**Problem 1** Convert to prose

- **P**: the set of all single-input functions
- **I**: the set of all inputs
- **C(p, i)**: p crashes when run on i

Convert the following to simple, readable English. Make sure your answer shows how the questions are different:

1. \( \exists p \in P . \forall i \in I . C(p, i) \)

   There’s a program that crashes no matter what input you give it.

2. \( \exists i \in I . \forall p \in P . C(p, i) \)

   There’s one special input that will crash any program you run it on.

3. \( \forall p \in P . \exists i \in I . C(p, i) \)

   Every program has some input it crashes on.

4. \( \forall i \in I . \exists p \in P . C(p, i) \)

   Every input has some program it crashes.

Convert the following to logic:

5. If a program crashes on any input, it crashes on more than one input.

   \( \forall p \in P . (\exists i \in I . C(p, i)) \rightarrow (\exists i, j \in I . i \neq j \land C(p, i) \land C(p, j)) \)

6. No program crashes on every input.

   \( \forall p \in P . \exists i \in I . \neg C(p, i) \)
   
   — or —
   
   \( \nexists p \in P . \forall i \in I . C(p, i) \)
PROBLEM 2 Identify domain and range

7. If the domain of \( f(x) = x^2 \) is \( \mathbb{R} \), it’s range is \( \mathbb{R}^+ \cup \{0\} \)

8. If the domain of \( f(x) = x^2 \) is \( \mathbb{N} \), it’s range is the perfect squares \( \{0, 1, 4, 9, 16, ...\} \)

9. If the domain of \( f(x) = x^3 \) is \( \mathbb{R} \), it’s range is \( \mathbb{R} \)

10. If the codomain of \( f(x) = \frac{1}{2x} \) is \( \mathbb{N} \) and \( f \) is total, \( \mathbb{Z} \cap \) its domain is \( \mathbb{Z}^+ \cup \{0\} \)

PROBLEM 3 Provide example functions

In each blank, define a total function \( f : \mathbb{Z} \rightarrow \mathbb{Z} \)

11. Give an example injective (1-to-1) and surjective (onto) function: \( f(x) = x + 1 \)

12. Give an example injective (1-to-1) but not surjective (not onto) function: \( f(x) = 2x \)

13. Give an example non-injective (not 1-to-1) but surjective (onto) function: \( f(x) = \left\lfloor \frac{x}{2} \right\rfloor \)

14. Give an example neither injective (not 1-to-1) not surjective (not onto) function: \( f(x) = x^2 \)
In each blank, define a function \( f : \mathbb{N} \rightarrow \mathbb{N} \) or relation \( R : \mathbb{N} \times \mathbb{N} \rightarrow \{\top, \perp\} \)

15. Give an example function that is not total: \( f(x) = x - 1 \)

16. Give an example function that is total but not invertable: \( f(x) = (x - 1)^2 \)

17. Give the relation corresponding to the function \( f(x) = 3x \): \( R(a, b) : a = 3b \)

18. Give an example relation that is not a function: \( R(x, y) = x < y \)

In each blank, define a function \( f : \mathbb{R} \rightarrow \mathbb{R} \)

Give an example function that is not total: \( f(x) = \sqrt{x} \)

Give an example function that is total but not invertable: \( f(x) = x^2 \)

Give an example function that is invertable: \( f(x) = x \)

See also §4 Problems 4.12–4.33