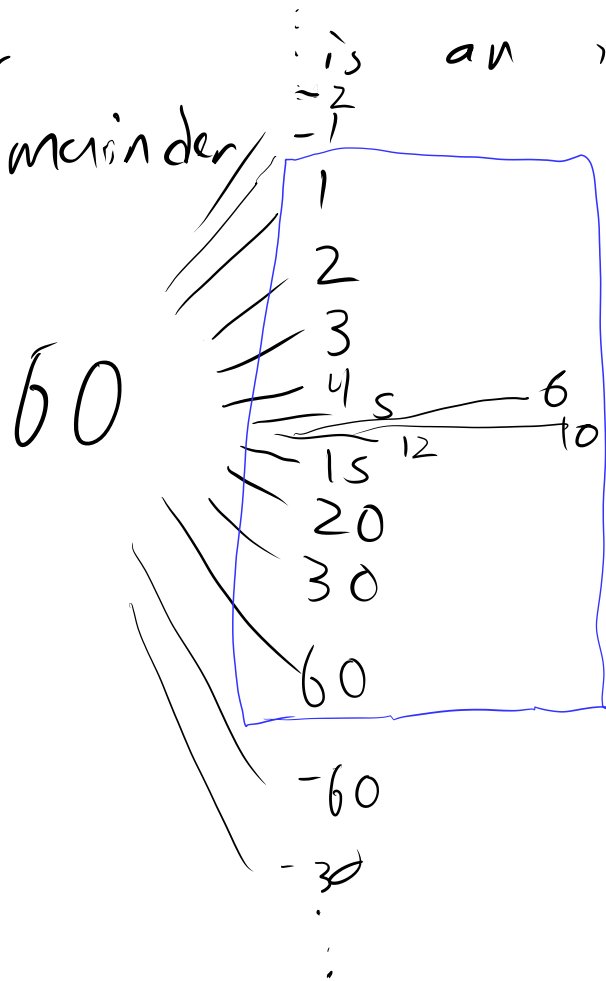


factor of integer
with no remainder

is an integer that divides it



natural num
more common

Prime: x is prim
iff only ^{natural} factors are
1 and x

2, 3, 5, 7, 11, 13, 17, 23, ...

Fundamental Theorem of Arithmetic

$\forall n \in \mathbb{N}. \exists$ factorization of n . all factors are prime

A factorization is unique

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

↓
multiplicity

$$2^2 \cdot 3^1 \cdot 5^1$$

gcd Greatest Common Divisor

$$\text{gcd}(60, 8) = 4 = \begin{array}{l} \boxed{2 \cdot 2 \cdot 3 \cdot 5} = 60 \\ \boxed{2 \cdot 2 \cdot 2} = 8 \end{array}$$

lcm Least Common Multiple

$$\text{lcm}(60, 8) = 120 = \begin{array}{l} \boxed{2 \cdot 2 \cdot 3 \cdot 5} \\ \boxed{2 \cdot 2 \cdot 2} \end{array}$$

relatively prime
 x, y relatively prime iff $\text{gcd}(x, y) = 1$
60, 77

proof by contradiction

assum \rightarrow Prove

$$\boxed{A} \vdash \perp$$

$$\therefore \neg A$$

thm: $\frac{1}{2} \notin \mathbb{N}$

assume $\frac{1}{2} \in \mathbb{N}$. thus $\exists x \in \mathbb{N}, \frac{1}{2} = x$

$$\frac{1}{2} = x$$

$$1 = 2x$$

by fund. Thm. of Arithm., exist unique prime factorizations

$$p \text{ factors}(1) = p \text{ factors}(2x)$$

$$\{\} = \{2\} \cup p \text{ factors}(x)$$

0 contradiction!

Since assumption $\frac{1}{2} \in \mathbb{N}$ led to a contradiction, it must be that $\frac{1}{2} \notin \mathbb{N}$

Proof.

We proceed by contradiction.

Assume $\frac{1}{2} \in \mathbb{N}$. Then $\exists x \in \mathbb{N} . x = \frac{1}{2}$. By algebra, that means $2x = 1$. By the fund. Thm. of Arith., 1 and $2x$ must have the same prime factorization. But the factors of $2x$ include 2, and the factors of 1 do not. This is a contradiction.

Buts Assm $\frac{1}{2} \in \mathbb{N}$ result in contradiction, $\frac{1}{2} \notin \mathbb{N}$ \square