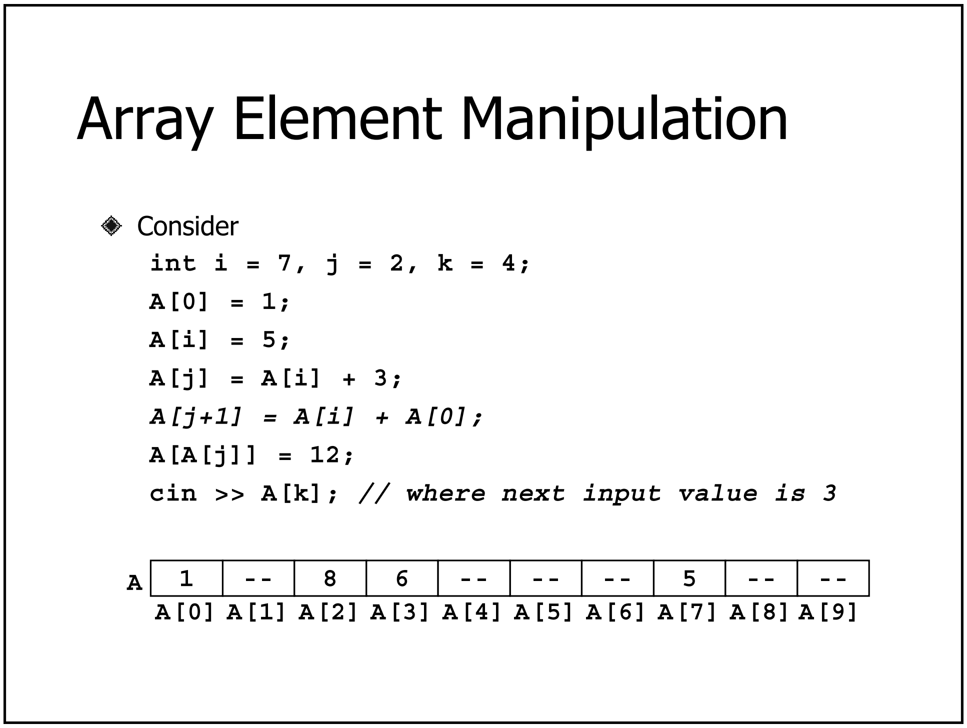
```

A	1	--	8	--	--	--	--	5	--	--
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]



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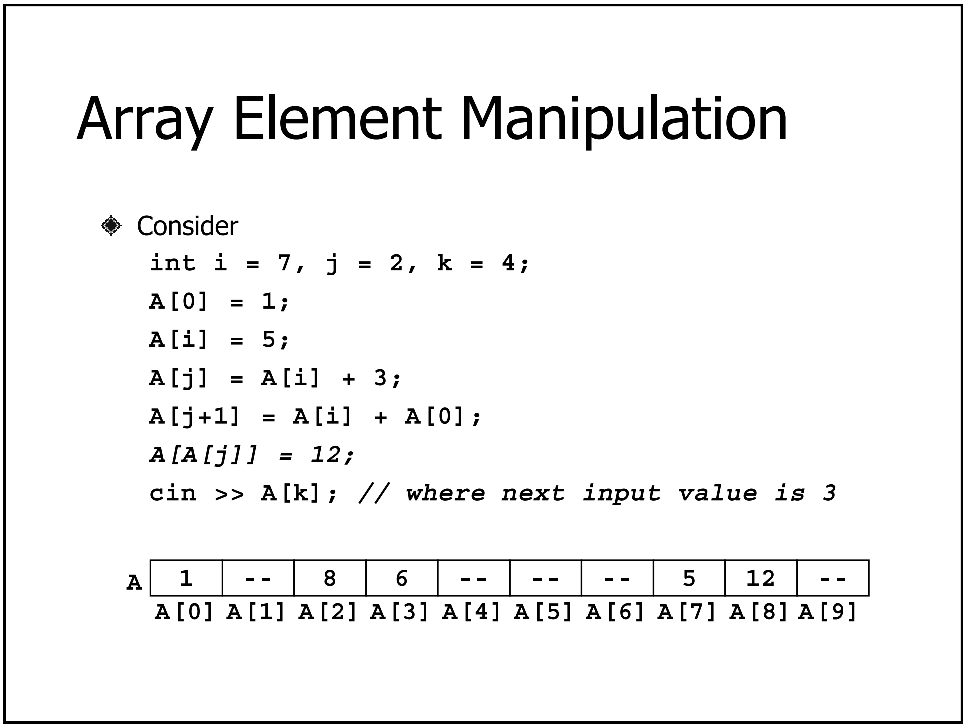
cin >> A[k]; // where next input value is 3
```

A	1	--	8	6	--	--	--	5	--	--
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]



Array Element Manipulation

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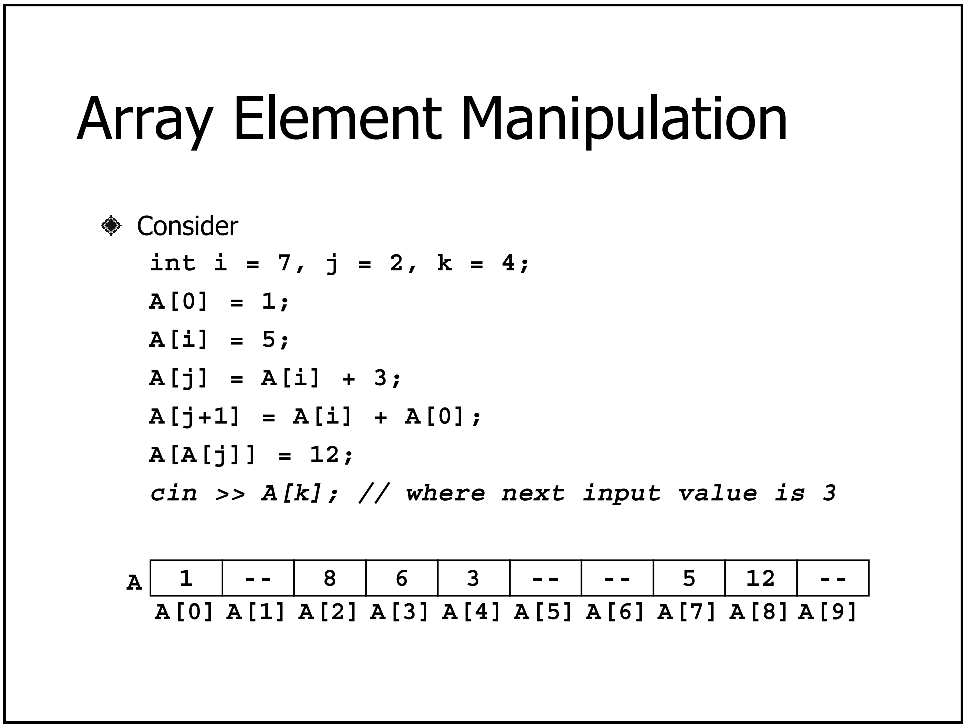
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]



Array Element Manipulation

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```

A	1	--	8	6	3	--	--	5	12	--
	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]

Extracting Values For A List

```
int A[MaxListSize];
int n = 0;
int CurrentInput;
while((n < MaxListSize) && (cin >> CurrentInput)){
  A[n] = CurrentInput;
  ++n;
}
```

Displaying A List

```
// List A of n elements has already been set
for (int i = 0; i < n; ++i) {
   cout << A[i] << " ";
}
cout << endl;
```

Smallest Value

- Problem
  - Find the smallest value in a list of integers
- Input
  - A list of integers and a value indicating the number of integers
- Output
  - Smallest value in the list
- Note
  - List remains unchanged after finding the smallest value!

Preliminary Design

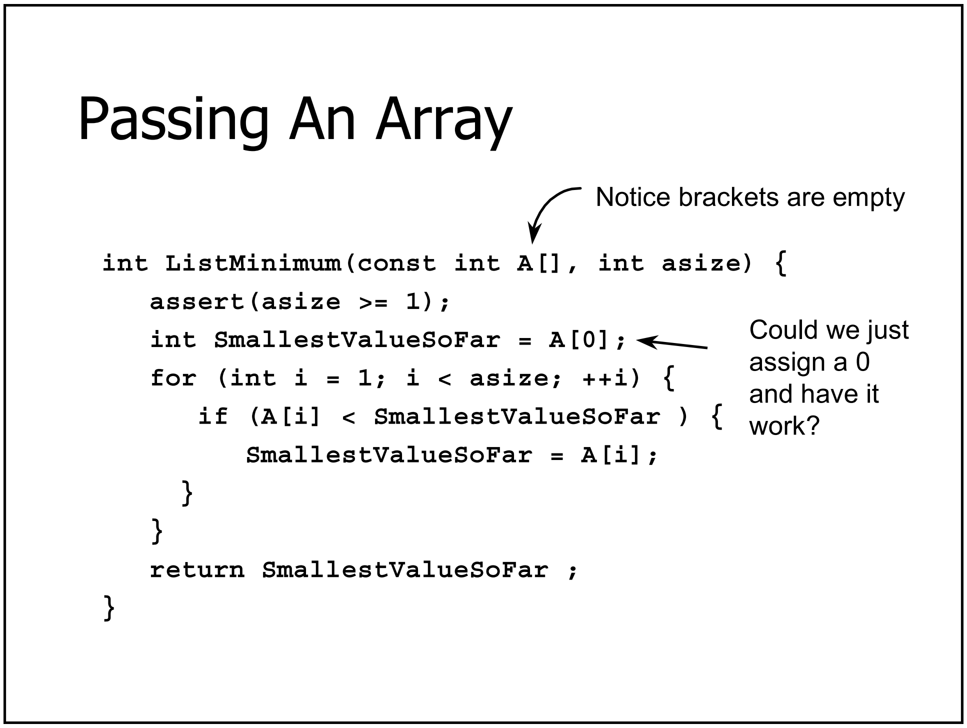
- Realizations
  - When looking for value with distinguishing characteristics, need a way of remembering best candidate found so far
    - Make it a function -- likely to be used often
- Design
  - Search array looking for smallest value
    - Use a loop to consider each element in turn
    - If current element is smallest so far, then update smallest value so far candidate
  - When done examining all of the elements, the smallest value seen so far is the smallest value

Necessary Information

- Information to be maintained
  - Array with values to be inspected for smallest value
  - Number of values in array
  - Index of current element being considered
  - Smallest value so far

A More Detailed Design

- Solution
  - Function that takes array of values and array size as its two in parameters; returns smallest value seen as its value
  - Initialize smallest value so far to first element
  - For each of the other elements in the array in turn
    - If it is smaller than the smallest value so far, update the value of the smallest value so far to be the current element
  - Return smallest value seen as value of function

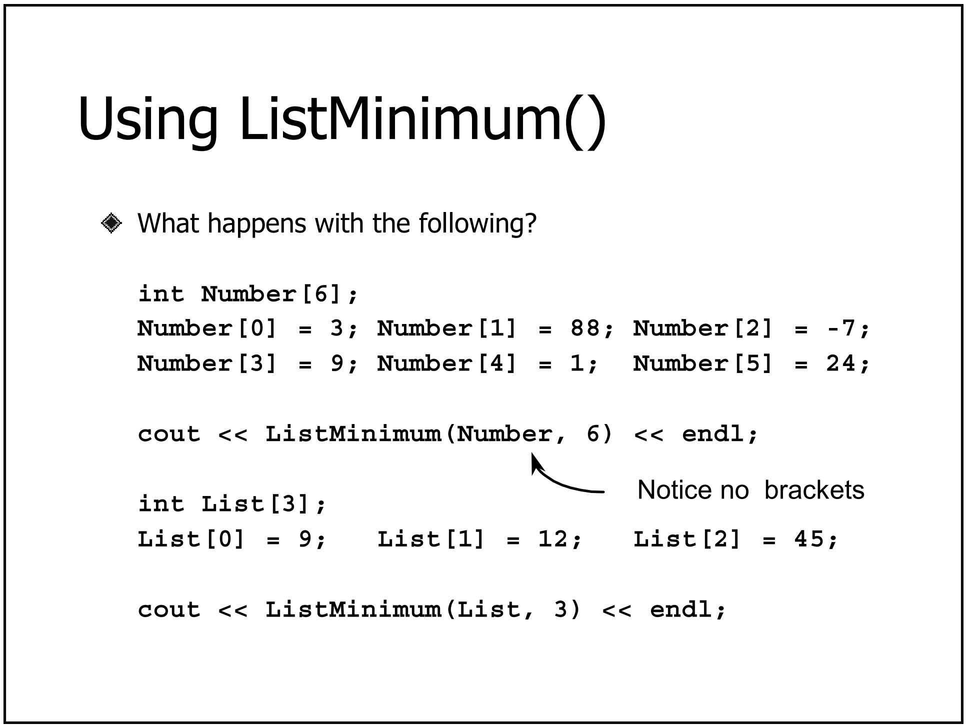


Passing An Array

Notice brackets are empty

```
int ListMinimum(const int A[], int asize) {
   assert(asize >= 1);
   int SmallestValueSoFar = A[0];
   for (int i = 1; i < asize; ++i) {
      if (A[i] < SmallestValueSoFar ) {
         SmallestValueSoFar = A[i];
     }
   }
   return SmallestValueSoFar ;
}
```

Could we just assign a 0 and have it work?



