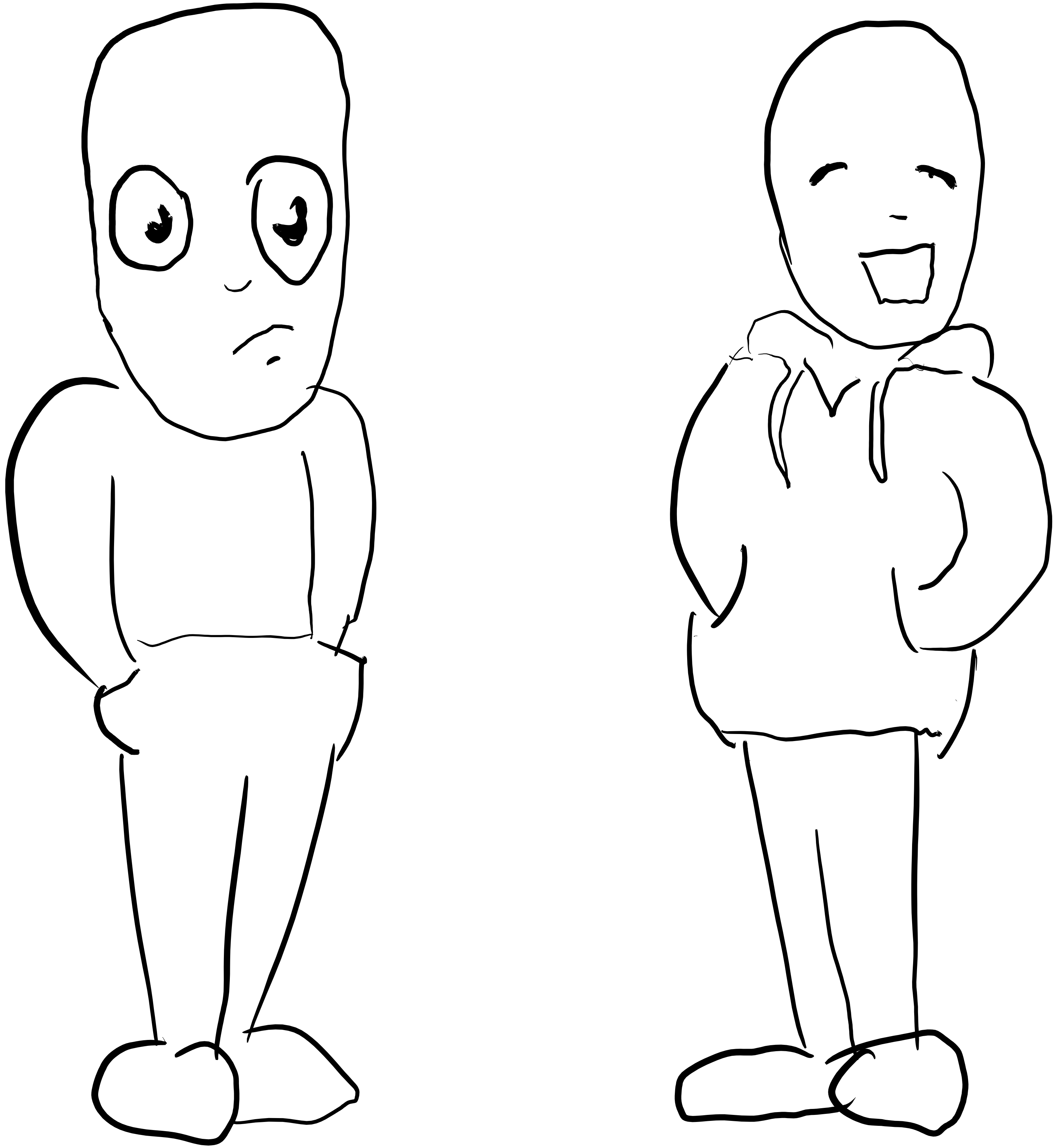

$\notin \mathbb{Z}$

$\notin \mathbb{Q}$

$|S| = \infty$

no largest/smallest elem

$\sum = \_\_\_\_$ (mostly next week)

