



$\neg(\neg X)$

A	\neg	\neg	A
0	0	1	0
1	1	0	1

$$\neg\neg Q \equiv Q$$
$$\neg\neg A \equiv A$$
$$\neg$$

$$\neg\neg(\neg\neg A \vee \neg\neg\neg B) \equiv (A \vee \neg B)$$

$$A \vee B \equiv \neg\neg A \vee B$$

$$\begin{array}{c|l} \downarrow & \\ \downarrow & \\ \downarrow & \\ \downarrow & \\ \hline \neg(\neg A) \wedge (\neg A) & \text{Given} \\ \downarrow & \\ \perp & \text{Simplification} \end{array}$$

$$\begin{array}{c|l} \neg\neg A \wedge \neg A & \text{Given} \\ \hline A \wedge \neg A & \text{double neg} \\ \downarrow & \\ \perp & \text{simplification} \end{array}$$

A	A	$\neg A$	$A \wedge \neg A$
0	0	1	0
1	1	0	0

P	Q	$P \rightarrow Q$	\equiv	$(\neg P) \vee Q$
0	0	1		1
0	1	1		1
1	0	0		0
1	1	1		1

$A \vee B$ | given
 $\neg(\neg A) \vee B$ | double neg
 $\neg A \rightarrow B$ | def. of imp

$A \vee B \equiv \overbrace{\neg A \rightarrow B}^{\text{given}} \equiv \overbrace{\neg B \rightarrow A}^{\text{goal}}$

$\neg A \rightarrow B$	given
$\neg(\neg A) \vee B$	def. of Imp
$A \vee B$	double neg
$B \vee A$	commut
$\neg(\neg B) \vee A$	double neg
$\neg B \rightarrow A$	def. of Imp

$P = \neg B$
 $Q = A$
 $\neg P \vee Q$
 $P \rightarrow Q$

De Morgan

$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

$$\neg(A \wedge B) \equiv (\neg A \vee \neg B)$$

A	B	$\neg(A \vee B)$	$(\neg A \wedge \neg B)$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$\neg(A \rightarrow B)$	given
$\neg(\neg A \vee B)$	def
$(\neg\neg A \wedge \neg B)$	de Morgan
$A \wedge \neg B$	double neg

$$3 \times (2 + 5) = (3 \times 2) + (3 \times 5)$$

$$2 + (3 \times 5) \neq (2 + 3) \times (2 + 5)$$

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C)$$

$$(A \wedge B) \vee (A \wedge C)$$

$$((A \wedge B) \vee A) \wedge ((A \wedge B) \vee C)$$

⋮

$$(A \vee B) \wedge A \equiv A$$

$$(A \wedge A) \vee (B \wedge A)$$

$$A \vee (B \wedge A)$$

$$(A \vee B) \wedge (A \vee A)$$

$$(A \vee B) \wedge A$$