Using ListMinimum()

- What happens with the following?

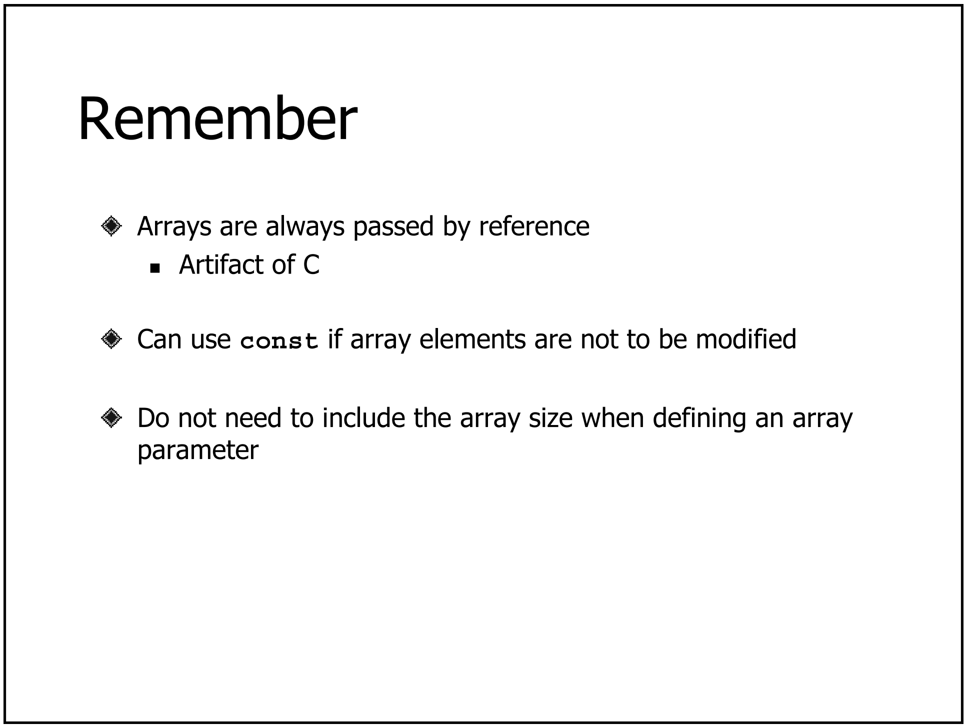
```
int Number[6];
Number[0] = 3; Number[1] = 88; Number[2] = -7;
Number[3] = 9; Number[4] = 1;  Number[5] = 24;

cout << ListMinimum(Number, 6) << endl;

int List[3];
List[0] = 9;   List[1] = 12;   List[2] = 45;

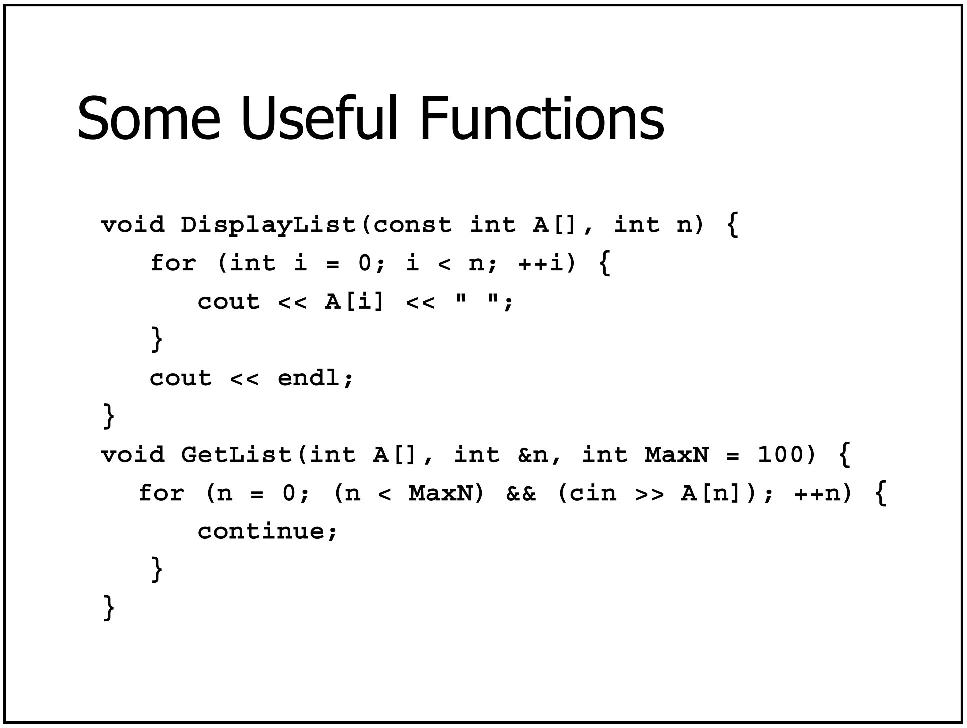
cout << ListMinimum(List, 3) << endl;
```

Notice no brackets



Remember

- Arrays are always passed by reference
  - Artifact of C
- Can use `const` if array elements are not to be modified
- Do not need to include the array size when defining an array parameter



Some Useful Functions

```
void DisplayList(const int A[], int n) {
   for (int i = 0; i < n; ++i) {
      cout << A[i] << " ";
   }
   cout << endl;
}
void GetList(int A[], int &n, int MaxN = 100) {
   for (n = 0; (n < MaxN) && (cin >> A[n]); ++n) {
      continue;
   }
}
```

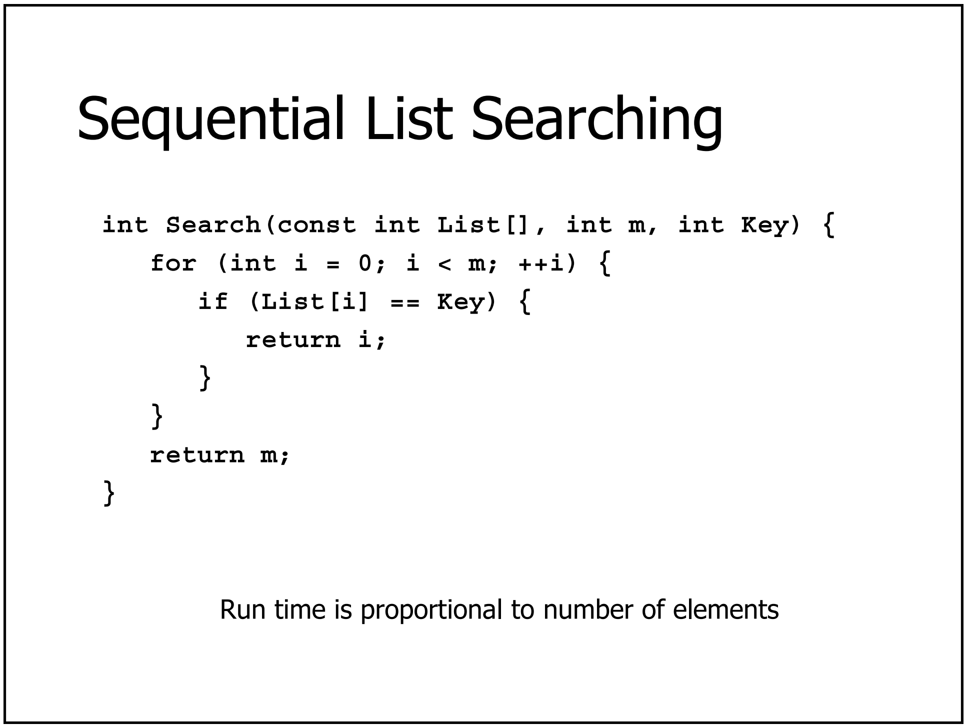
Useful Functions Being Used

```
const int MaxNumberValues = 25;
int Values[MaxNumberValues];
int NumberValues;

GetList(Values, NumberValues, MaxNumberValues);
DisplayList(Values, NumberValues);
```

Searching

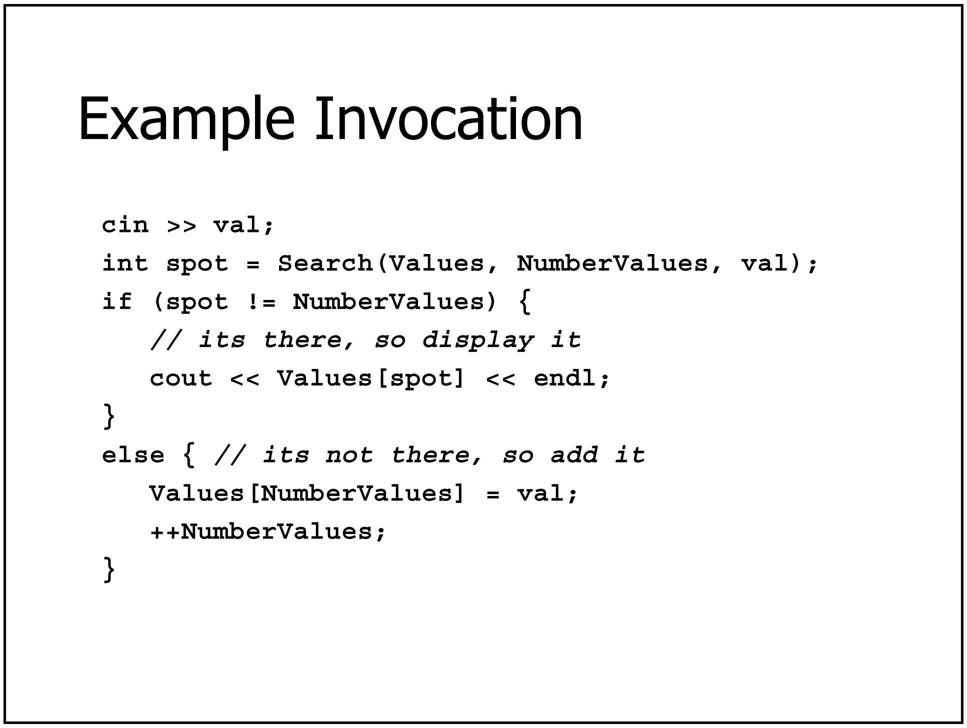
- Problem
  - Determine whether a value key is one of the element values
- Does it matter if
  - Element values are not necessarily numbers
  - Element values are not necessarily unique
  - Elements may have key values and other fields



Sequential List Searching

```
int Search(const int List[], int m, int Key) {
   for (int i = 0; i < m; ++i) {
      if (List[i] == Key) {
         return i;
      }
   }
   return m;
}
```

Run time is proportional to number of elements



Example Invocation

```
cin >> val;
int spot = Search(Values, NumberValues, val);
if (spot != NumberValues) {
   // its there, so display it
   cout << Values[spot] << endl;
}
else { // its not there, so add it
   Values[NumberValues] = val;
   ++NumberValues;
}
```

