PROBLEM 1 Convert to prose

Convert the following symbolic proof that \( f(x) = x^2 \) to prose.

1. let \( f(x) \) be computed as
   if \( x \leq 0 \) then return 0
   else return \((2 \times x - 1) + f(x-1)\)

Symbolic Proof.

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<tbody>
<tr>
<td>1</td>
<td>( f(0) = 0 = 0^2 ) definition</td>
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<tr>
<td>2</td>
<td>( f(x) = (x-1)^2 ) assumption</td>
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<td>3</td>
<td>( f(x) = 2x - 1 + f(x-1) ) definition</td>
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<td>4</td>
<td>( f(x) = 2x - 1 + (x-1)^2 ) combine line 2 and 3</td>
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<td>5</td>
<td>( f(x) = 2x - 1 + (x^2 - 2x + 1) ) algebra on line 4</td>
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<td>6</td>
<td>( f(x) = x^2 ) simplify line 5</td>
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3. \( \forall x \geq 0 . f(x) = x^2 \) principle of induction on lines 1 and 2

Proof.
Problem 2 Code termination

Prove by induction that each of the following functions terminate given any integer argument.

2. let \( f(x) \) be computed as
   \[
   \begin{align*}
   &\text{if } x \leq 0 \text{ then return } x \\
   &\text{otherwise return } 1 + f(x-1)
   \end{align*}
   \]

   Proof.

3. let \( f(x) \) be computed as
   \[
   \begin{align*}
   &\text{if } x \leq 1 \text{ then return } x \\
   &\text{otherwise return } 1 + f(x-1) + f(x-2)
   \end{align*}
   \]

   Proof.
4. let \( f(x) \) be computed as
   \[
   \begin{align*}
   & \text{if } x \geq -1 \text{ then return } x \\
   & \text{otherwise return } 1 + f(x+1)
   \end{align*}
   \]
   \text{Proof.}

\textbf{PROBLEM 3 Code property}

Prove by induction each of the following functions returns an even number given any non-negative integer argument.

5. let \( f(x) \) be computed as
   \[
   \begin{align*}
   & y = 0 \\
   & \text{repeat } x \text{ times:} \\
   & \quad y += 2 \\
   & \text{return } y
   \end{align*}
   \]
   \text{Proof.}
6. Let \( f(x) \) be computed as  
   \[
   \text{if } x \leq 0 \text{ then return } 0 \\
   \text{else return } 4 \ast f(x-1)
   \]

   Proof.

7. Let \( f(x) \) be computed as  
   \[
   \text{if } x \leq 0 \text{ then return } 2 \\
   \text{else return } 2 \ast f(x-2)
   \]

   Proof.