Upper: set B, prop, pred

lower: variables

$B : \{T, \bot\}$

S : set of 2102 students

G : set of module 1 groups

$U(s,g)$: s uploaded a PDF for g

$\forall g \in G.\ g \subseteq S$

$\rightarrow \forall g \in G.\ \exists s \in g.\ U(s,g)$

$\cancel{\exists s \in S.\ \forall g \in G.\ U(s,g)}$

$\forall x \quad \exists x$

$\exists x \in \mathbb{N}.\ x^2 - 4x + 3 = 0$

$\forall p \in B.\ \forall q \in B. \rightarrow \forall p, q \in B.\ (p \land q) \rightarrow (p \lor q)$

$\forall p \in B.\ \exists q \in B.\ (p \oplus q) \leftrightarrow p$

$\exists p \in B.\ \forall q \in B.\ (p \land q) \leftrightarrow q$

Consider $p = T$. Then we have

$\forall q \in B.\ (T \land q) \leftrightarrow q$

Consider $(T \land x) \Leftrightarrow x$ for unknown x. By simplification, $\equiv x \Leftrightarrow x \equiv T$.

Because $(T \land x) \Leftrightarrow x$ for unknown x, it is also the case that $\forall x \in B.\ (T \land x) \Leftrightarrow x$

Therefore, $\exists p \in B$ (in particular, $p = T$) $.\ \forall q \in B\ (p \land q) \Leftrightarrow q$

Existential instantiation

instantiate

Proof.

Consider $x = \boxed{3}$. $\boxed{3^2 - 4(3) + 3 = 0}$ $= 9 - 12 + 3 = 0$

Therefore, $\exists x.\ x^2 - 4x + 3 = 0$ $\blacksquare$ $\square$

q.e.d.

Q.E.D.

Universal Instantiation (Skolemization)

$(p \land q) \rightarrow (p \lor q)$

$\neg(p \land q) \lor (p \lor q)$

$\neg p \lor \neg q \lor p \lor q$

$\neg p \lor p \quad \lor \neg q \lor q$

$T \quad \lor \quad T$

T

Proof.

$(p \land q) \rightarrow (p \lor q)$ is equivalent to $\neg(p \lor q) \lor (p \lor q)$ by def of imp. By De Morgan's law and simplification, this is equiv to T.

Bcs $(p \land q) \rightarrow (p \lor q)$ regardless of the values of p & q, $\forall p, q \in B.\ (p \land q) \rightarrow (p \lor q)$ $\blacksquare$

x	$T \oplus x$
$\bot$	T
T	$\bot$

$\forall p \in B.\ \exists q \in B.\ (p \oplus q) \Leftrightarrow p$

$\exists q \in B.\ (x \oplus q) \Leftrightarrow x$ Univ inst $p \doteq x$

$(x \oplus \bot) \Leftrightarrow x$ Exis inst $q = \bot$

$x \Leftrightarrow x$ Simp

T Simp

$\forall p \in B.\ \exists q \in B.\ (p \oplus q)$

$\exists q \in B.\ (x \oplus q)$ $p = x$

$(x \oplus \neg x)$ $q = \neg x$

T Simp

$\forall x \in \{\top, \bot\}.$

$x = \top$

$x = \bot$

y