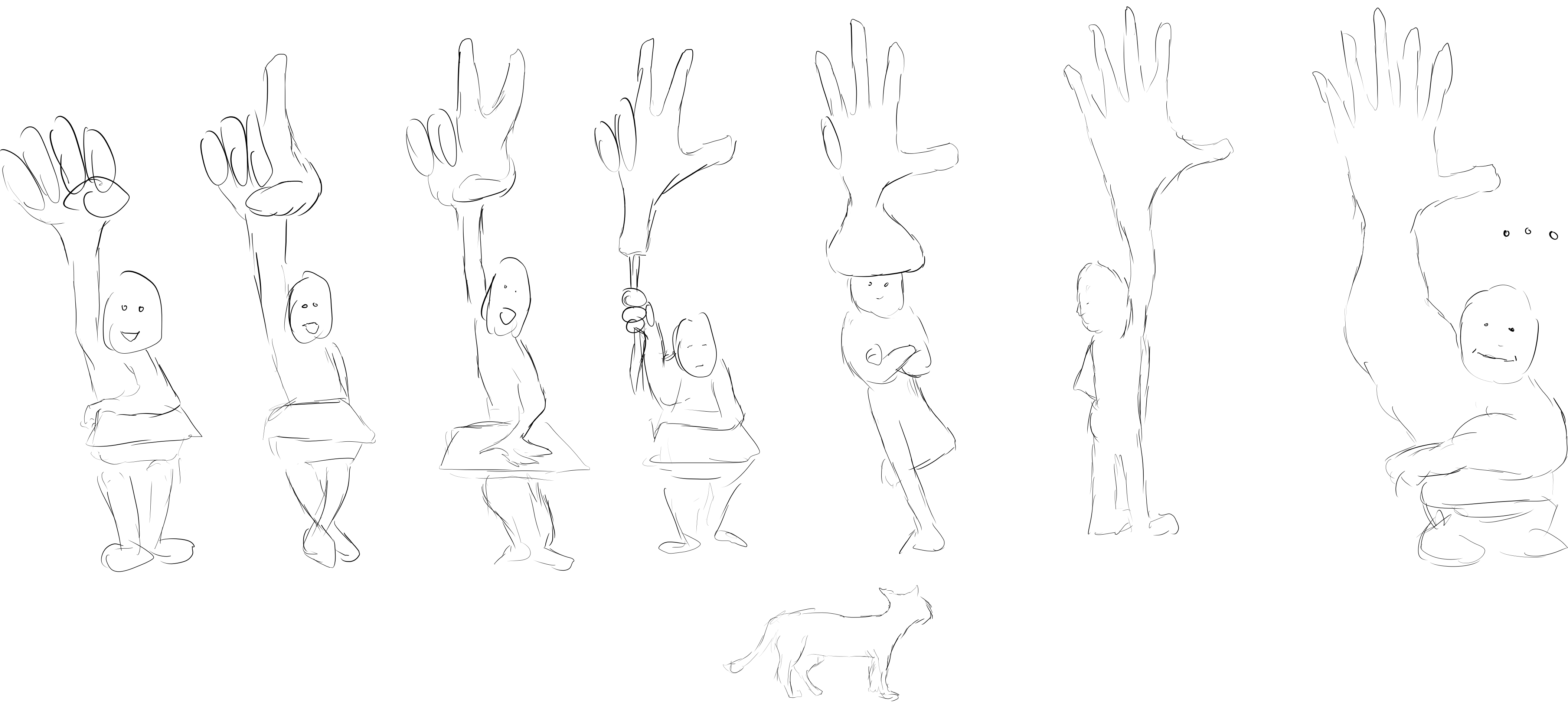

Counting

\# times an instruction runs

\# passwords that can exist

bytes used by DS

⋮

$$\sum_{i=1}^{n} i = 1+2+3+\dots+n-1+n$$

$$i \in \{x \mid x \in \mathbb{Z} \wedge x \geq 1 \wedge x \leq n\}$$

$$\sum_{i=2}^{1} i = 0$$

$$\sum_{i=2}^{2} x^i = x^2$$

$$\sum_{i \in \mathbb{N}} \frac{1}{2^i} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \dots$$

