1. Write simple, succinct English that means $(P \land Q) \lor (\lnot P \land \lnot Q)$, where $P$ means “I like potatoes”, $Q$ means “I’m on a quest”

I like potatoes when on a quest, but not when not.
— or —
I’m on a quest if and only if I like potatoes.

etc.

2. Rewrite “I’ll win if neither hurt nor tired” as an expression over atomic propositions. Include both a mapping from symbols to propositions and the final expression.

W: I’ll win
H: I’m hurt
T: I’m tired

$\neg (H \lor T) \rightarrow W$

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**Problem Group 2** Direct Proof

For each of the following claims, write out a series of steps, one per line, where the first and last lines are given in the problem and each line other than the first is an application of one equivalence rule to the line above it (a list of equivalence rules is on the next page). Write the name of each rule next to the line it creates.

3. Prove that \((P \rightarrow Q) \equiv \lnot (\lnot Q \rightarrow \lnot P)\)

counterexample: if $P$ and $Q$ are both \(\bot\), the left-hand side is \(\top\) and the right-hand side is \(\bot\). Ergo, this cannot be proven.

4. Prove that \(A \rightarrow (B \rightarrow C) \equiv (A \land B) \rightarrow C\)

\[ A \rightarrow (B \rightarrow C) \]
\[ A \lor (B \rightarrow C) \] definition
\[ A \lor (B \lor C) \] definition
\[ (A \lor B) \lor C \] associativity
\[ (A \land B) \lor C \] De Morgan
\[ (A \land B) \rightarrow C \] definition
Symbols

<table>
<thead>
<tr>
<th>Concept</th>
<th>Java/C</th>
<th>Python</th>
<th>This class</th>
<th>Bitwise</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>True</td>
<td>T or 1</td>
<td>-1</td>
<td>T, tautology</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
<td>False</td>
<td>⊥ or 0</td>
<td>0</td>
<td>F, contradiction</td>
</tr>
</tbody>
</table>
| not P   | !p     | not p  | ~P or P
| P and Q | p && q | p and q | P & Q      | p & q   | P, · Q       |
| P or Q  | p || q | p or q | P ∨ Q      | p | q    | P + Q       |
| P xor Q | p != q | p != q | P ⊕ Q      | p ^ q   | P ⊻ Q       |
| P implies Q | P → Q | P ⊃ Q, P ⇒ Q |
| P iff Q | p == q | p == q | P ↔ Q      | P ⇔ Q, P xor Q |

Axioms: Equivalence rules

- associativity and commutativity of ∧, ∨, and ⊕; commutativity of ↔
- double negation: \( \neg \neg P \equiv P \)
- simplification: \( P \land \perp \equiv \perp \), \( P \lor \perp \equiv P \), \( P \lor \top \equiv \top \), and \( P \land P \equiv P \lor P \equiv P \)
- distribution: \( A \land (B \lor C) \equiv (A \land B) \lor (A \land C) \) and \( A \lor (B \land C) \equiv (A \lor B) \land (A \lor C) \)
- De Morgan: \( \neg (A \land B) \equiv (\neg A) \lor (\neg B) \) and \( \neg (A \lor B) \equiv (\neg A) \land (\neg B) \)
- definitions: \( A \rightarrow B \equiv (\neg A) \lor B \) and \( A \leftrightarrow B \equiv (A \rightarrow B) \land (B \rightarrow A) \) and \( A \oplus B \equiv (A \lor B) \land \neg (A \land B) \)

You may use the space below for scratchwork. It will not be graded.