Sorting

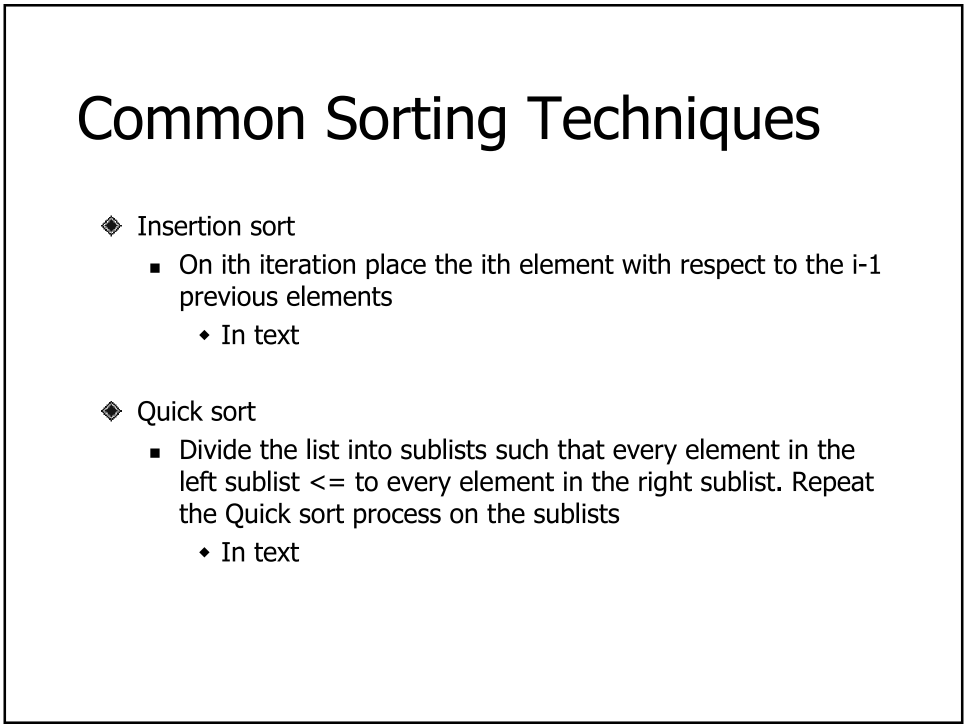
- Problem
  - Arranging elements so that they are ordered according to some desired scheme
    - Standard is non-decreasing order
      - Why don't we say increasing order?

- Major tasks
  - Comparisons of elements
  - Updates or element movement

Common Sorting Techniques

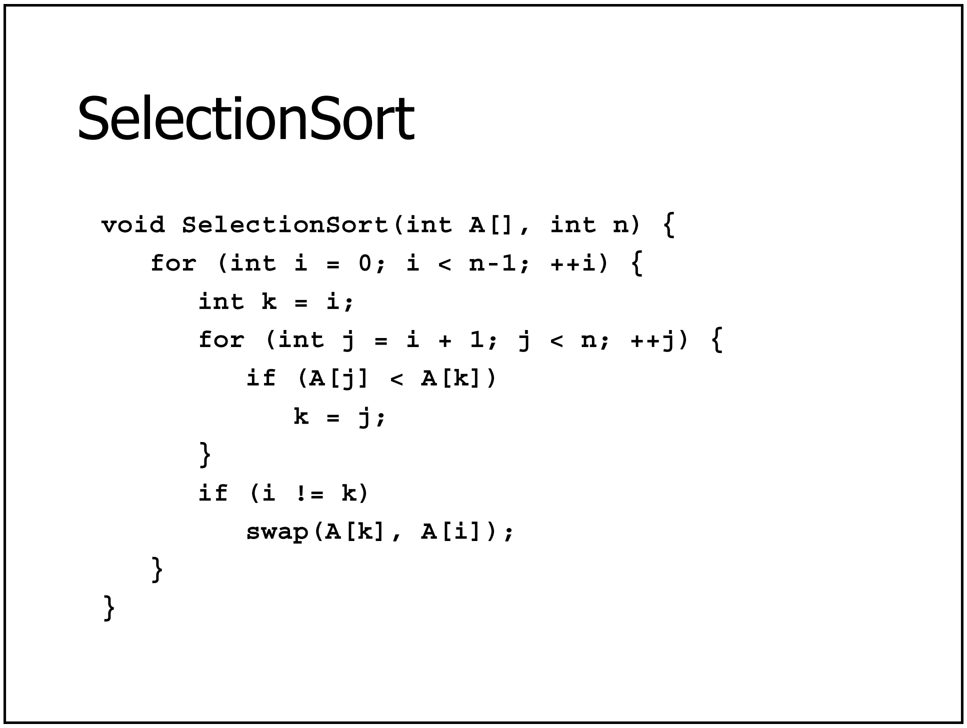
- Selection sort
  - On ith iteration place the ith smallest element in the ith list location

- Bubble sort
  - Iteratively pass through the list and examining adjacent pairs of elements and if necessary swap them to put them in order. Repeat the process until no swaps are necessary



Common Sorting Techniques

- Insertion sort
  - On ith iteration place the ith element with respect to the i-1 previous elements
    - In text
- Quick sort
  - Divide the list into sublists such that every element in the left sublist <= to every element in the right sublist. Repeat the Quick sort process on the sublists
    - In text



SelectionSort

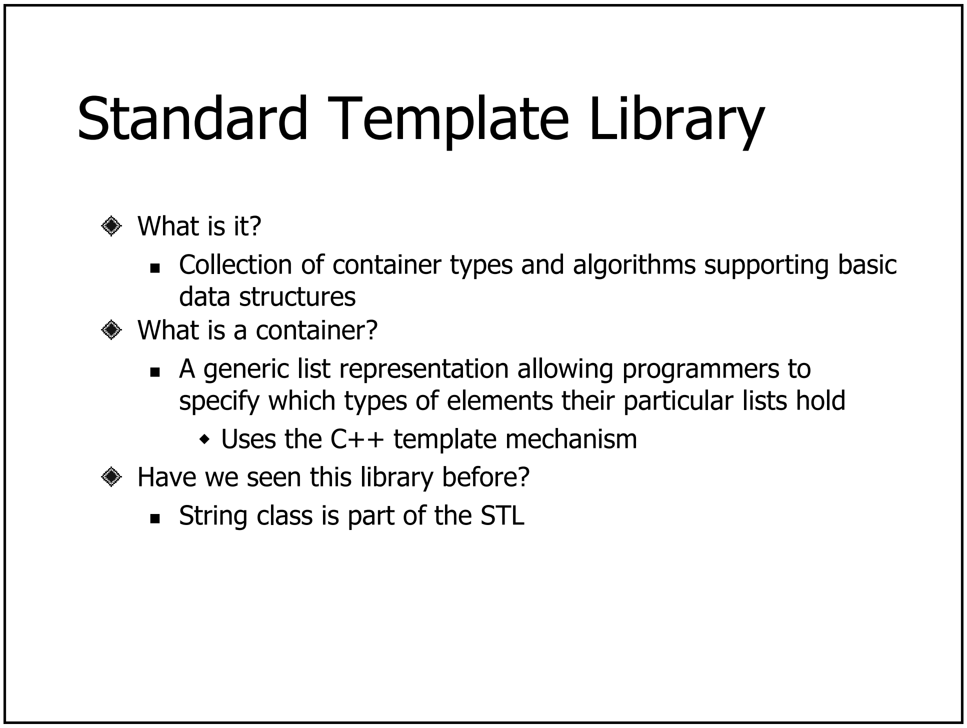
```
void SelectionSort(int A[], int n) {
   for (int i = 0; i < n-1; ++i) {
      int k = i;
      for (int j = i + 1; j < n; ++j) {
         if (A[j] < A[k])
            k = j;
      }
      if (i != k)
         swap(A[k], A[i]);
   }
}
```

Complexity

- SelectionSort() Question
  - How long does the function take to run
    - Proportional to n*n time units, where n is the number of elements in the list
- General question
  - How fast can we sort using the perfect comparison-based method
    - The best possible worst case time is proportional to n log n time units

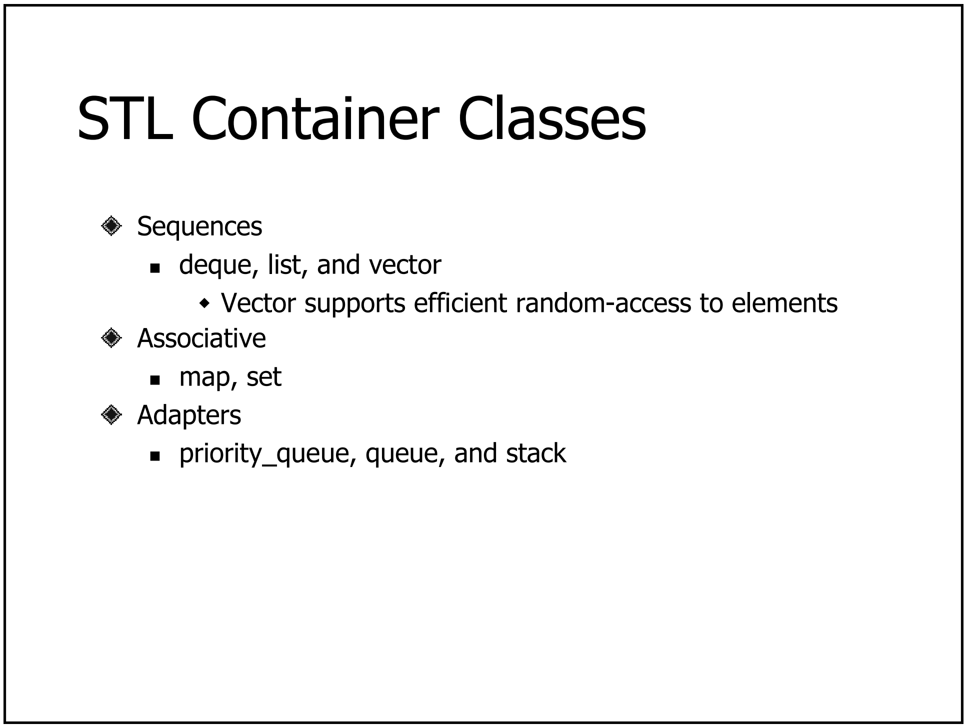
Vectors

First-class mechanism for representing lists



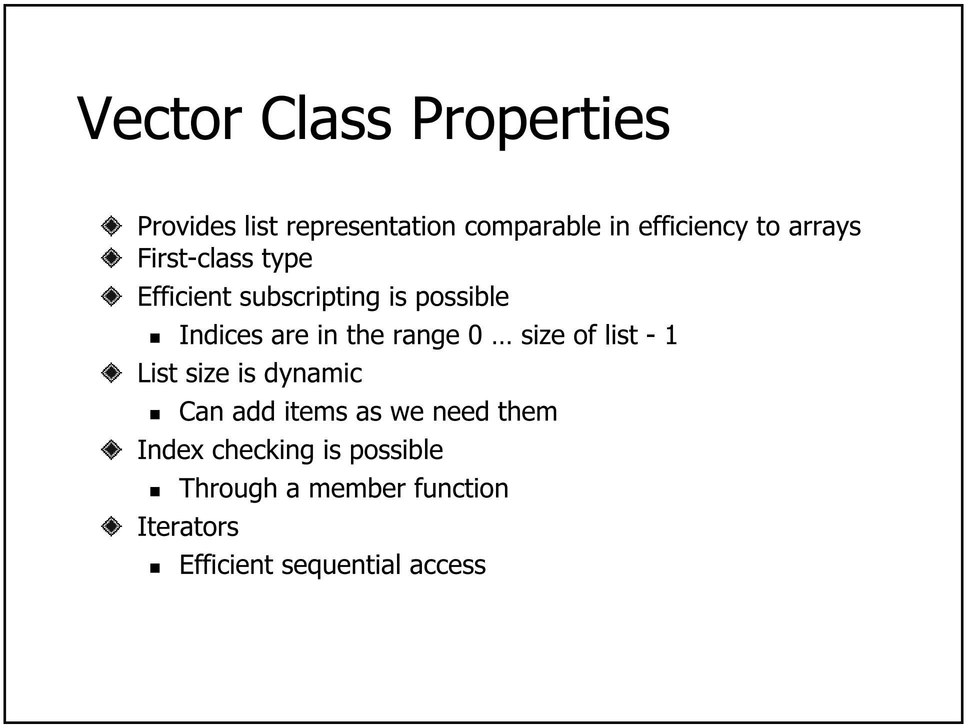
Standard Template Library

- What is it?
  - Collection of container types and algorithms supporting basic data structures
- What is a container?
  - A generic list representation allowing programmers to specify which types of elements their particular lists hold
    - Uses the C++ template mechanism
- Have we seen this library before?
  - String class is part of the STL



