$$\neg A \vdash \bot$$

$$\therefore A$$

$$A \vdash \bot$$

$$\therefore \neg A$$

Proof

1. Assum $\neg A$.

2. | —
   | —
   | —
   | —

Contradiction

3. Bks assum $\neg A$ led to cont, A.

$$\neg A \vdash \bot$$

$$\therefore A$$

$$A \vdash \bot$$

$$\therefore \neg A$$

Proof

1. Assum $\neg A$.

2.

Contradiction

3. Bks assum $\neg A$ led to contr, A.

Well-ordering Principle

$\forall X \subseteq \mathbb{N} \,.\, \exists$ smallest $x \in X$

Prime $\subseteq \mathbb{N}$ 2

Composite $\subseteq \mathbb{N}$ 0

Programs that computes sqrt

smallest

Bytes

2.4 digits

1 KB ~ 2400 digit number

thm: All natural numbers are finite

nothing

Proof:

1. assum som nat num are infinite
let $F \subseteq \mathbb{N}$ be the infinite natural numbers
by assumption, $|F| > 0$
by w.o.p, $\exists$ smallest $x \in F$.

2. Consider $y = x - 1$
y is smaller than x and x is smallest infinit num
means y is finite, so y+1 must be finite,
and $y + 1 = x$. But we kn x is infite, a contradiction

Bks infinit - 1 is infinite, y is infinite. But y is smaller than x, mean x is not the smallest infinit nat num
Cont

3. Becaus assum infit nat num led to cont, all nat num must be finite

$$|\mathbb{N}| = \infty$$

1. assume $|\mathbb{N}|$ is finite.

   consider the biggest element of $\mathbb{N}$, x

   $\boxed{\text{let } y = x + 1.}$

2. Because adding two nat num creates a nat num, $y \in \mathbb{N}$.

   $y = x + 1 > x$, but that contradicts x being the biggest

3. Because assume $|\mathbb{N}|$ is finite led to contr, $|\mathbb{N}| = \infty$

$\sum_{y \in P} y$ sum

let P be the set of all primes

assum $|P|$ is finite

$x = \left(\prod_{y \in P} y\right) + 1$

$\prod$ product

$1 + \left(\prod_{y \in P} y\right)$

$x \notin P$

∞ many prime numbers

x is prime iff
1. $x \in \mathbb{N}$
2. 1 is a factor of x
3. x is a factor of x
4. x has no other natural number factors

assum finite number n of primes, $P_1, P_2 \ldots P_n$

consider $x = \boxed{P_1 \cdot P_2 \cdot P_3 \cdot \ldots \cdot P_n} + 1$

let x be 1+ the product of all members of P

no P_i is a factor of x

FToA

either x is prime or x has a prime factor not in $P_1, P_2 \ldots P_n$

Case 1: x is prime

But x is not $P_1, P_2 \ldots P_n$, which are all the primes so x is not prime.
Contradiction

Case 2: x has a ~~smaller~~ prime factor not in $P_1 \ldots P_n$ $\notin P$

then that factor is a prime not in $P_1 \ldots P_n$

which means it is not prime (Becau not $P_1 P_2 \ldots P_n$)
contradiction

Bks both case result in cont, cont in general.

Bks assum finit Prime led to Cont, ∞ Primes.

0	2	4	6	8	10
0	3	6	9	12	15
0	5	10	15	20	25

$30 + 1 = 31$

1 3 5 7 9 ..
1 4 7 10 13 ...
1 6 11 16 21