Problem 1 Convert to prose

Convert the following symbolic proof that \( f(x) = (x)(x + 1) \) to prose.

1. let \( f(x) \) be computed as
   - if \( x \leq 0 \) then return 0
   - else return \( 2x + f(x-1) \)

Symbolic Proof.

\[
\begin{array}{c|cc}
1 & f(0) = 0 = (0)(0 + 1) & \text{definition} \\
2 & f(x-1) = (x-1)(x) & \text{assumption} \\
3 & f(x) = 2x + f(x-1) & \text{definition} \\
4 & f(x) = 2x + (x - 1)(x) & \text{combine line 2 and 3} \\
5 & f(x) = 2x + (x^2 - x) & \text{algebra on line 4} \\
6 & f(x) = x^2 + x & \text{algebra on line 5} \\
7 & f(x) = (x)(x + 1) & \text{simplify line 6} \\
3 & \forall x \geq 0 . f(x) = (x)(x + 1) & \text{principle of induction on lines 1 and 2} \\
\end{array}
\]

Proof.

We proceed by induction on \( x \).

**Base case:** Assume \( x = 0 \). Then \( f(x) = 0 \), which is equal to \( (x)(x + 1) \) when \( x = 0 \).

**Inductive step:** Assume that \( x > 0 \) and that \( f(x-1) = (x-1)\times(x) \). Then \( f(x) = 2x + f(x-1) \) which, by the assumption, is \( 2x + (x-1)\times(x) \); rearranging, that becomes \( xx + x \), which is \( (x)(x + 1) \).

By the principle of induction, it follows that \( f(x) \) always returns \( (x)(x + 1) \). \( \square \)
**Problem 2 Code termination**

Prove by induction that each of the following function terminates given any natural number argument.

2. let \( f(x) \) be computed as
   
   ```plaintext
   if \( x = 0 \) then return 1
   otherwise return \( 2 \times f(x-1) \)
   ```
   
   Proof.

   We proceed by induction on \( x \).

   **Base case:** Assume \( x = 0 \). Then the function terminates immediately by taking the first branch of the `if` statement.

   **Inductive step:** Assume that \( x > 0 \) and that \( f(x-1) \) terminates. Then the function takes the second branch of the `if` statement and terminates after invoking \( f(x-1) \) and performing one multiplication.

   By the principle of induction, it follows that \( f(x) \) terminates for all integer \( x \). \( \square \)