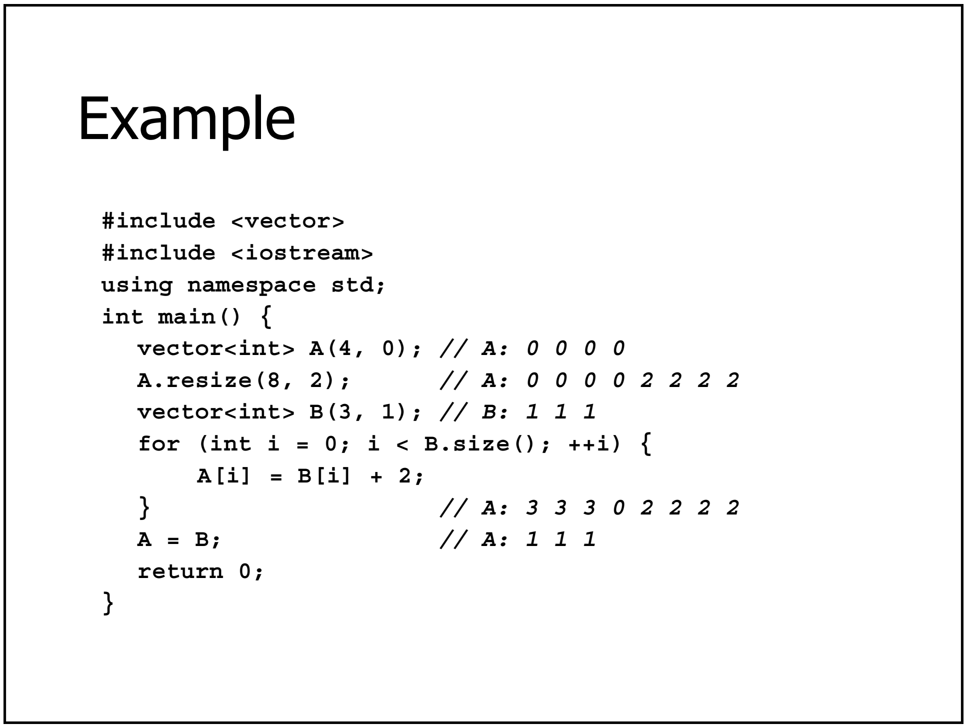
STL Container Classes

- Sequences
  - deque, list, and vector
    - Vector supports efficient random-access to elements
- Associative
  - map, set
- Adapters
  - priority_queue, queue, and stack



Vector Class Properties

- Provides list representation comparable in efficiency to arrays
- First-class type
- Efficient subscripting is possible
  - Indices are in the range 0 … size of list - 1
- List size is dynamic
  - Can add items as we need them
- Index checking is possible
  - Through a member function
- Iterators
  - Efficient sequential access



Example

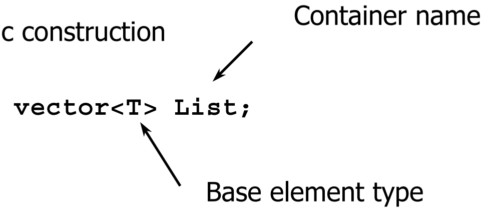
```
#include <vector>
#include <iostream>
using namespace std;
int main() {
   vector<int> A(4, 0); // A: 0 0 0 0
   A.resize(8, 2);      // A: 0 0 0 0 2 2 2 2
   vector<int> B(3, 1); // B: 1 1 1
   for (int i = 0; i < B.size(); ++i) {
       A[i] = B[i] + 2;
   }                    // A: 3 3 3 0 2 2 2 2
   A = B;               // A: 1 1 1
   return 0;
}
```

Some Vector Constructors

- `vector()`
  - The default constructor creates a vector of zero length
- `vector(size_type n, const T &val = T())`
  - *Explicit* constructor creates a vector of length `n` with each element initialized to `val`
- `vector(const T &V)`
  - The copy constructor creates a vector that is a duplicate of vector `V`.
    - Shallow copy!

Construction

- Basic construction



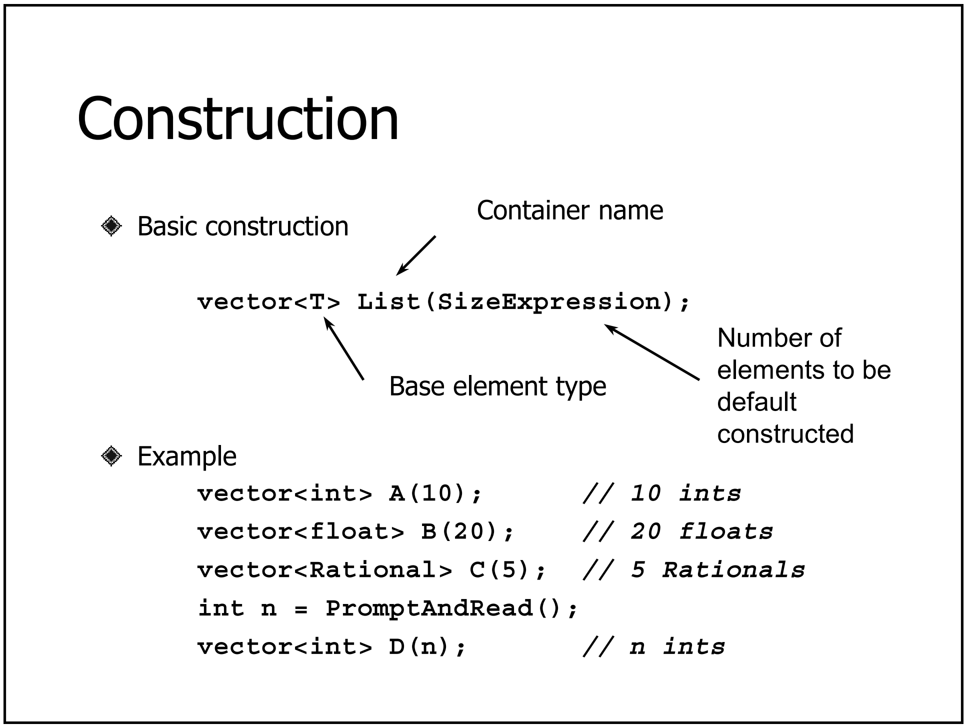
Container name

```
vector<T> List;
```

Base element type

- Example

```
vector<int> A;          // 0 ints
vector<float> B;        // 0 floats
vector<Rational> C;     // 0 Rationals
```



Construction

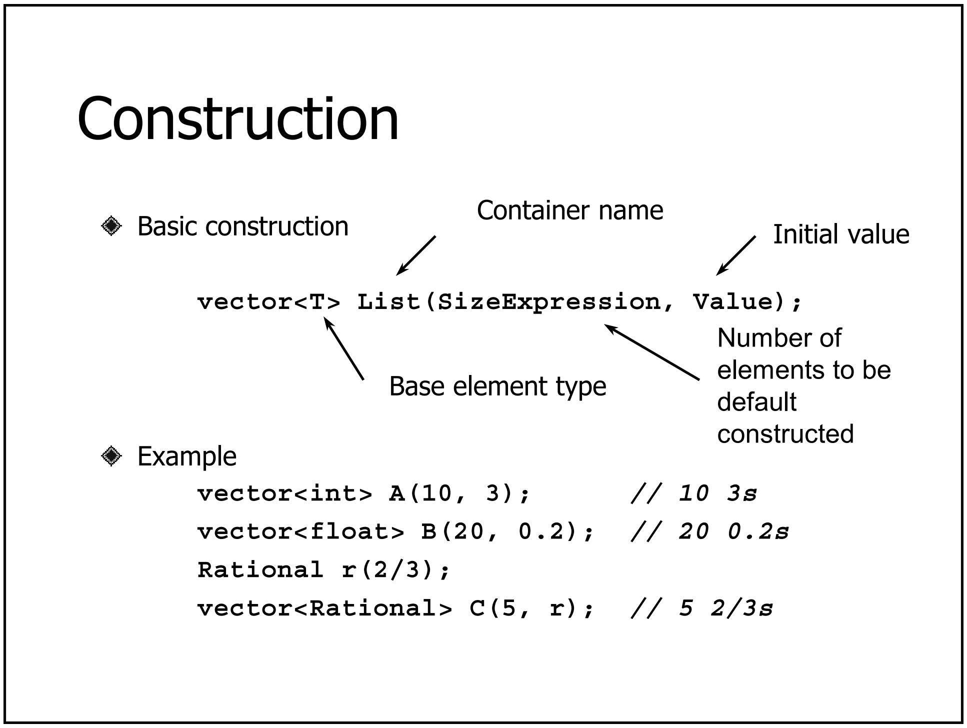
- Basic construction

```
vector<T> List(SizeExpression);
```

Container name
Base element type
Number of elements to be default constructed

- Example

```
vector<int> A(10);       // 10 ints
vector<float> B(20);     // 20 floats
vector<Rational> C(5);   // 5 Rationals
int n = PromptAndRead();
vector<int> D(n);        // n ints
```



Construction

- Basic construction

```
vector<T> List(SizeExpression, Value);
```

Container name
Initial value
Base element type
Number of elements to be default constructed

- Example

```
vector<int> A(10, 3);       // 10 3s
vector<float> B(20, 0.2);   // 20 0.2s
Rational r(2/3);
vector<Rational> C(5, r);   // 5 2/3s
```

Vector Interface

- **`size_type size() const`**
  - Returns the number of elements in the vector

```
        cout << A.size();     // display 3
```

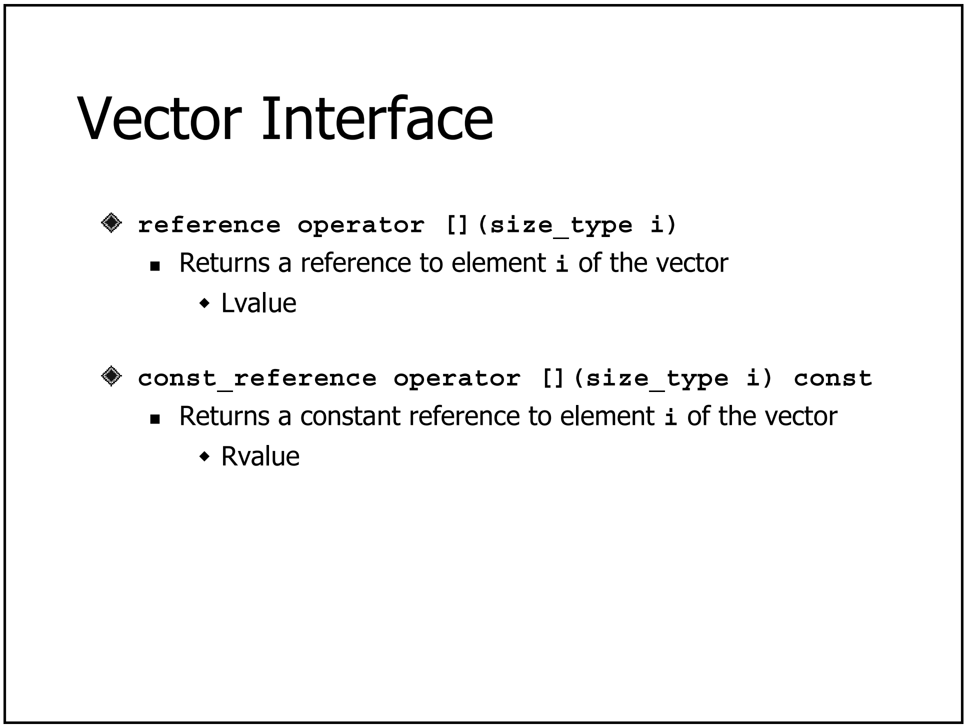
- **`bool empty() const`**
  - Returns true if there are no elements in the vector; otherwise, it returns false

```
if (A.empty()) {
    // ...
```

