$f(456\ 789) = 0$

$f(0) = 0$

$f(1) = 0$

$f(2) = 0$

$f(x)$

if $x < 1$ then return $x$

else return $f(x-1)$

Theorem $f(x)$ terminates for all $x \in \mathbb{N}$

Proof. we proceed by induction on $x$

Base case: $f(0)$ terminates because $0 < 1$ so the function returns $0$

Inductive step: assume $f(x-1)$ terminates. Then $f(x)$ returns $f(x-1)$, which terminates, so $f(x)$ also terminates.

By principle of induction, $\forall x \in \mathbb{N}$. $f(x)$ terminates proof by cases

$A \vee B$

$\boxed{A \vdash C}$

$\boxed{B \vdash C}$

$\therefore C$

subproof

assume $A$

prove $C$

induction

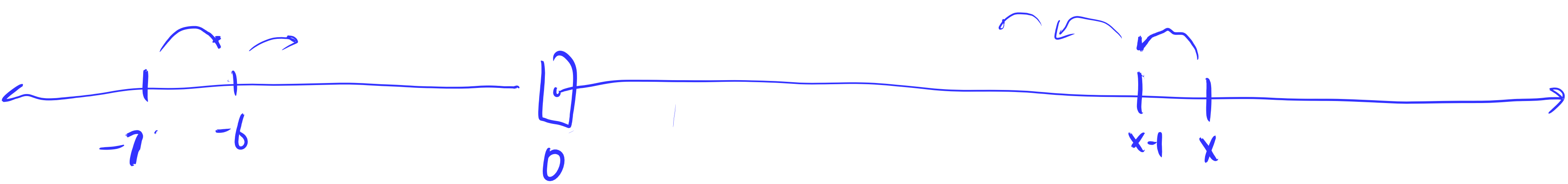
$P(0)$

assume it

$\boxed{P(x-1)} \vdash P(x)$

$\therefore \forall x \in \mathbb{N} .\ P(x)$

$P(x) \vdash P(x+1)$



$f(x)$

if $\boxed{x \le 1}$ $\boxed{\text{ret } x}$

else ret $f(x-1)$

prove term $\forall x \in \mathbb{Z}$

Base : $\underline{x < 1}$. term by returning $x$

ind : assum $f(x-1)$ term

then $f(x)$ term by return $f(x-1)$.

By induction, done.

$f(x)$

$x < 1$ , $x$

else $f(x-1)$

Prove return $0$ $\forall x \in \mathbb{N}$

Base case $x = 0$ return $x = 0$ ✓

Inductive step assume $f(x-1) = 0$. then $f(x)$ returns $f(x-1) = 0$, so $f(x) = 0$.

by principle of induction, $\forall x \in \mathbb{N}.\ f(x) = 0$

$\forall x \in \mathbb{R}^{+}.\ f(x) \neq x + 1 - \lceil x \rceil$

$= x - \lfloor x \rfloor$

$\lfloor x \rfloor$ floor

$\lceil x \rceil$ ceiling

$f(x)$

if $x < 1$ ret $x$

else ret $f(x-1)$

$\lfloor 2.1 \rfloor = 2$ $\lceil 2.1 \rceil = 3$

$\lfloor 2.9 \rfloor = 2$ $\lceil 2.9 \rceil = 3$

Base $0 \le x < 1$ return $x = x - 0 = x - \lfloor x \rfloor$ ✓

Inductive assum $f(x-1) = x-1 - \lfloor x-1 \rfloor$

then $f(x)$ returns $f(x-1) = x-1 - \lfloor x-1 \rfloor$

$(x-1+1) - \lfloor x-1+1 \rfloor = x - \lfloor x \rfloor$