PROBLEM 1 Convert to prose

$S$: the set of all snakes

$R$: the set of all rabbits

$E(x, y)$: $x$ eats $y$

$Y(x)$: $x$ is yellow

Convert the following to simple, readable English:

1. $\exists r \in R . \forall s \in S . (E(s, r) \rightarrow \neg Y(s))$

   There is a rabbit that no yellow snake eats.

PROBLEM 2 Primes and factors

2. $2 \cdot 3^2$ is the prime factorization of 18

3. $3^4$ is the prime factorization of 81

4. $2^{20} \cdot 3^{40}$ is the prime factorization of $9^{10} \cdot 6^{20}$

5. $(1, 3, 7, 9)$ is the set of positive 1-digit numbers relatively prime with 10
PROBLEM 3  Proof by contradiction

Prove the following using proof-by-contradiction. You may use prose or symbols or any readable mix of the two.
6. \( \frac{7}{3} \notin \mathbb{Z} \)

Proof.

We proceed by contradiction.
Assume \( \frac{7}{3} \in \mathbb{Z} \); let \( x \in \mathbb{Z} \) be the element of \( \mathbb{Z} \) that equals \( \frac{7}{3} \). Thus, \( \frac{7}{3} = x \), which can be re-written as \( 7 = 3x \). By the fundamental theorem of arithmetic, both must have the same prime factors, but 3 is a factor of \( x \) and is not a factor of 7, a contradiction.
Because assuming \( \frac{7}{3} \in \mathbb{Z} \) led to a contradiction, it must be the case that \( \frac{7}{3} \notin \mathbb{Z} \). \( \square \)