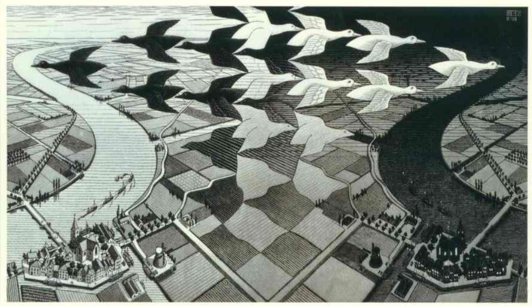

# Lecture 18: The Story So Far

CS150: Computer Science
University of Virginia
Computer Science

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## Menu

- Finish insert-sort-tree
- Course roadmap
- Introducing Mutation

A few people have extensions on Exam 1, so
no talking about the exam questions until Wednesday.

If you have an extension on Exam 1, don't read Chapter 9
until you turn in the exam.



## insert-one-tree

```
(define (insert-one-tree cf el tree)
  (if (null? tree)
      (make-tree null el null)
      (if (cf el (get-element tree))
          (make-tree
           (insertel-tree cf el (get-left tree))
           (get-element tree) (get-right tree))
          (make-tree (get-left tree)
           (get-element tree)
           (insertel-tree cf el (get-right tree))))))
```

Each time we call insert-one-tree, the size of the tree approximately halves (if it is well balanced).

Each application is constant time.

The running time of insert-one-tree is in $\Theta(\log n)$ where $n$ is the number of elements in the input tree, which must be well-balanced.



## insert-sort-helper

```
(define (insert-sort-helper cf lst)
   (if (null? lst) null
       (insert-one-tree
        cf (car lst)
        (insert-sort-helper cf (cdr lst)))))
```

No change (other than using insert-one-tree)…but evaluates to a tree not a list!

((((() 1 ()) 2 ()) 5 (() 8 ()))



## extract-elements

We need to make a list of all the tree elements, from left to right.

```
(define (extract-elements tree)
  (if (null? tree) null
      (append (extract-elements (get-left tree))
              (cons
                 (get-element tree)
                 (extract-elements (get-right tree))))))
```



## Running time of insert-sort-tree

```
(define (insert-one-tree cf el tree)
  (if (null? tree)
    (make-tree null el null)
    (if (cf el (get-element tree))
      (make-tree (insert-one-tree cf el (get-left tree))
                 (get-element tree)
                 (get-right tree))
      (make-tree (get-left tree)
                 (get-element tree)
                 (insert-one-tree cf el (get-right tree))))))
```

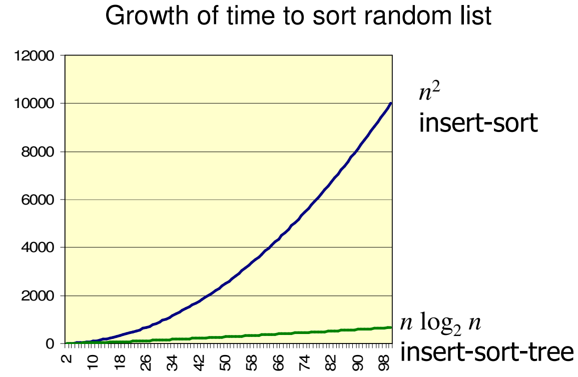
$n$ = number of elements in tree

$\Theta(\log n)$

```
(define (insert-sort-tree cf lst)
  (define (insert-sort-helper cf lst)
    (if (null? lst) null
      (insert-one-tree
       cf (car lst)
       (insert-sort-helper cf (cdr lst)))))
  (extract-elements (insert-sort-helper cf lst)))
```

$n$ = number of elements in lst

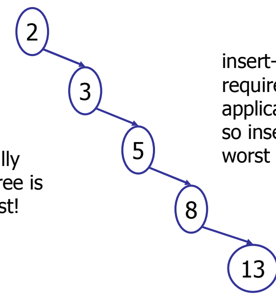
$\Theta(n \log n)$






## What if tree is not well-balanced?

A pathologically unbalanced tree is as bad as a list!




