

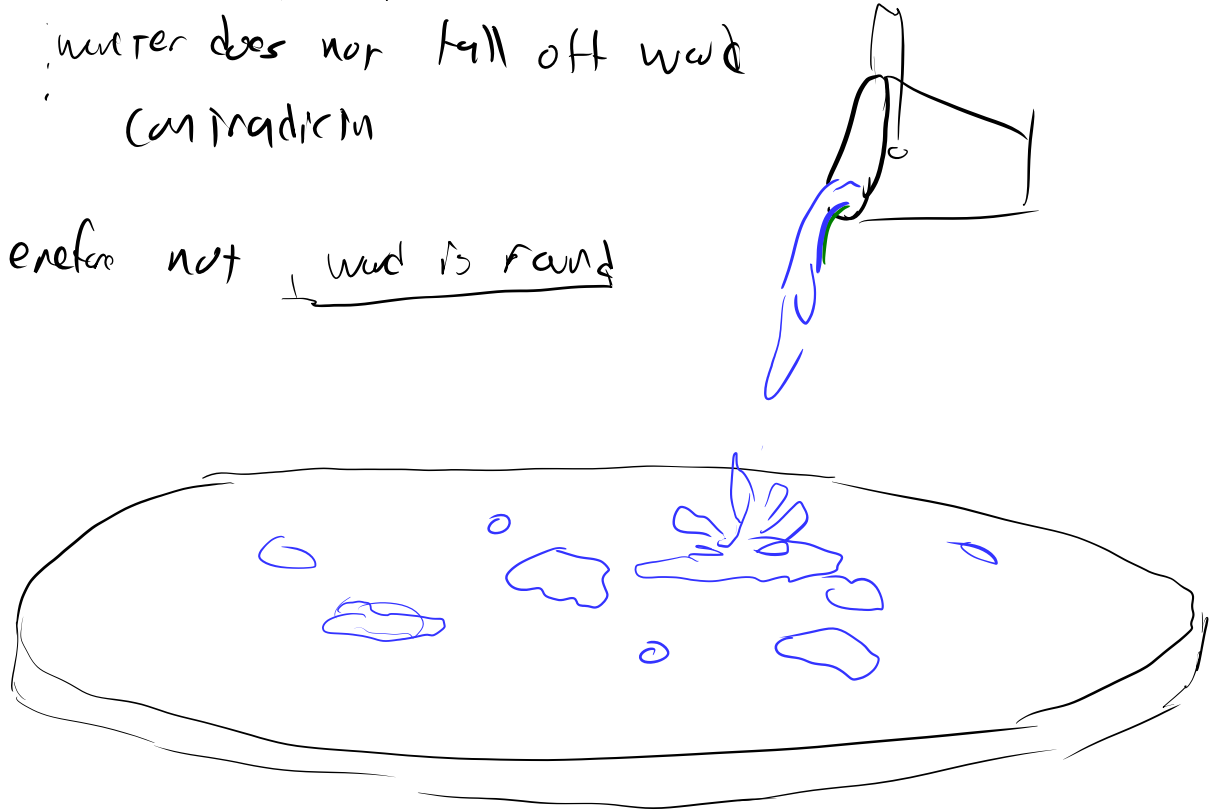
Proof

assume world is round

- water falls off round globe
  - water does not fall off world
- contradiction

Therefore not world is round

small ball + big balls  
New seen water behavior





$|x|$   
 $|x|$   
 $|x|$

{ 1 }

$x|y$   
means  
 $x$  divides  $y$   
 $x$  is a factor of  $y$

Factor = divisors  
Primality  
Fund. Thm of Arith

$$z = \text{GCD}(x, y)$$

$c = \text{LCM}(a, b)$

$a|c \quad c \bmod a = 0$   
 $b|c \quad a|d \implies b|d$

$\exists d < c. a|d \wedge b|d$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

$$8 = 2 \cdot 2 \cdot 2$$

$$\text{lcm}(60, 8) = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 120$$

$$x \equiv 0 \pmod{z}$$

$$x \bmod z = 0$$

$$y \bmod z = 0$$

$$2 = 4 \pmod{2}$$

$$7 = 10 \pmod{3}$$

$$\exists w > z. w|x \wedge w|y$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

$$8 = 2 \cdot 2 \cdot 2$$

$$\text{gcd}(60, 8) = 2 \cdot 2 = 4$$

$$\text{gcd}(2^3 \cdot 3^2 \cdot 7^4 \cdot 11 \cdot 13^2, 3^5 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17)$$