Vector Interface

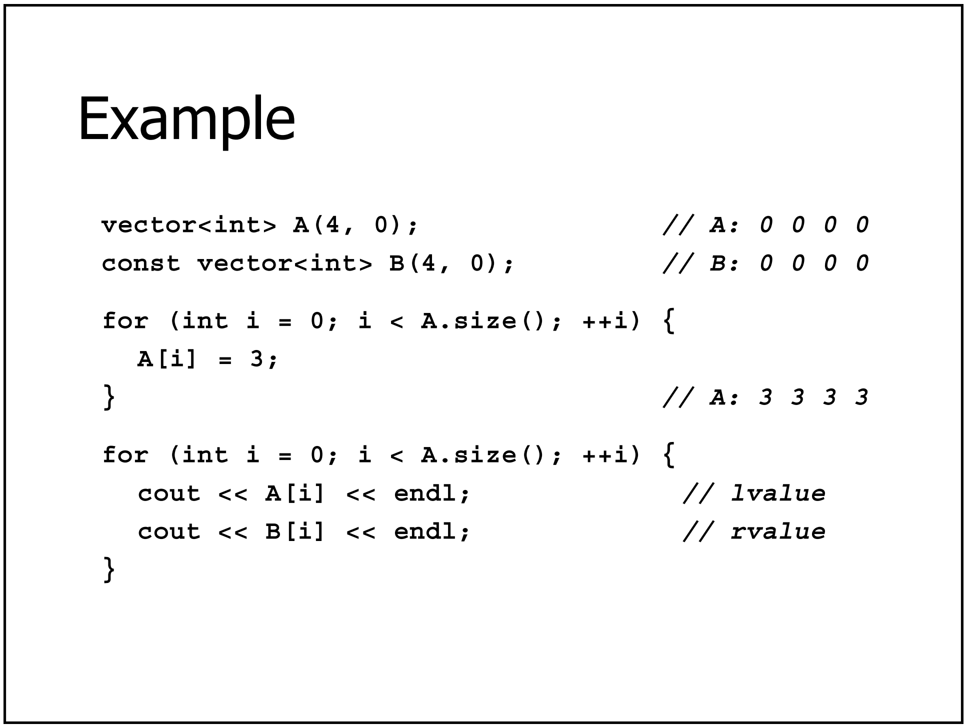
- **`vector<T>& operator = (const vector<T> &V)`**
  - The member assignment operator makes its vector representation an exact duplicate of vector V.
    - Shallow copy
  - The modified vector is returned

```
vector<int> A(4, 0); // A: 0 0 0 0
vector<int> B(3, 1); // B: 1 1 1
A = B;               // A: 1 1 1
```



Vector Interface

- **`reference operator [](size_type i)`**
  - Returns a reference to element `i` of the vector
    - Lvalue
- **`const_reference operator [](size_type i) const`**
  - Returns a constant reference to element `i` of the vector
    - Rvalue

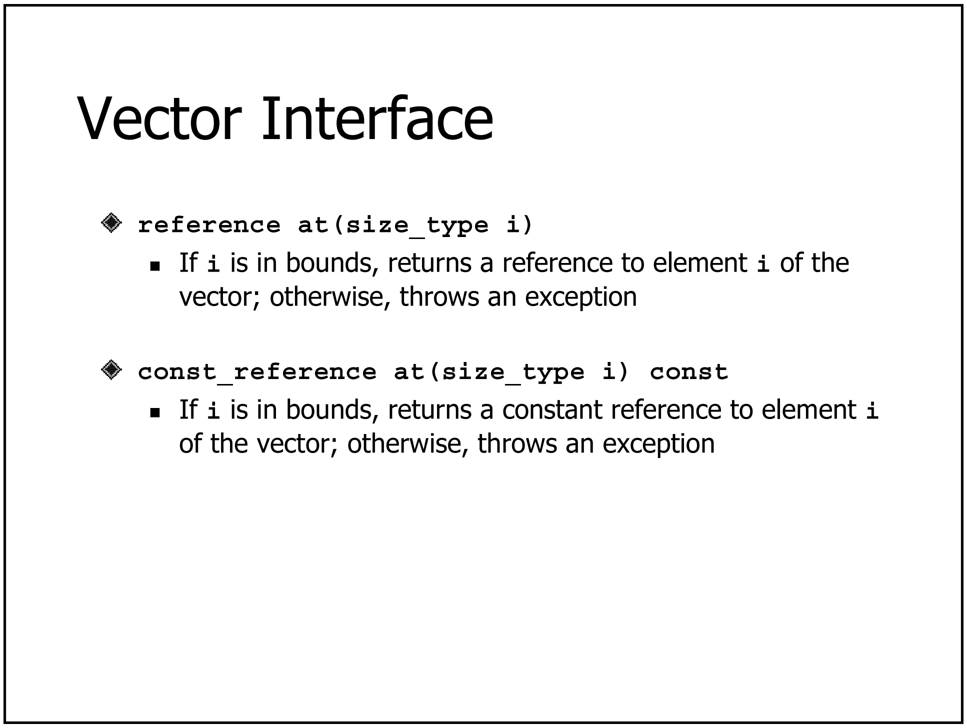


Example

```
vector<int> A(4, 0);              // A: 0 0 0 0
const vector<int> B(4, 0);        // B: 0 0 0 0

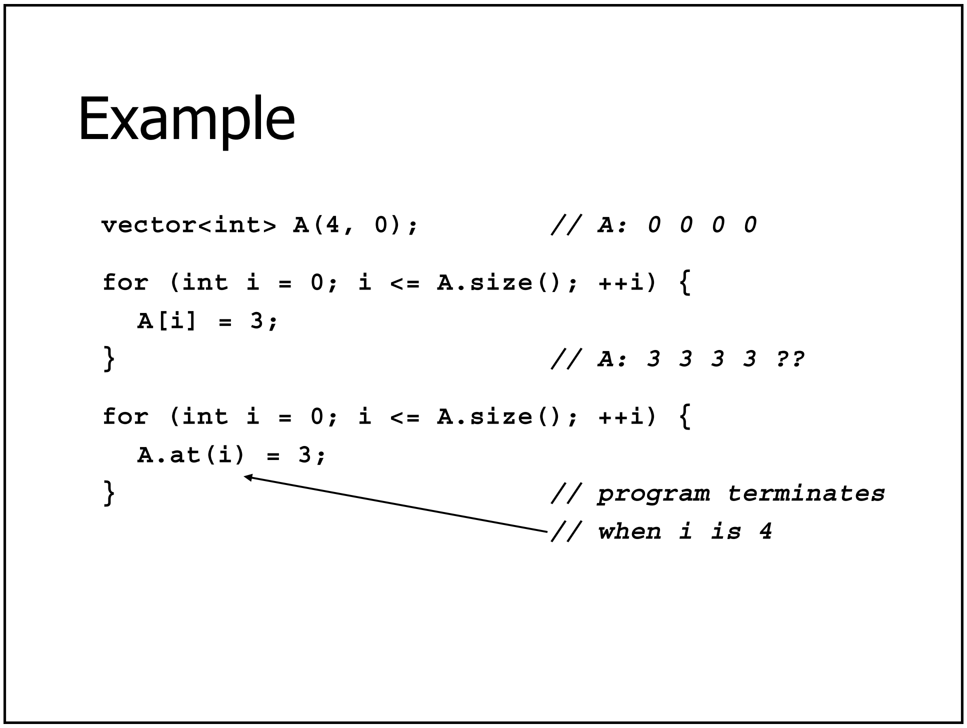
for (int i = 0; i < A.size(); ++i) {
  A[i] = 3;
}                                 // A: 3 3 3 3

for (int i = 0; i < A.size(); ++i) {
  cout << A[i] << endl;            // lvalue
  cout << B[i] << endl;            // rvalue
}
```



Vector Interface

- **`reference at(size_type i)`**
  - If `i` is in bounds, returns a reference to element `i` of the vector; otherwise, throws an exception
- **`const_reference at(size_type i) const`**
  - If `i` is in bounds, returns a constant reference to element `i` of the vector; otherwise, throws an exception



Example

```
vector<int> A(4, 0);          // A: 0 0 0 0

for (int i = 0; i <= A.size(); ++i) {
  A[i] = 3;
}                             // A: 3 3 3 3 ??

for (int i = 0; i <= A.size(); ++i) {
  A.at(i) = 3;
}                             // program terminates
                              // when i is 4
```

Vector Interface

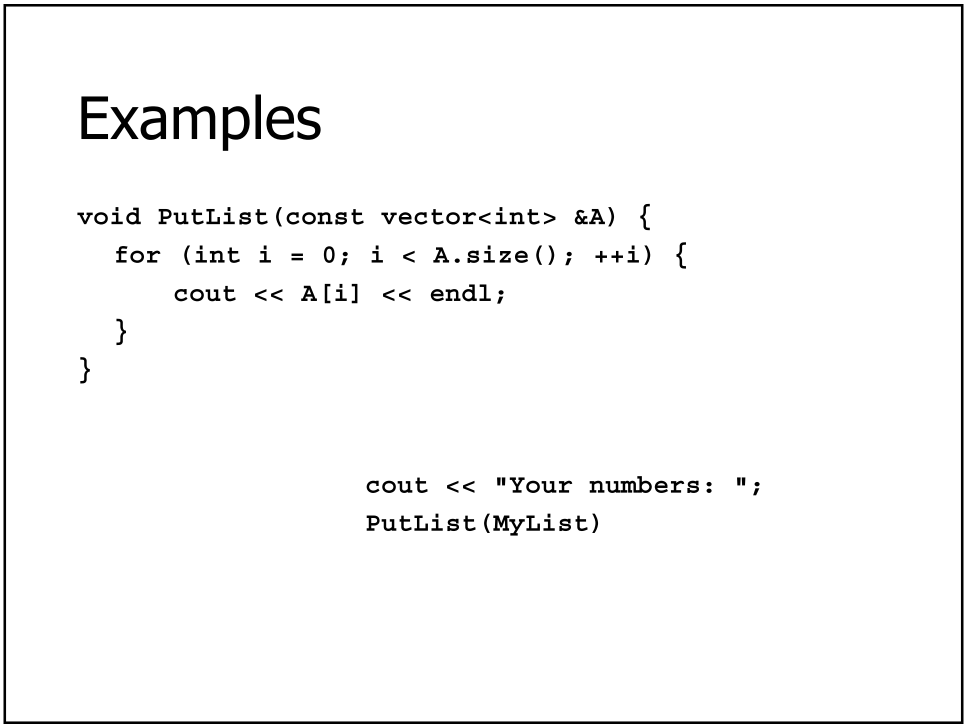
- `void resize(size_type s, T val = T())`
  - The number of elements in the vector is now `s`.
    - To achieve this size, elements are deleted or added as necessary
      - Deletions if any are performed at the end
      - Additions if any are performed at the end
      - New elements have value `val`

```
vector<int> A(4, 0); // A: 0 0 0 0
A.resize(8, 2);      // A: 0 0 0 0 2 2 2 2
A.resize(3,1);       // A: 0 0 0
```

Function Examples

```
void GetList(vector<int> &A) {
  int n = 0;
  while ((n < A.size()) && (cin >> A[n])) {
     ++n;
  }
  A.resize(n);
}
```

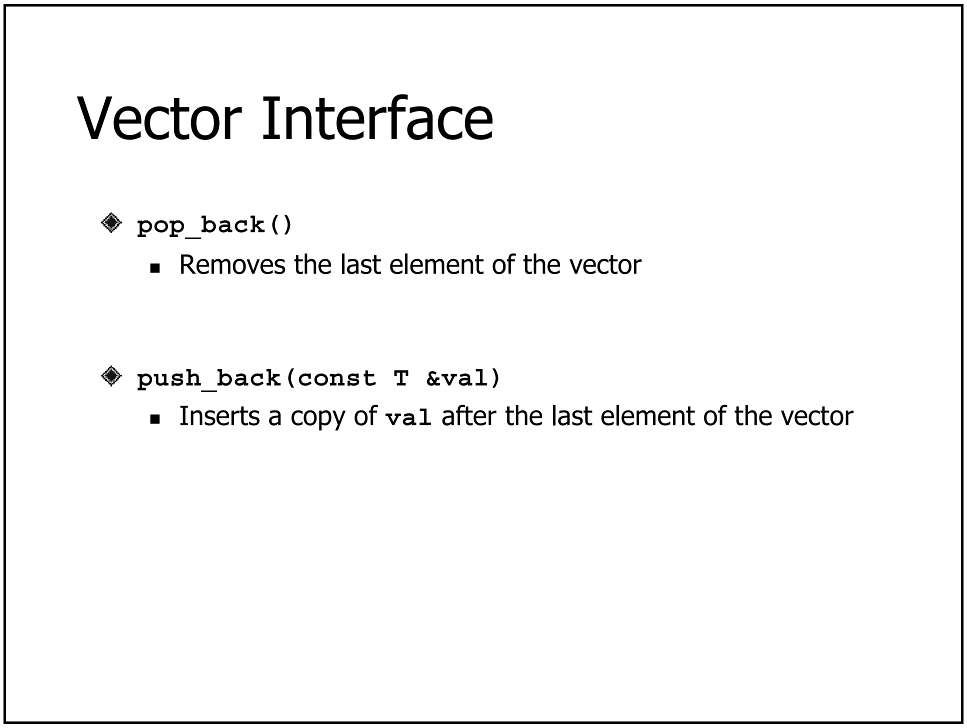
```
vector<int> MyList(3);
cout << "Enter numbers: ";
GetList(MyList);
```