$$\sum_{i \in \emptyset} i = 0$$

$$\{\}$$

True when $n \in \{-1, 0, 1, \dots \infty\}$

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

$$\sum_{i=1}^{1} i = 1 \qquad \frac{1\overbrace{(1+1)}^{2}}{2} = 1$$

$$\sum_{i=1}^{-1} i = 0 \qquad \frac{\overbrace{(-1)(\overbrace{-1+1}^{0})}^{0}}{2} = 0$$

$$\sum_{i=1}^{2} i = 1+2 = 3 \qquad \frac{\cancel{2}(2+1)}{\cancel{2}} = 3$$

$$\sum_{i=1}^{-2} i = 0 \qquad \frac{\overbrace{(-2)(\overbrace{-2+1}^{-1})}^{2}}{2} = 1$$

Thm $\boxed{\forall n \in \mathbb{N}}$. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Proof. by induction

base case

$n=0$ $\quad \sum_{i=1}^{0} i = 0 \quad \frac{0(0+1)}{2} = 0$

$n=1$ $\quad \sum_{i=1}^{1} i = 1 \quad \frac{1(1+1)}{2} = 1$

inductive step

assume true for $n=k$

$\rightarrow \boxed{\sum_{i=1}^{k} i = \frac{k(k+1)}{2}}$

consider case $n = k+1$

$$\underbrace{k+1 + \sum_{i=1}^{k} i}_{\sum_{i=1}^{k+1} i} = \frac{k(k+1)}{2} + k+1$$

$$\sum_{i=1}^{k+1} i = \underbrace{\frac{k^2+k}{2} + \frac{2k+2}{2}}_{\frac{k^2+3k+2}{2}}$$

$$\rightarrow \sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

$$\sum_{i=1}^{k} i = 1 + 2 + 3 + \dots + k-1 + k$$

$-1 \quad -2$

$$(k+1) + \sum_{i=1}^{k} i = \sum_{i=1}^{k+1} i$$

$$-2 + -1 + \sum_{i=1}^{k} i = \sum_{i=3}^{k} i$$

$$\begin{matrix} +k+2 \\ +k+1 \\ -1 \\ -2 \end{matrix} + \sum_{i=1}^{k} i = \sum_{i=3}^{k+2} i$$

$$\sum_{i=1}^{k+1} i = 1 + 2 + 3 + \dots + k-1 + k + \boxed{(k+1)}$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = 1$$

$$\boxed{\forall n \in \mathbb{Z}^+.\ \sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}}$$

$i=1 \rightarrow \frac{1}{2}$

$i=2 \rightarrow \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$i=3 \rightarrow \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

$2^{k+1} = 2^1 \cdot 2^k$

Base case: $i=1$

$$\sum_{i=1}^{n} \frac{1}{2^i} = \frac{1}{2} \qquad 1 - \frac{1}{2^n} = \frac{1}{2} \checkmark$$

Induclve

assum $\sum_{i=1}^{k} \frac{1}{2^i} = 1 - \frac{1}{2^k}$

$$\underbrace{\frac{1}{2^{k+1}} + \sum_{i=1}^{k} \frac{1}{2^i}}\ \underbrace{= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}}$$

$$\sum_{i=1}^{k+1} \frac{1}{2^i} = 1 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$$

$$\underbrace{1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}}$$

$$1 - \frac{2 \cdot 1}{2 \cdot 2^k} + \frac{1}{2^{k+1}}$$

$$1 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}}$$

$$1 + \frac{-2+1}{2^{k+1}}$$

$$1 + \frac{-1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$$