factor of integer ⋮ is an integer that divides it with no remainder

60

⋮
-2
-1
1
2
3
4
5
6
10
12
15
20
30
60
-60
-30
⋮

natural num more common

Prime: x is prim iff only natural factors are 1 and x

2, 3, 5, 7, 11, 13, 17, 23, ...

Fundamental Theorem of Arithmatic

$\forall n \in \mathbb{N}.\ \exists$ factorization of n. all factors are prime

$\wedge$ factorization is unique

$$60 = \underbrace{2 \cdot 2}_{\text{multiplicity}} \cdot 3 \cdot 5$$

$$2^2 \cdot 3^1 \cdot 5^1$$

gcd Greatest Common Divisor

$$\gcd(60, 8) = 4 = \begin{matrix} \boxed{2 \cdot 2} \cdot 3 \cdot 5 & = 60 \\ \boxed{2 \cdot 2} \cdot 2 & = 8 \end{matrix}$$

lcm Least Common Multiple

$$\text{lcm}(60, 8) = 120 = \begin{matrix} 2 \cdot 2 \cdot 3 \cdot 5 \\ 2 \cdot 2 \cdot 2 \end{matrix}$$

relatively Prime

x, y relatively prime iff $\gcd(x, y) = 1$

60, 77

proof by contradiction

assum to prove

$$\boxed{A} \vdash \bot$$

$$\therefore \neg A$$

thm: $\frac{1}{2} \notin \mathbb{N}$

assume $\frac{1}{2} \in \mathbb{N}$. thus $\exists x \in \mathbb{N} .\ \frac{1}{2} = x$

$$\frac{1}{2} = x$$

$$1 = 2x$$

by fund. Thm. of Arithm., exist unique prime factorizations

$$\text{pfactors}(1) = \text{pfactors}(2x)$$

$$\{\} = \{2\} \cup \text{pfactors}(x)$$

$$0 \quad \text{contradiction}$$

Since assuming $\frac{1}{2} \in \mathbb{N}$ led to a contradiction, it must be that $\frac{1}{2} \notin \mathbb{N}$

Proof.

We proceed by contradiction.

Assume $\frac{1}{2} \in \mathbb{N}$. Then $\exists x \in \mathbb{N}$. $x = \frac{1}{2}$. By algebra, that means $2x = 1$. By the fund. Thm. of Arith, 1 and $2x$ must have the same prime factorization. But the factors of $2x$ include 2, and the factors of 1 do not. This is a contradiction.

Bc Asm $\frac{1}{2} \in \mathbb{N}$ result in contradiction, $\frac{1}{2} \notin \mathbb{N}$ $\square$