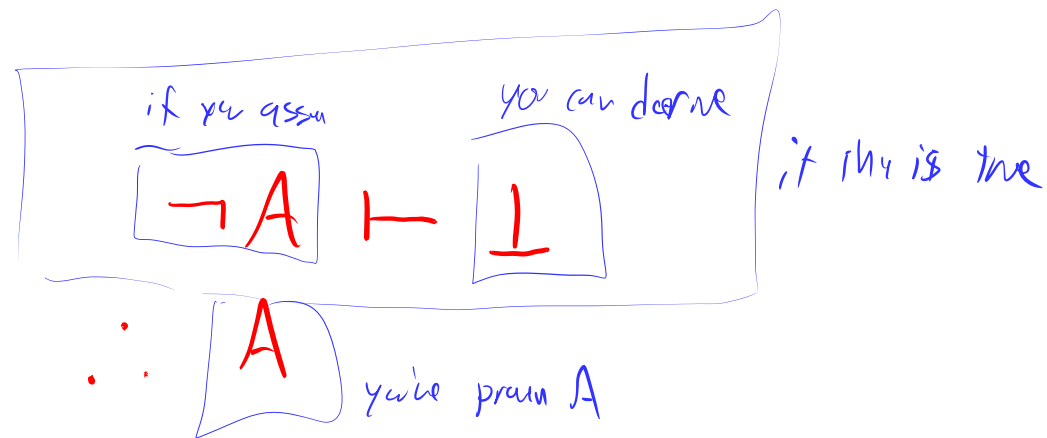
$$= 3^2 \cdot 7^2 \cdot 11 \cdot 13$$

$\text{GCD}(x, y) = 1 \iff x$  and  $y$  are coprime relatively prime

$\mathbb{Q}$

$$\frac{-33}{-11} = 3$$

$$\forall x \in \mathbb{Q} \quad \exists d \in \mathbb{N}^+, n \in \mathbb{Z} \quad x = \frac{n}{d} \wedge \gcd(n, d) = 1$$



thm.  $\frac{1}{2} \notin \mathbb{Z}$

Proof.

assume  $\frac{1}{2} \in \mathbb{Z}$ . That means  $\exists x \in \mathbb{Z} . \frac{1}{2} = x$ .

thus,  $1 = 2x$

By fund thm of Arith, 1 and  $2x$  must have same prime factors.

But 1 has no prime factors and  $2x$  has at least one (2).

That is a contradiction

Because assuming  $\frac{1}{2} \in \mathbb{Z}$  led to a contradiction, <sup>it must be that</sup>  $\frac{1}{2} \notin \mathbb{Z} \quad \square$

$$x = \underbrace{2^k \cdot w}$$

$$x^2 = 2^{2k} \cdot w^2$$

$$\sqrt{2} \notin \mathbb{Q}$$

$$\text{then } \boxed{\nexists q \in \mathbb{Q} \cdot q^2 = 2}$$

$$x = 14$$

$$y = 10$$

$$14^2 = (2 \cdot 7)^2 = 2^2 \cdot 7^2 = 196$$

$$10^2 = (2 \cdot 5)^2 = 2^2 \cdot 5^2 = 100$$

$$8^2 = (2 \cdot 2 \cdot 2)^2 = 2^6$$

Pf. assume  $\boxed{\exists q \in \mathbb{Q} \cdot q^2 = 2}$

that means  $\exists x \in \mathbb{Z}, y \in \mathbb{Z}^+ \cdot \frac{x}{y} = q$  and  $\gcd(x, y) = 1$

which mean  $\frac{x^2}{y^2} = 2$

$$x^2 = 2y^2$$

$$60 = 2^2 \cdot 3 \cdot 5$$

multiplicity of factor 7 in 60 is 0

ftoa:  $x^2$  and  $2y^2$  have same prime factors

multiplicity of factor 2 in  $x^2$  is even

multiplicity of factor 2 in  $2y^2$  is odd

⊥

$2 \cdot 2^{\text{even}} \cdot w$

$2^{\text{odd}} \cdot w$

Mult of factor 3 is 1

$2 \rightarrow 1$

Because assumption  $\rightarrow$  then led to ⊥, then is no  $\square$