PROBLEM 1  Convert to prose

$P$: the set of all single-input functions

$I$: the set of all inputs

$C(p, i)$: $p$ crashes when run on $i$

Convert the following to simple, readable English. Make sure your answer shows how the questions are different:

1. $\exists p \in P . \forall i \in I . C(p, i)$

2. $\exists i \in I . \forall p \in P . C(p, i)$

3. $\forall p \in P . \exists i \in I . C(p, i)$

4. $\forall i \in I . \exists p \in P . C(p, i)$

Convert the following to logic:

5. If a program crashes on any input, it crashes on more than one input.

6. No program crashes on every input.
Problem 2 Identify domain and range

7. If the domain of \( f(x) = x^2 \) is \( \mathbb{R} \), its range is ____________________________

8. If the domain of \( f(x) = x^2 \) is \( \mathbb{N} \), its range is ____________________________

9. If the domain of \( f(x) = x^3 \) is \( \mathbb{R} \), its range is ____________________________

10. If the codomain of \( f(x) = \frac{1}{2^x} \) is \( \mathbb{N} \) and \( f \) is total, \( \mathbb{Z} \cap \) its domain is ____________________________

Problem 3 Provide example functions

In each blank, define a total function \( f : \mathbb{Z} \to \mathbb{Z} \)

11. Give an example injective (1-to-1) and surjective (onto) function: ____________________________

12. Give an example injective (1-to-1) but not surjective (not onto) function: ____________________________

13. Give an example non-injective (not 1-to-1) but surjective (onto) function: ____________________________

14. Give an example neither injective (not 1-to-1) nor surjective (not onto) function: ____________________________
In each blank, define a function $f : \mathbb{N} \to \mathbb{N}$ or relation $R : \mathbb{N} \times \mathbb{N} \to \{\top, \bot\}$

15. Give an example function that is not total: $f(x) =$ ______________

16. Give an example function that is total but not invertable: $f(x) =$ ______________

17. Give the relation corresponding to the function $f(x) = 3x$: $R(a, b):$ ______________

18. Give an example relation that is not a function: $R(x, y) =$ ______________

In each blank, define a function $f : \mathbb{R} \to \mathbb{R}$

    Give an example function that is not total: $f(x) =$ ______________

    Give an example function that is total but not invertable: $f(x) =$ ______________

    Give an example function that is invertable: $f(x) =$ ______________

See also §4 Problems 4.12–4.33