do this by __

$|\mathbb{N}|$

$$|\mathbb{Z}| = \infty$$

$x \in \mathbb{R}$

$k \in \mathbb{Z}$

WOP

$\boxed{\cancel{S = \{k - x \mid x \in \mathbb{Z}\}}}$

Cont assume $|\mathbb{Z}| = k$ where $k \in \mathbb{N}$

then there is a ~~largest~~ smallest $x \in \mathbb{Z}$

Consider $x + 1 > x$ ($x - 1$)

$x + 1 \in \mathbb{Z}$

$\perp$

Consider $y = \sum_{n \in \mathbb{N}} n$

$\forall n \in \mathbb{N} .\ y > n$

$y \in \mathbb{N}$

$y > y$

$\perp$

assume $\mathbb{Z}$ is finite. Then $\mathbb{Z}$ must have a largest element; call that element x. Consider $y = x + 1$. Because y is the sum of two integers, it is an integer. But $y > x$, contradicting the assumption that x was the largest integer. Because assum $\mathbb{Z}$ is finite led to contradiction, $\mathbb{Z}$ must be infinite.

- $\mathbb{N}$ is finite
- consider $y = \sum_{n \in \mathbb{N}} n$
- becau y is the sum of a finite number of int, it is int ($y \in \mathbb{N}$)
- becau $y > x$ $\forall x \in \mathbb{N}$, $y \notin \mathbb{N}$

W.O.P $\boxed{S \subseteq \mathbb{N} \rightarrow S \text{ has a smallest member}}$

$\mathbb{R}^+$

$\mathbb{Q}^-$ has no largest number

ass x is largest in $\mathbb{Q}^-$

$x < 0$

$y = \boxed{\frac{x}{2}} \in \mathbb{Q}^-$

$y > x$ $\quad x < 0$ and $2 > 1$

$\boxed{\mathbb{Q}^+}$ has no smallest number

assume x is the smallest $\mathbb{Q}^+$. Then $x > 0$

Consider $y = \boxed{\frac{x}{2}}$

$y \in \mathbb{Q}^+$

- $y > 0$ because $x > 0$ and $2 > 0$
- y is rational because x and 2 are rational

$\cancel{x - 1}$

$\cancel{x}$

$y < x$ because $x > 0$ and $2 > 1$

Contradict x being smallest $\in \mathbb{Q}^+$

no closest rational to 2

That is $\neq 2$

assume closest rational exists. call it x.

$x \neq 2$

$x \in \mathbb{Q}$

consider $y = \frac{x+2}{2}$

rational

y closer to 2 than x

difference btw x and 2

divide by 2 ?

$\boxed{\frac{x-2}{2}} + 2 = \frac{x+2}{2}$

1.8 $\quad \frac{201}{100}$

$y = \frac{x}{2}$

$\frac{3/2}{2} = \frac{3}{4}$

1.5 $\quad$ 0.75

$x < 2$ vs $x > 2$

average $\quad \boxed{\frac{x+2}{2}}$

assume x is closest.

consider $y = 4 - x$

$|x-2| = |4-x-2|$

equally close

$4-x \neq x$

$4 \neq 2x$

$2 \neq x$

$|x-2| > |y-2|$

$(x-2)^2 > (y-2)^2$

$(x-2)^2 > \left(\frac{x+2}{2} - 2\right)^2$

$x^2 - 4x + 4 > \frac{x^2 + 4x + 4}{4}$

pos $\quad$ pos

no closest x in $\mathbb{Q}$ to 2, $\neq 2$

Assume x is the closest rational to 2.

Consider $y = \dfrac{x+2}{2}$

y is rational (because x and 2 are rational)

Because y is the average of x and 2, y is between x and 2, meaning y is closer to 2 than x is.

Therein lies the contradiction.

Because assum... , there is no closest rational to 2