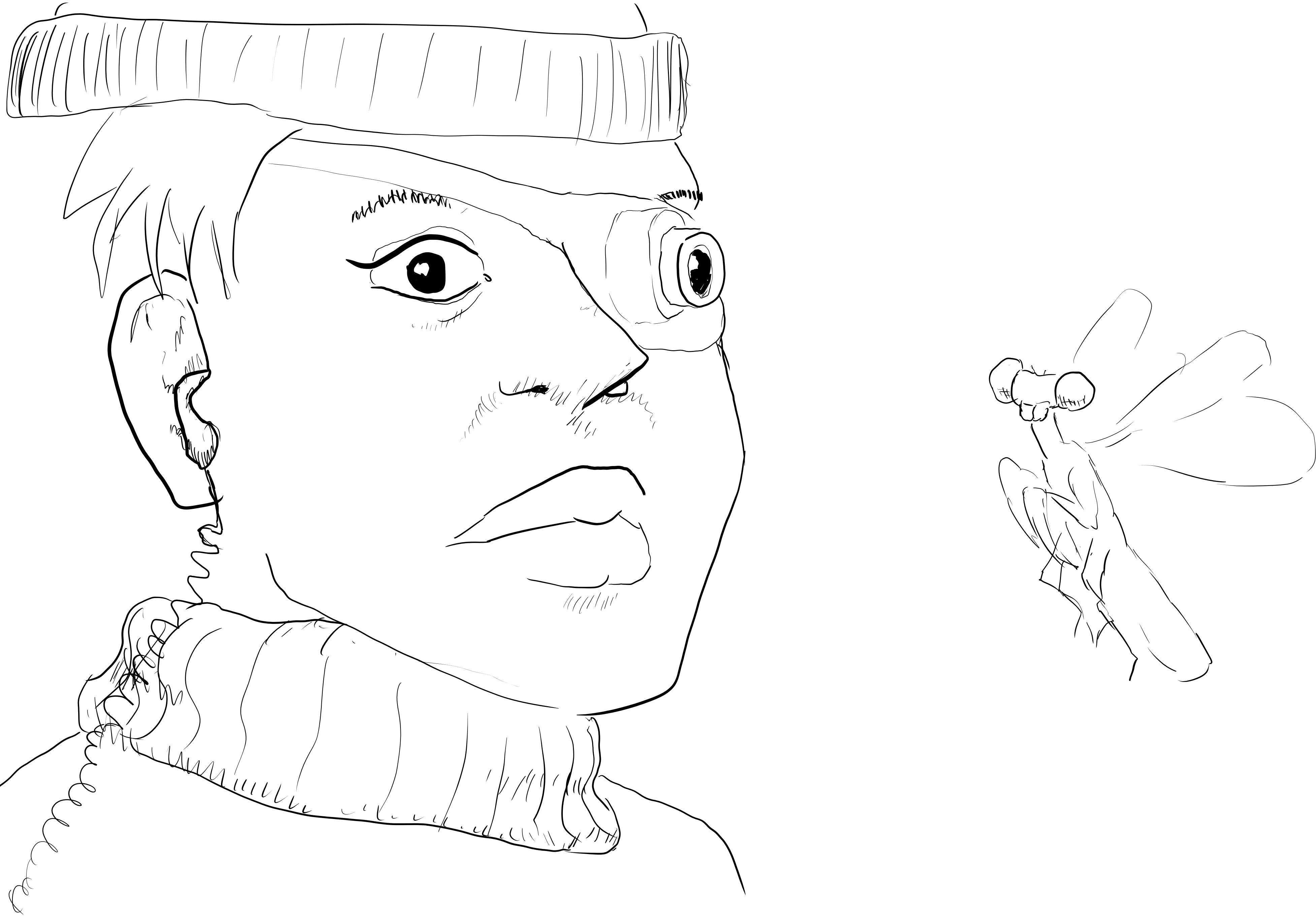

Proof by Contradiction

- $A \to \perp \vDash \neg A$
- $A \vdash B$ (assum, prov)

$\therefore A \to B$

want to prove A.

assume $\neg A$

prove $\perp$

Because assum $\neg A$ led to a contradiction, A.

theorem

$R(x,y): x = |y|$ is not symm $\mathbb{Z}$

Counter: $\underbrace{R(1,-1) \text{ but not } R(-1,1)}$

theorem: $R(x,y): x = |y|$ is not symmetric over $\mathbb{Z}$.

that is, $\neg \forall x,y \in \mathbb{Z}.\ R(x,y) \to R(y,x)$

Proof. We proceed by contradiction.

Assume that R is symmetric. That is assum $\forall x,y \in \mathbb{Z}.\ R(x,y) \to R(y,x)$. Consider $R(1,-1)$. Becaus $1 = |-1|$, $R(1,-1)$ is true. Becaus $R(x,y) \to R(y,x)$, it must be the case that $R(-1,1)$. But $-1 \neq |1|$, so $R(-1,1)$ is false. this is a contradiction.

Because assuming $R(x,y)$ was symmetric led to a contradiction, R must not be symmetric $\square$

$R(x,y): x = |y|$ is not symmetric

Contradiction

is sym

$R(1,-1)$ but not $R(-1,1)$

know

- $\underbrace{\forall x\, y}\,{}^{\in \mathbb{Z}}.\; R(x,y) \to R(y,x)$
- $R(1,-1)$
- $\neg R(-1,1)$

$x = 1, \; y = -1$

$$\underbrace{\underbrace{R(1,-1)}_{T} \to \underbrace{R(-1,1)}_{\perp}}_{\perp}$$

:)

$\cancel{x = -1, \; y = 1}$

$$\cancel{\underbrace{\underbrace{R(-1,1)}_{\perp} \to \underbrace{R(1,-1)}_{T}}_{T}}$$

$R(x,y): x = |y|$ is not symmetric

$$\neg \forall x,y \in \mathbb{Z} .\ R(x,y) \rightarrow R(y,x)$$

$$\exists x,y \in \mathbb{Z} .\ \neg(\neg R(x,y) \vee R(y,x))$$

$$\exists x,y \in \mathbb{Z} .\ R(x,y) \wedge \neg R(y,x)$$

$x = 1$
$y = -1$ $\in \mathbb{Z}$

$$\underbrace{\overset{T}{R(1,-1)} \wedge \overset{T}{\neg \overset{\perp}{\overline{R(-1,1)}}}}_{T}$$

Proof. Consider 1 and -1. $R(1,-1)$ is true because $1 = |-1|$; but $R(-1,1)$ is false because $-1 \neq |1|$. Thus, $R(1,-1) \wedge \neg R(-1,1)$. Because both 1 & -1 are integers, by existential instantiation we have $\exists x,y \in \mathbb{Z} .\ \boxed{R(x,y) \wedge \neg R(y,x)}$. Using double negation, De Morgan's law, and the def of Imp, this is equiv to $\neg \forall x,y \in \mathbb{Z} .\ R(x,y) \rightarrow R(y,x)$, which is the negation of the definition of R being symmetric. Thus, R is not symmetric.

$$P(1) \models \exists x \in \mathbb{Z} . P(x)$$

$$P(3) \models \exists x \in \mathbb{Z} . P(x)$$

$$\models \exists x \in \mathbb{N} . P(x)$$

$$R(x, y) \land \neg R(y, x)$$

(x, y)

$$P(x)$$

$$x \in S$$

$$\therefore \exists x \in S . P(x)$$

Existential Instantiation