PROBLEM 1  Products and Powers

Write out the following in full.

1. \(\{1, 2\} \times \{3\} \times \{1, 4\} = \) 

2. \((56)^3 = \) 

3. \(\{1, 2\} \times \mathcal{P}(\{1\}) = \) 

4. \(\{(1, 2)\} \times \mathcal{P}((1, 2, 3, 4)) = \) 

5. \(\{a, b\}^2 = \) 

6. \(\{4, 1\} \times \{1, 2\} = \) 

7. \(\{4\} \times \{1, 2\} \times \{3\}^3 = \) 

8. \(\mathcal{P}(\{}^2 = \) 

PROBLEM 2  Members of Products and Powers

Give two different example members of each of the following sets. Make them different from one another: different lengths, different internal patterns, etc., is the set allows that. If there are not enough elements of the set to give two different elements, leave some blanks blank.

9. \(\{a, b, c\}^4 \) contains _______ and _______ 

10. \(\{a, b, c\}^1 \) contains _______ and _______ 

11. \(\{a, b, c\}^0 \) contains _______ and _______ 

12. \(\{a, b, c\}^* \) contains _______ and _______
13. \( \{ \text{good}, \text{fun} \} \)^2 contains \_____________ and \_____________

Give two strings of length 3 belonging to

14. \( \{ \text{a}, \text{ok} \} \)^*: \____________ and \__________

15. \( \{ \text{a}, \text{bb}, \text{ccc} \} \)^*: \____________ and \__________

**Problem 3 Subsequences**

**Definition 1** A subsequence is a sequence that can be derived from another sequence by deleting zero or more elements without changing the order of the remaining elements.

What are the subsequences of the string “OK”?
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What is the longest subsequence shared by “MATHEMATICS” and “COMPUTERS”? 
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**Problem 4 Summation proofs**

Prove the following theorems by induction.

16. \( \forall n \in \mathbb{N} \cdot \sum_{i=0}^{n} i = \frac{n(n + 1)}{2} \)

Proof.
17. \( \forall n \in \mathbb{N} . \sum_{x=0}^{n} \frac{1}{2^x} = \frac{2^{n+1} - 1}{2^n} \)

Proof.
18. \( \forall n \in \mathbb{N} \). \( \sum_{x=n}^{2n} x = \frac{3(n + 1)n}{2} \)

Proof.
19. \( \forall x \in \{a \mid a \in \mathbb{Z} \land a \geq -1\} \cdot \sum_{k=-1}^{x} 12 - 2k = 26 + 11x - x^2 \)

Proof.
You might also try doing inductive proofs with other summation formulae, such as

\[
\sum_{i=0}^{n} i^2 = \frac{(n+1)(2n+1)(n)}{6} \\
\sum_{i=1}^{n+1} i^2 = \frac{(n+2)(2n+3)(n+1)}{6} \\
\sum_{i=2}^{n+2} i^2 = \frac{(n+3)(2n+5)(n+2)}{6} \\
6 \sum_{i=0}^{n} i^3 - i = \left(\frac{n+2}{4}\right) \\
\sum_{x=0}^{n} \frac{x^2 - 1}{x + 1} = \frac{(n+1)(n-1)}{2} \\
\sum_{x=0}^{n} x^3 - x^2 = \frac{(n+1)(3n+2)(n)(n-1)}{12} \\
\sum_{i=0}^{n} 3i^2 + 2i = \frac{(2n+3)(n+1)(n)}{2} \\
\sum_{x=n}^{x} x = \frac{n + n^4}{2} \\
\sum_{x=0}^{2n} (-1)^x = n \\
\sum_{i=1}^{n} \frac{1}{2^i} = \frac{2^n - 1}{2^n} \\
\sum_{k=-n}^{0} k = \frac{(n+1)n}{-2} \\
\sum_{i=1}^{n} \frac{1}{3i} = \frac{3^n - 1}{3^n} \\
\forall k \neq 1, \left(\sum_{i=1}^{n} \frac{1}{k^i} = \frac{k^n - 1}{k^n(k-1)}\right) \\
\]

Note: at least one of the above formulae is false. In the process of proving it you should find the normal methods not working, revealing the non-truth.