Practice 02

problem group 1 *English and Math*

1. Rewrite "life is unexpected, but death is not unless it is sudden" as an expression over atomic propositions. Include both a mapping from symbols to propositions and the final expression.

L: Life is unexpected
D: Death is unexpected
S: Death is sudden

$L \wedge (D \to S)$
(or $L \wedge (\neg D \vee S)$
or $(L \wedge \neg D) \vee (L \wedge S)$
etc.)

2. Write simple, succinct English that means $P \wedge (\neg Q \to R)$, where P means "I like potatoes", Q means "I'm on a quest", and R means "I like radishes"

I like potatoes; also radishes if I'm not on a quest.

problem group 2 *Direct Proof*

For each of the following claims, write out a series of steps, one per line, where the first and last lines are given in the problem and each line other than the first is an application of **one** equivalence rule to the line above it (a list of equivalence rules is on the last page). Write the name of each rule next to the line it creates.

3. Prove that $\boxed{(P \wedge \neg Q) \equiv \neg(P \to Q)}$

$(P \wedge \neg Q)$	
$(\neg\neg P \wedge \neg Q)$	double negation
$\neg(\neg P \vee Q)$	De Morgan
$\neg(P \to Q)$	definition

4. Prove that $\boxed{P \to (A \vee Q) \equiv (P \wedge \neg A) \to Q}$

$P \to (A \vee Q)$	
$(\neg P) \vee (A \vee Q)$	definition
$(\neg P \vee A) \vee Q$	associativity
$\neg\neg(\neg P \vee A) \vee Q$	double negation
$\neg(\neg P \vee A) \to Q$	definition
$(\neg\neg P \wedge \neg A) \to Q$	De Morgan
$(P \wedge \neg A) \to Q$	double negation

5. Prove that $\boxed{(A \oplus B) \equiv \neg(A \leftrightarrow B)}$ *note: this is longer than we'd consider in-scope for an in-class quiz*

$A \oplus B$	
$(A \vee B) \wedge \overline{(A \wedge B)}$	definition of $\oplus$
$((A \wedge \overline{(A \wedge B)}) \vee (B \wedge \overline{(A \wedge B)}))$	distribute
$((A \wedge (\overline{A} \vee \overline{B})) \vee (B \wedge (\overline{A} \vee \overline{B})))$	De Morgan
$((A \wedge \overline{A}) \vee (A \wedge \overline{B})) \vee (B \wedge (\overline{A} \vee \overline{B})))$	distribute
$((A \wedge \overline{A}) \vee (A \wedge \overline{B})) \vee ((B \wedge \overline{A}) \vee (B \wedge \overline{B}))$	distribute
$(\bot \vee (A \wedge \overline{B})) \vee ((B \wedge \overline{A}) \vee (B \wedge \overline{B}))$	simplify
$(\bot \vee (A \wedge \overline{B})) \vee ((B \wedge \overline{A}) \vee \bot)$	simplify
$(A \wedge \overline{B}) \vee ((B \wedge \overline{A}) \vee \bot)$	simplify
$(A \wedge \overline{B}) \vee (B \wedge \overline{A})$	simplify
$(\overline{\overline{A}} \wedge \overline{B}) \vee (\overline{\overline{B}} \wedge \overline{A})$	double negation
$\overline{(\overline{A} \vee B)} \vee \overline{(\overline{B} \vee A)}$	De Morgan
$\overline{(\overline{A} \vee B) \wedge (\overline{B} \vee A)}$	De Morgan
$\overline{(A \rightarrow B) \wedge (B \rightarrow A)}$	definition of $\rightarrow$
$\overline{A \leftrightarrow B}$	definition of $\leftrightarrow$

Want more practice? Try $\boxed{A \oplus B \oplus C \equiv (A \wedge \overline{B} \wedge \overline{C}) \vee (\overline{A} \wedge B \wedge \overline{C}) \vee (\overline{A} \wedge \overline{B} \wedge C) \vee (A \wedge B \wedge C)}$*, any axiom using only the other axioms, or* $\forall x$ *chapter 15's exercise C; chapter 17 exercise B; and chapter 19 all exercises*

Symbols

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction
not P	`!p`	`not p`	$\neg P$ or $\overline{P}$	`~p`	
P and Q	`p && q`	`p and q`	$P \wedge Q$	`p & q`	PQ, $P \cdot Q$
P or Q	`p || q`	`p or q`	$P \vee Q$	`p | q`	$P + Q$
P xor Q	`p != q`	`p != q`	$P \oplus Q$	`p ^ q`	$P \veebar Q$
P implies Q			$P \rightarrow Q$		$P \supset Q$, $P \Rightarrow Q$
P iff Q	`p == q`	`p == q`	$P \leftrightarrow Q$		$P \Leftrightarrow Q$, P xnor Q

Axioms: Equivalence rules

- associativity and commutativity of $\wedge$, $\vee$, and $\oplus$; commutativity of $\leftrightarrow$
- double negation: $\boxed{\neg\neg P \equiv P}$
- simplification: $\boxed{P \wedge \bot \equiv \bot}$, $\boxed{P \wedge \top \equiv P}$, $\boxed{P \vee \bot \equiv P}$, $\boxed{P \vee \top \equiv \top}$, and $\boxed{P \wedge P \equiv P \vee P \equiv P}$
- distribution: $\boxed{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)}$ and $\boxed{A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)}$
- De Morgan: $\boxed{\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)}$ and $\boxed{\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)}$
- definitions: $\boxed{A \rightarrow B \equiv (\neg A) \vee B}$, $\boxed{(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)}$ and $\boxed{(A \oplus B) \equiv (A \vee B) \wedge \neg(A \wedge B)}$

You may use the space below for scratchwork. It will not be graded.