Examples

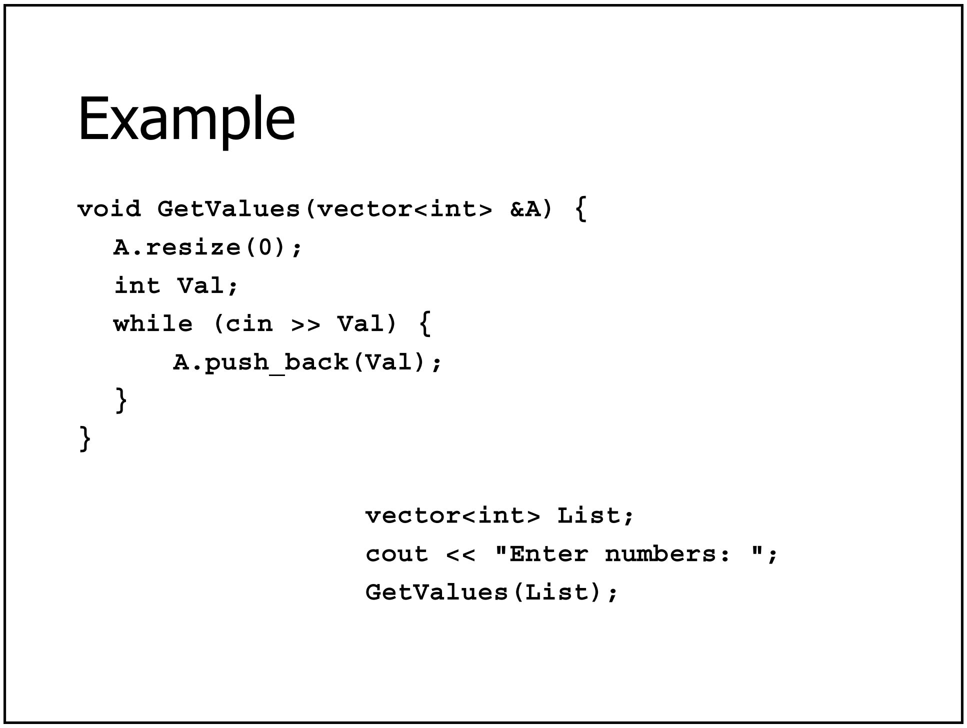
```
void PutList(const vector<int> &A) {
  for (int i = 0; i < A.size(); ++i) {
      cout << A[i] << endl;
  }
}
```

```
                    cout << "Your numbers: ";
                    PutList(MyList)
```



Vector Interface

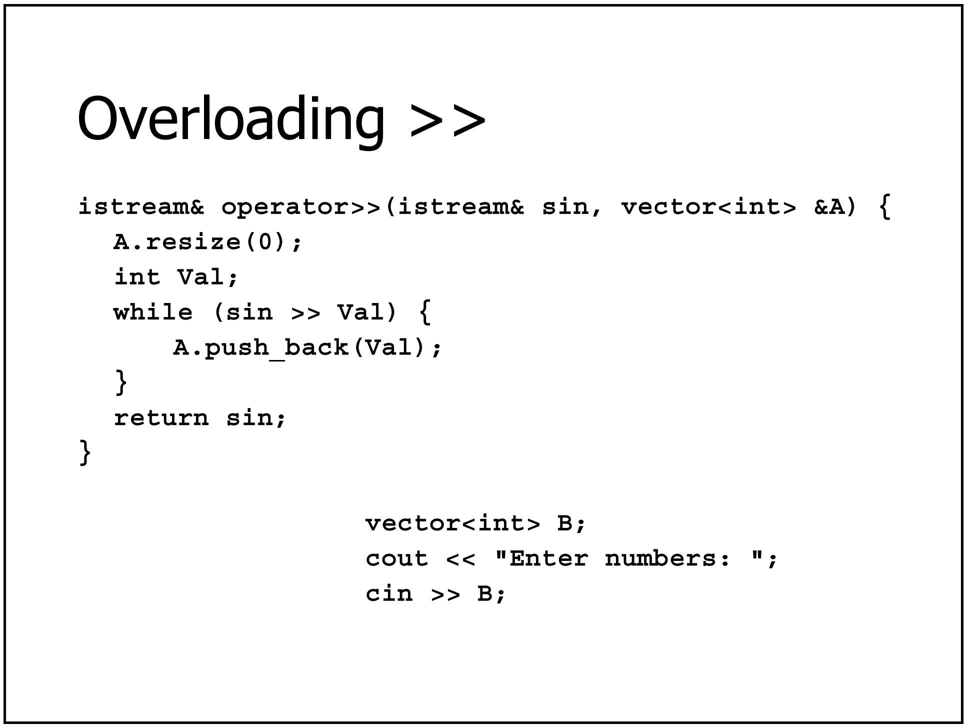
- **`pop_back()`**
  - Removes the last element of the vector
- **`push_back(const T &val)`**
  - Inserts a copy of **`val`** after the last element of the vector



Example

```
void GetValues(vector<int> &A) {
  A.resize(0);
  int Val;
  while (cin >> Val) {
      A.push_back(Val);
  }
}

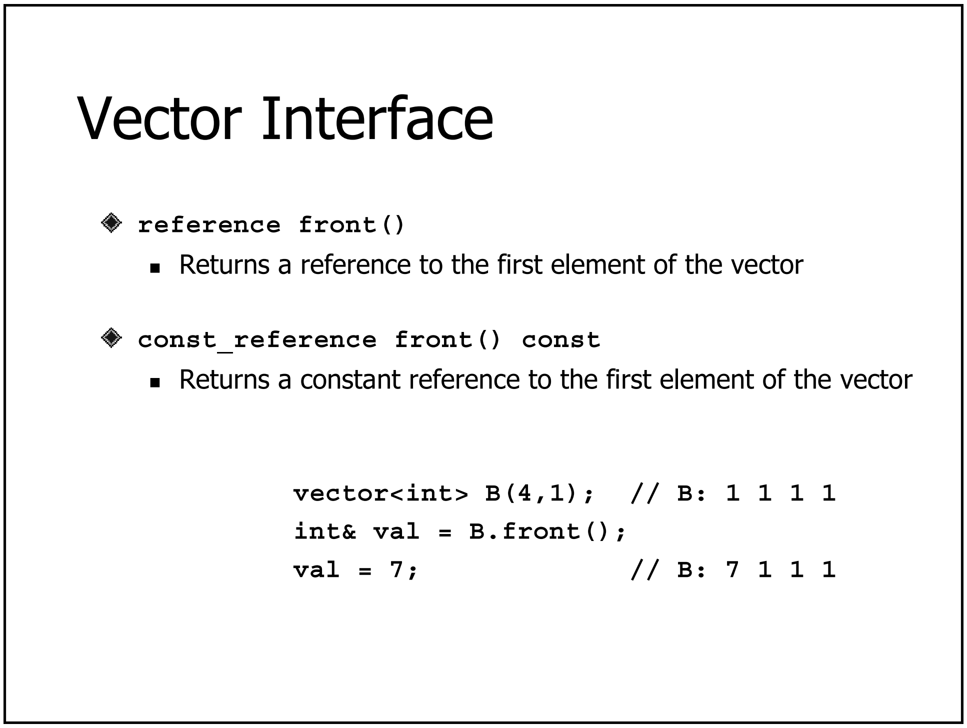
                  vector<int> List;
                  cout << "Enter numbers: ";
                  GetValues(List);
```



Overloading >>

```
istream& operator>>(istream& sin, vector<int> &A) {
  A.resize(0);
  int Val;
  while (sin >> Val) {
      A.push_back(Val);
  }
  return sin;
}

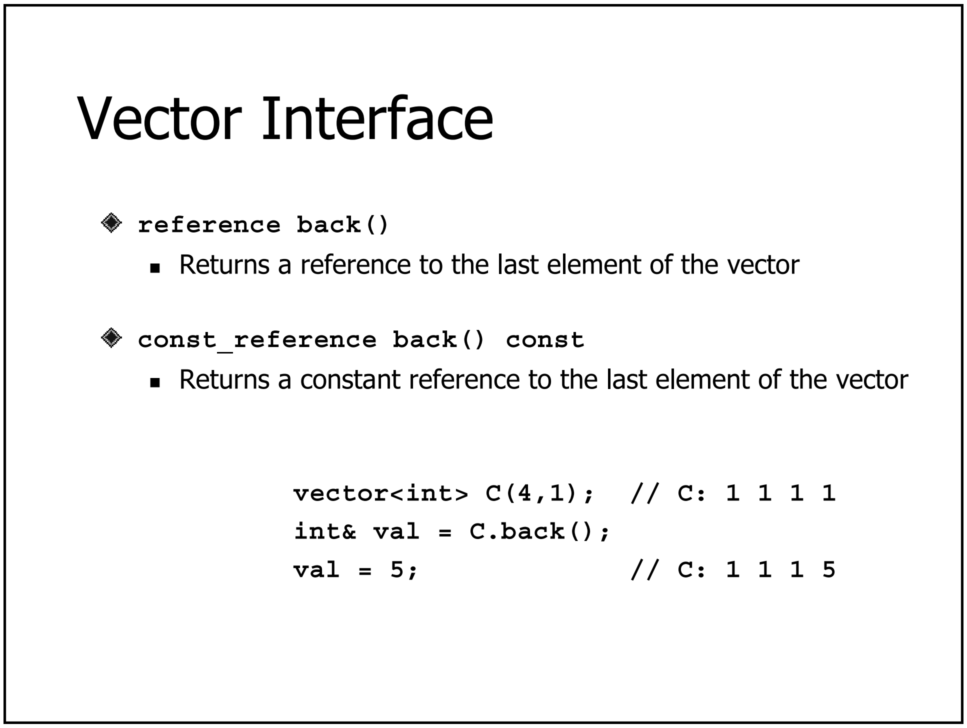
                  vector<int> B;
                  cout << "Enter numbers: ";
                  cin >> B;
```



Vector Interface

- `reference front()`
  - Returns a reference to the first element of the vector
- `const_reference front() const`
  - Returns a constant reference to the first element of the vector

```
vector<int> B(4,1);  // B: 1 1 1 1
int& val = B.front();
val = 7;             // B: 7 1 1 1
```



Vector Interface

- `reference back()`
  - Returns a reference to the last element of the vector
- `const_reference back() const`
  - Returns a constant reference to the last element of the vector

```
vector<int> C(4,1);  // C: 1 1 1 1
int& val = C.back();
val = 5;             // C: 1 1 1 5
```

Iterators

- Iterator is a pointer to an element
  - Really pointer abstraction
- Mechanism for sequentially accessing the elements in the list
  - Alternative to subscripting
- There is an iterator type for each kind of vector list
- Notes
  - Algorithm component of STL uses iterators
  - Code using iterators rather than subscripting can often be reused by other objects using different container representations

Vector Interface

- **`iterator begin()`**
  - Returns an iterator that points to the first element of the vector
- **`iterator end()`**
  - Returns an iterator that points to immediately *beyond* the last element of the vector

```
vector<int> C(4);  // C: 0 0 0 0
C[0] = 0; C[1] = 1; C[2] = 2; C[3] = 3;
vector<int>::iterator p = C.begin();
vector<int>::iterator q = C.end();
```