insert-one worst case requires $n$ recursive applications, so insert-sort-tree worst case is in $\Theta(n^2)$



## Comparing sorts

```
> (testgrowth best-first-sort)
n = 250, time = 110
n = 500, time = 371
n = 1000, time = 2363
n = 2000, time = 8162
n = 4000, time = 31757
(3.37 6.37 3.45 3.89)
> (testgrowth insert-sort)
n = 250, time = 40
n = 500, time = 180
n = 1000, time = 571
n = 2000, time = 2644
n = 4000, time = 11537
(4.5 3.17 4.63 4.36)
```

```
> (testgrowth insert-sort-halves)
n = 250, time = 251
n = 500, time = 1262
n = 1000, time = 4025
n = 2000, time = 16454
n = 4000, time = 66137
(5.03 3.19 4.09 4.02)
> (testgrowth insert-sort-tree)
n = 250, time = 30
n = 500, time = 250
n = 1000, time = 150
n = 2000, time = 301
n = 4000, time = 1001
(8.3 0.6 2.0 3.3)
```



## Can we do better?

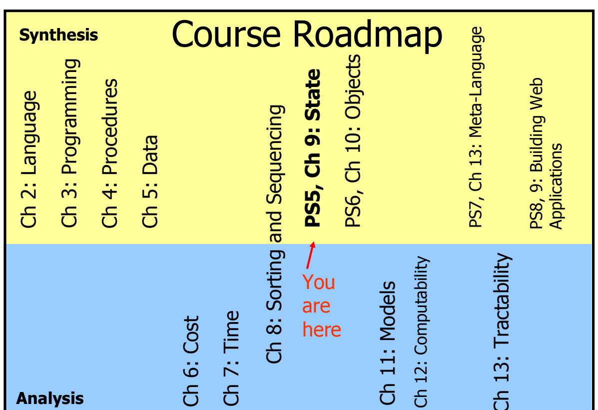
- Making all those trees is a lot of work
- Can we divide the problem in two halves, without making trees?

This is the famous "Quicksort" algorithm invented by Sir Tony Hoare. See Chapter 8.

There are lots of ways to do a little bit better, but **no** way to do asymptotically better. **All** possible sort procedure have running times in $\Omega(n \log n)$. (We'll explain why later in the course...)



## Course Roadmap






## Computer Science: CS150 so far

- How to describe information processes by defining procedures
  - Programming with procedures, lists, recursion
  - Chapters 3, 4, 5
- How to predict properties about information processes
  - Predicting running time, Θ, *O*, Ω
- How to elegantly and efficiently implement information processes
  - Chapter 3 (rules of evaluation)

# CS150 upcoming

- How to describe information processes by defining procedures
  - Programming with state, objects, networks
- How to predict properties about information processes
  - What is the fastest process that can solve a given problem?
  - Are there problems which can't be solved by algorithms?
- How to elegantly and efficiently implement information processes
  - How to implement a Scheme interpreter



From Lecture 1:

# The Liberal Arts

language — numbers

Trivium (3 roads): Grammar, Rhetoric, Logic

Quadrivium (4 roads): Arithmetic, Geometry, Music, Astronomy



# Liberal Arts Checkup

Trivium

- Grammar: study of meaning in written expression — BNF replacement rules for describing languages, rules of evaluation for meaning
- Rhetoric: comprehension of verbal and written discourse — Not much yet… interfaces between components (PS6-9), program and user (PS8-9)
- Logic: argumentative discourse for discovering truth — Rules of evaluation, if, recursive definitions

Quadrivium

- Arithmetic: understanding numbers — Not much yet… wait until April
- Geometry: quantification of space — Curves as procedures, fractals (PS3)
- Music: number in time — Yes, listen to "Hey Jude!"
- Astronomy — Friday: read Neil deGrasse Tyson's essay