Easy

English
Boolean arithmetic

Hard

$+$

$-$

$\times$

$\div$

Concepts

Pacing

$\wedge$

$\vee$

$\neg$

$\rightarrow$

$\oplus$

$\leftrightarrow$

Boolean   Algebra

x	y	
		$3(x+y) \overset{?}{=} 3x + 3y$
0	0	$\underline{3(0+0)} = \underline{3\cdot 0 + 3\cdot 0}$ ✓
		$0 \qquad\qquad 0$
0	1	$\underline{3(0+1)} = \underline{3\cdot 0 + 3\cdot 1}$ ✓
		$3 \qquad\qquad 3$
		$\vdots$

equiv

double negation

$$\neg \neg \neg P \equiv P$$

assertion about expressions

$$\neg \neg P \leftrightarrow P \qquad \longrightarrow \quad \top \text{ or } \bot$$

P	$\neg (\neg \; P)$	
$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\top$

$\neg R \Rightarrow \neg \neg (P \wedge Q)$
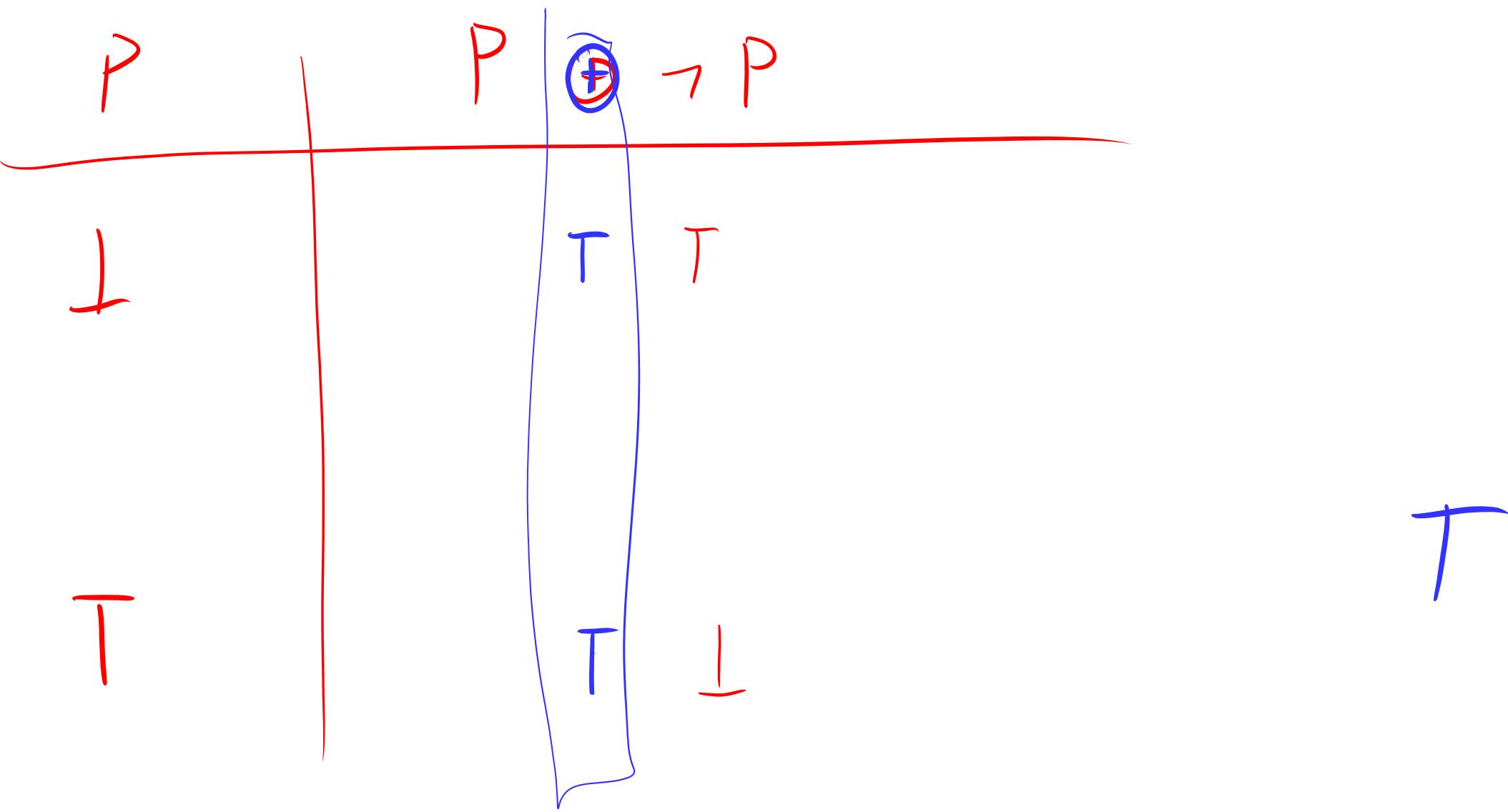
$\neg R \Rightarrow (P \wedge Q)$

$\neg \neg (\neg \neg \neg R \Rightarrow \neg \neg (\neg \neg \neg \neg P \wedge \neg \neg Q))$

$\times\ 0$

$\times\ 1$

$+\ 0$

P

$\bot$

T

$P \quad \neg P$

$T \quad T$

$T \quad \bot$

T

$$x + x \qquad\qquad 2x$$

$$x - x \qquad\qquad 0$$

$$x \times x \qquad\qquad x^2$$

associative

+

paren can add / rem

commutative

+

rearrange

$2+3$

$(2+3)+5$

$2-3$

$3+2$

$(\quad)$

$3-2$

$(2-3)-5 \quad \rightarrow \quad -6$

$2-(3-5) \quad \rightarrow \quad 4$

	Assoc ~~nonsensical~~	Comm
$\neg$		
$\wedge$	✓	✓
$\vee$	✓	✓
$\oplus$	✓	✓
$\rightarrow$	✗	✗
$\leftrightarrow$	✗	✓
$+$	✓	✓
$\times$	✓	✓

$$(A \leftrightarrow B) \leftrightarrow C \neq A \leftrightarrow (B \leftrightarrow C)$$

$$A \leftrightarrow B$$
$$B \leftrightarrow A$$

$$A \vee (B \wedge C) \overset{?}{\neq} (A \wedge C) \vee B$$

$$x + (y \times 2) \overset{?}{\neq} (x \times 2) + y$$

$$P \rightarrow Q \equiv \neg P \lor Q$$

De Morgan's Law

$$\neg(A \land B) \equiv (\neg A \lor \neg B) \qquad \neg(A \lor B) \equiv (\neg A \land \neg B)$$

Distributive laws

$$3(x+y) \hookrightarrow 3x + 3y$$

$$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$$
$$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$$