Iterators

- To avoid unwieldy syntax programmers typically use typedef statements to create simple iterator type names

```
typedef vector<int>::iterator iterator;
typedef vector<int>::reverse_iterator reverse_iterator;
typedef vector<int>::const_reference const_reference;

           vector<int> C(4);  // C: 0 0 0 0
           iterator p = C.begin();
           iterator q = C.end();
```

Iterator Operators

- `*` dereferencing operator
  - Produces a reference to the object to which the iterator **`p`** points

    `*p`

- `++` point to next element in list
  - Iterator **`p`** now points to the element that followed the previous element to which p points

    `++p`

- `--` point to previous element in list
  - Iterator **`p`** now points to the element that preceded the previous element to which p points

    `--p`

```
typedef vector<int>::iterator iterator;
typedef vector<int>::reverse_iterator reverse_iterator;
vector<int> List(3);

List[0] = 100; List[1] = 101; List[0] = 102;

iterator p = List.begin();
cout << *p;                         // 100
++p;
cout << *p;                         // 101
--p;
cout << *p;                         // 100
reverse_iterator q = List.rbegin();
cout << *q;                         // 102
++q;
cout << *q;                         // 101
--q;
cout << *q;                         // 102
```

Vector Interface

- **`insert(iterator pos, const T &val = T())`**
  - Inserts a copy of `val` at position `pos` of the vector and returns the position of the copy into the vector

- **`erase(iterator pos)`**
  - Removes the element of the vector at position `pos`

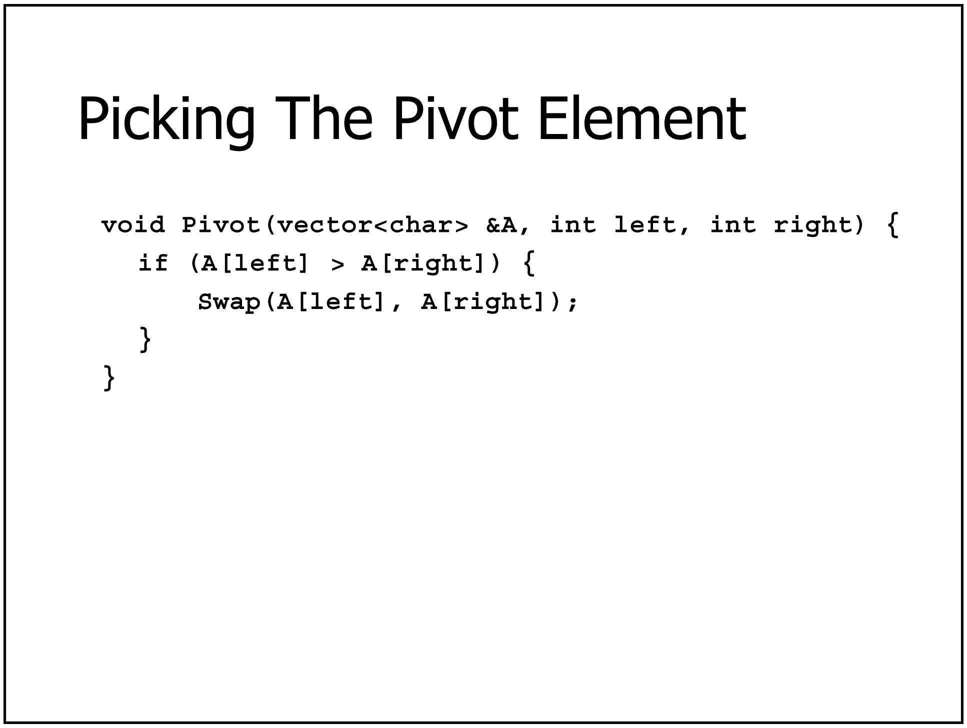
SelectionSort Revisited

```
void SelectionSort(vector<int> &A) {
   int n = A.size();
   for (int i = 0; i < n); ++i) {
      int k = i;
      for (int j = i + 1; j < n; ++j) {
         if (A[j] < A[k])
            k = j;
      }
      if (i != k)
         swap(A[k], A[i]);
   }
}
```

QuickSort

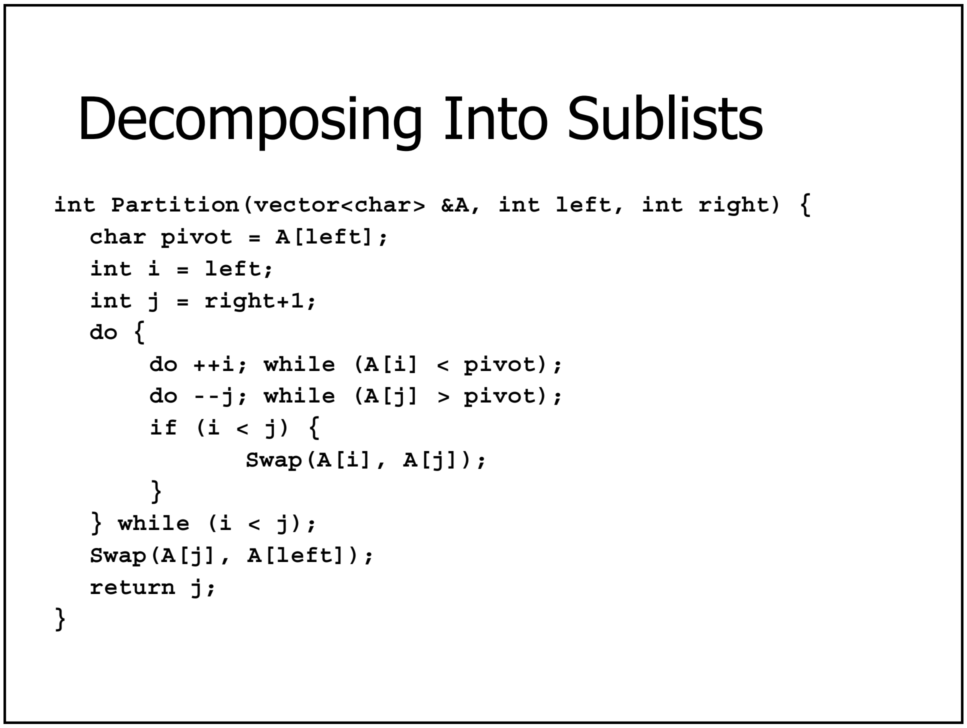
- QuickSort
  - Divide the list into sublists such that every element in the left sublist <= to every element in the right sublist
  - Repeat the QuickSort process on the sublists

```
void QuickSort(vector<char> &A, int left, int right) {
   if (left < right) {
       Pivot(A, left, right);
       int k = Partition(A, left, right);
       QuickSort(A, left, k-1);
       QuickSort(A, k+1, right);
   }
}
```



Picking The Pivot Element

```
void Pivot(vector<char> &A, int left, int right) {
  if (A[left] > A[right]) {
      Swap(A[left], A[right]);
  }
}
```



Decomposing Into Sublists

```
int Partition(vector<char> &A, int left, int right) {
   char pivot = A[left];
   int i = left;
   int j = right+1;
   do {
        do ++i; while (A[i] < pivot);
        do --j; while (A[j] > pivot);
        if (i < j) {
               Swap(A[i], A[j]);
        }
   } while (i < j);
   Swap(A[j], A[left]);
   return j;
}
```

Sorting Q W E R T Y U I O P

Q W E R T Y U I O P
I O E P T Y U R W Q
E O I P T Y U R W Q
E O I P T Y U R W Q
E I O P T Y U R W Q
E I O P T Y U R W Q
E I O P T Y U R W Q
E I O P Q Y U R W T
E I O P Q Y U R W T
E I O P Q R T U W Y
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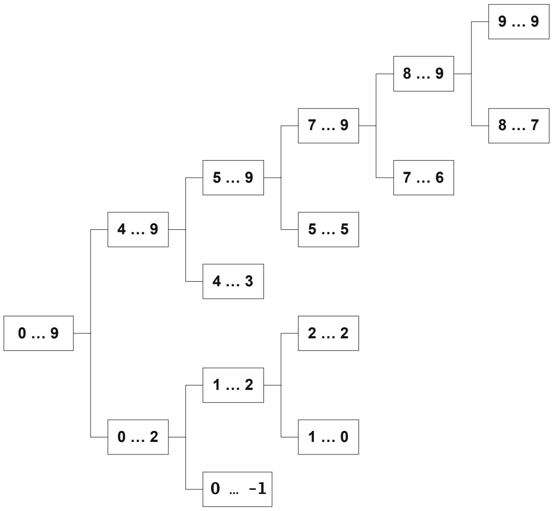
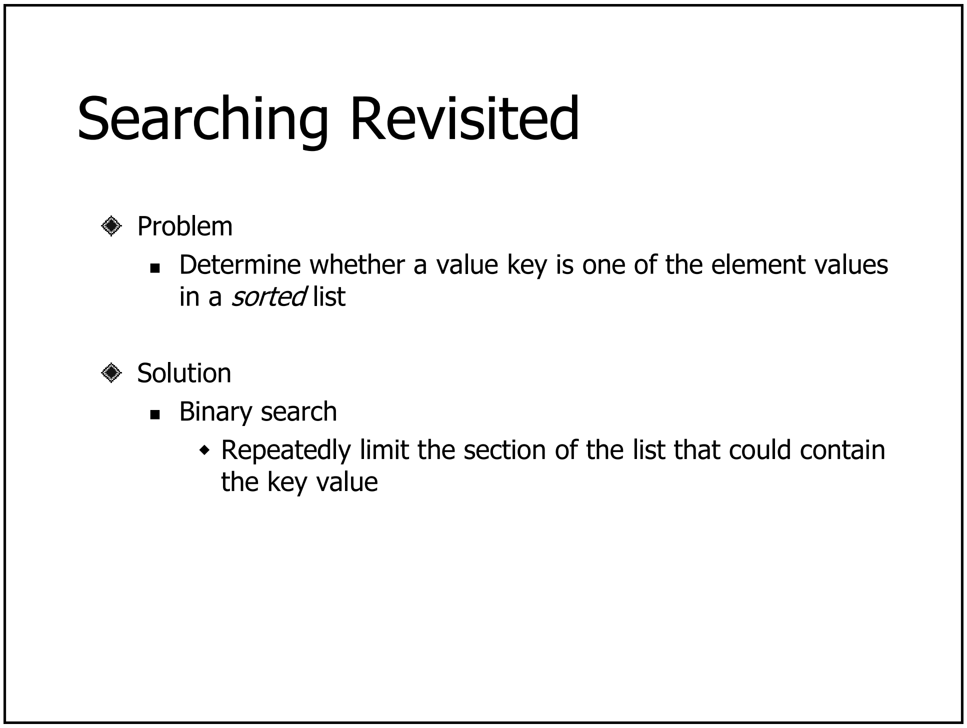


InsertionSort

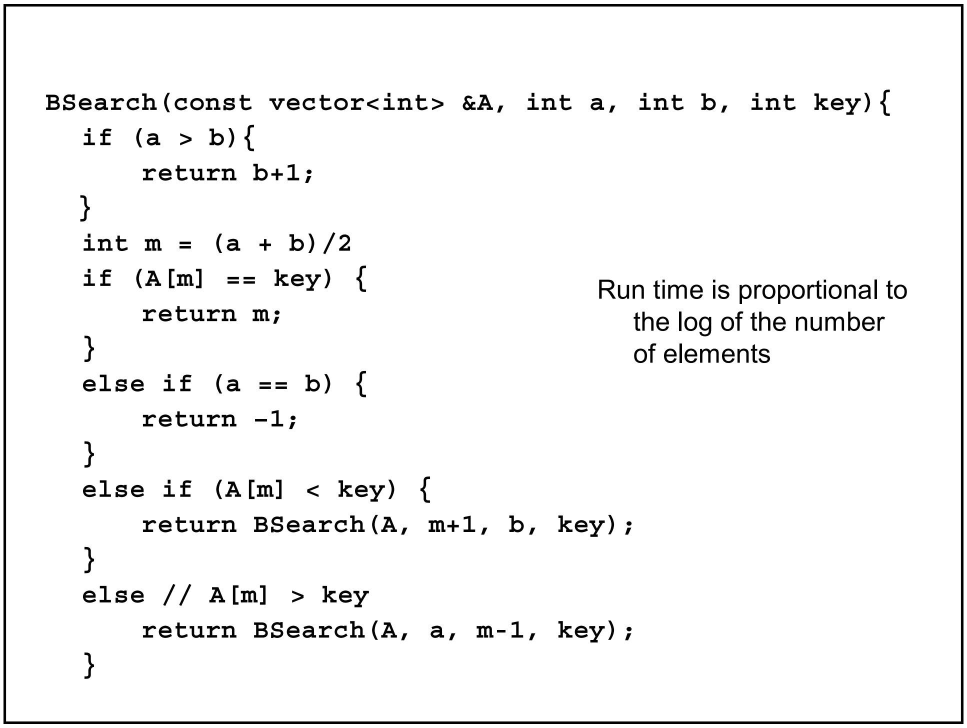
```
void InsertionSort(vector<int> &A) {
   for (int i = 1; i < A.size(); ++i) {
      int key = A[i]
      int j = i - 1;
      while ((j > 0) && (A[j] > key)) {
         A[j+1] = A[j]
         j = j - 1
      }
      A[j+1] = key
   }
```



