



$\forall x. P(x)$

$\exists x. P(x)$ — I pick my favorite x

$P(x)$: x is the sum of its factors

$$1 = 1 \quad \checkmark$$

$$6 = 1 + 2 + 3$$

$\exists x. x > 6 \wedge P(x)$

$\forall x. P(x)$

$-2^{31} \dots 2^{31}$

Prove $P(x)$ assuming nothing about x

$P(x): \underbrace{x+x \text{ is even}}$

$x+x = 2x$ (is even)

Equivalences

entailments

\exists instantiation

\forall instantiation

$$\begin{array}{l} \vdash A \equiv B \\ \vdash A \\ \therefore B \end{array}$$

$$\begin{array}{l} \vdash A \equiv B \\ \vdash A \\ \therefore B \end{array}$$

By Cases

$$\begin{array}{l} \vdash A \vee B \\ A \vdash C \\ B \vdash C \\ \therefore C \end{array}$$

$$\begin{array}{l} \vdash P(a) \text{ for an } a \\ \therefore \exists x. P(x) \end{array}$$

$$\begin{array}{l} \vdash P(a) \text{ with no know of } a \\ \therefore \forall x. P(x) \end{array}$$

Raining or not Raining

assume raining,
I use an umbrella
 \therefore not wet

assume not raining
 \therefore not ~~wet~~

o^c o^c I am not wet

$$a \oplus b \equiv b \oplus a$$

and prove

1. state ¹ disjunction
either a is \top or \perp

2. prove for each case
Case $a = \top$

$$\begin{aligned} a \oplus b &\equiv \top \oplus b \\ &\equiv \neg b \\ &\equiv b \oplus \top \\ &\equiv b \oplus a \end{aligned}$$

combine results

3. Since $a \oplus b \equiv b \oplus a$ in all cases, it is true in general

Show

and

$$\begin{aligned} a \oplus b &\equiv b \oplus a \\ b \oplus a &\equiv a \oplus b \end{aligned}$$

Case $a = \perp$

$$\begin{aligned} a \oplus b &\equiv \perp \oplus b \\ &\equiv b \\ &\equiv b \oplus \perp \\ &\equiv b \oplus a \end{aligned}$$

□