Searching Revisited

- Problem
  - Determine whether a value key is one of the element values in a *sorted* list

- Solution
  - Binary search
    - Repeatedly limit the section of the list that could contain the key value



```
BSearch(const vector<int> &A, int a, int b, int key){
  if (a > b){
      return b+1;
  }
  int m = (a + b)/2
  if (A[m] == key) {
      return m;
  }
  else if (a == b) {
      return -1;
  }
  else if (A[m] < key) {
      return BSearch(A, m+1, b, key);
  }
  else // A[m] > key
      return BSearch(A, a, m-1, key);
  }
```

Run time is proportional to the log of the number of elements

String Class Revisited

```
void GetWords(vector<string> &List) {
  List.resize(0);
  string s;
  while (cin >> s) {
      List.push_back(s);
  }
}
```

Using GetWords()

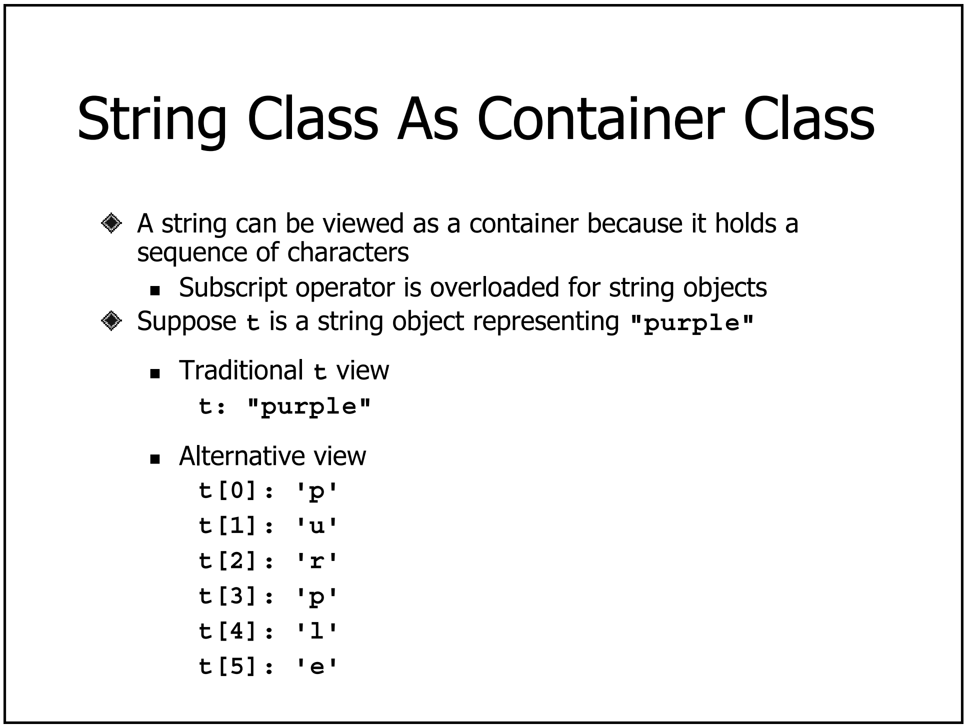
- Suppose standard input contains

  A list of words to be read.

```
vector<string> A;
GetWords(A);
```

- Would set **`A`** in the following manner:

```
A[0]: "A"
A[1]: "list"
A[2]: "of"
A[3]: "words"
A[4]: "to"
A[5]: "be"
A[6]: "read."
```



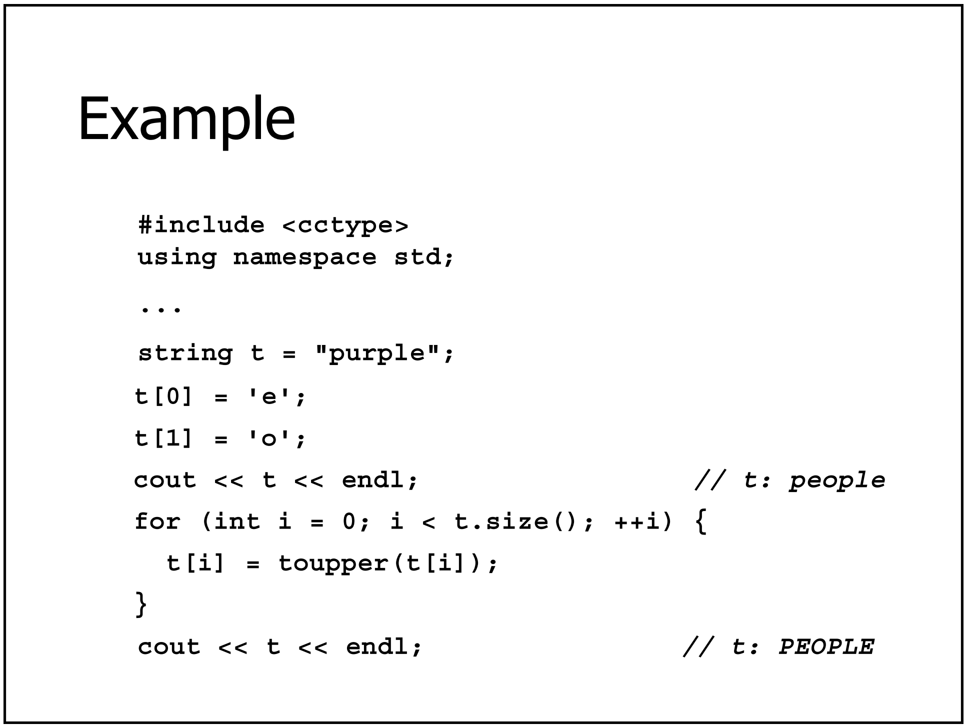
String Class As Container Class

- A string can be viewed as a container because it holds a sequence of characters
  - Subscript operator is overloaded for string objects
- Suppose `t` is a string object representing **`"purple"`**
  - Traditional `t` view

    ```
    t: "purple"
    ```

  - Alternative view

    ```
    t[0]: 'p'
    t[1]: 'u'
    t[2]: 'r'
    t[3]: 'p'
    t[4]: 'l'
    t[5]: 'e'
    ```



Example

```
#include <cctype>
using namespace std;
...
string t = "purple";
t[0] = 'e';
t[1] = 'o';
cout << t << endl;                  // t: people
for (int i = 0; i < t.size(); ++i) {
  t[i] = toupper(t[i]);
}
cout << t << endl;                  // t: PEOPLE
```

Reconsider A

- Where
  ```
  vector<string> A;
  ```

  Is set in the following manner
  ```
  A[0]: "A"
  A[1]: "list"
  A[2]: "of"
  A[3]: "words"
  A[4]: "to"
  A[5]: "be"
  A[6]: "read."
  ```

Counting o’s

- The following counts number of o’s within **`A`**

```
count = 0;
for (int i  = 0; i < A.size(); ++i) {
   for (int j = 0; A[i].size(); ++j) {
      if (A[i][j] == 'o') {
         ++count;
      }
   }
}
```

Size of **`A`**

Size of **`A[i]`**

To reference `j`th character of **`A[i]`** we need double subscripts

Explicit Two-Dimensional List

- Consider definition

  `vector< vector<int> > A;`

- Then

  `A` is a **`vector< vector<int> >`**

  - It is a vector of vectors

  `A[i]` is a **`vector<int>`**

  - **`i`** can vary from `0` to **`A.size() - 1`**

  `A[i][j]` is a **`int`**

  - **`j`** can vary from `0` to **`A[i].size() - 1`**

Multi-Dimensional Arrays

- Syntax

  `btype mdarray[size_1][size_2] ... [size_k]`

- Where
  - k - dimensional array
  - **`mdarray`**: array identifier
  - **`size_i`**: a positive constant expression
  - **`btype`**: standard type or a previously defined user type and is the base type of the array elements

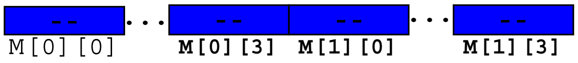
- Semantics
  - **`mdarray`** is an object whose elements are indexed by a sequence of **`k`** subscripts
  - the **`i`**-th subscript is in the range `0 ...` **`size_i - 1`**

Memory Layout

- Multidimensional arrays are laid out in row-major order
- Consider

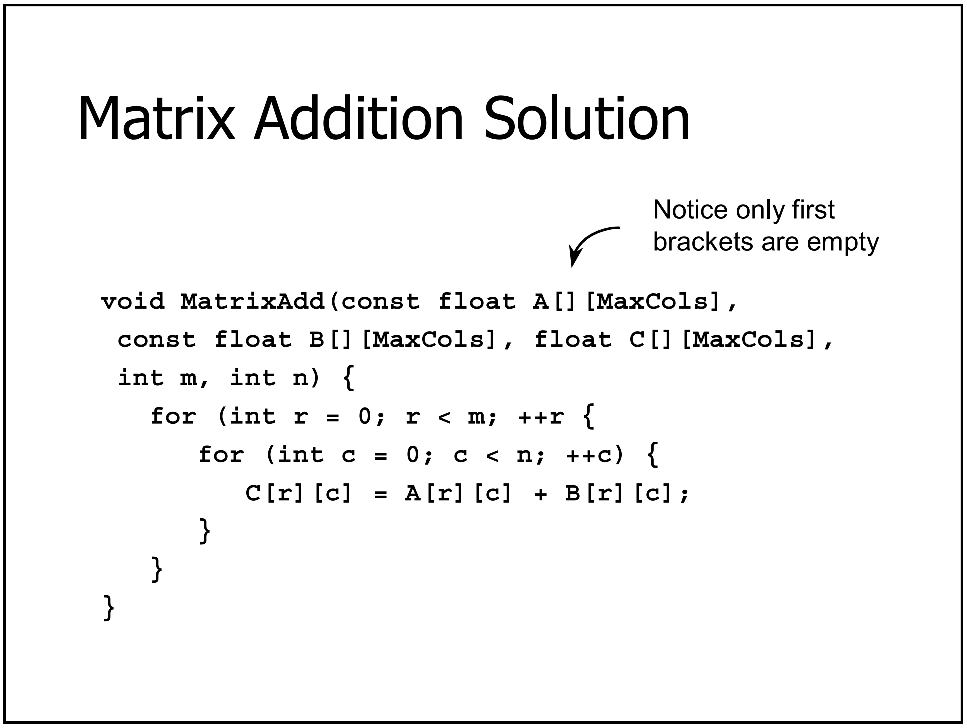
```
int M[2][4];
```

- `M` is two-dimensional array that consists of 2 subarrays each with 4 elements.
  - 2 rows of 4 elements
- The array is assigned to a contiguous section of memory
  - The first row occupies the first portion
  - The second row occupies the second portion



Identity Matrix Initialization

```
const int MaxSize = 25;
float A[MaxSize][MaxSize];
int nr = PromptAndRead();
int nc = PromptAndRead();
assert((nr <= MaxSize) && (nc <= MaxSize));
for (int r = 0; r < nr; ++r) {
   for (int c = 0; c < nc; ++c) {
      A[r][c] = 0;
   }
   A[r][r] = 1;
}
```



Matrix Addition Solution

Notice only first brackets are empty

```
void MatrixAdd(const float A[][MaxCols],
 const float B[][MaxCols], float C[][MaxCols],
 int m, int n) {
   for (int r = 0; r < m; ++r {
      for (int c = 0; c < n; ++c) {
         C[r][c] = A[r][c] + B[r][c];
      }
   }
}
```