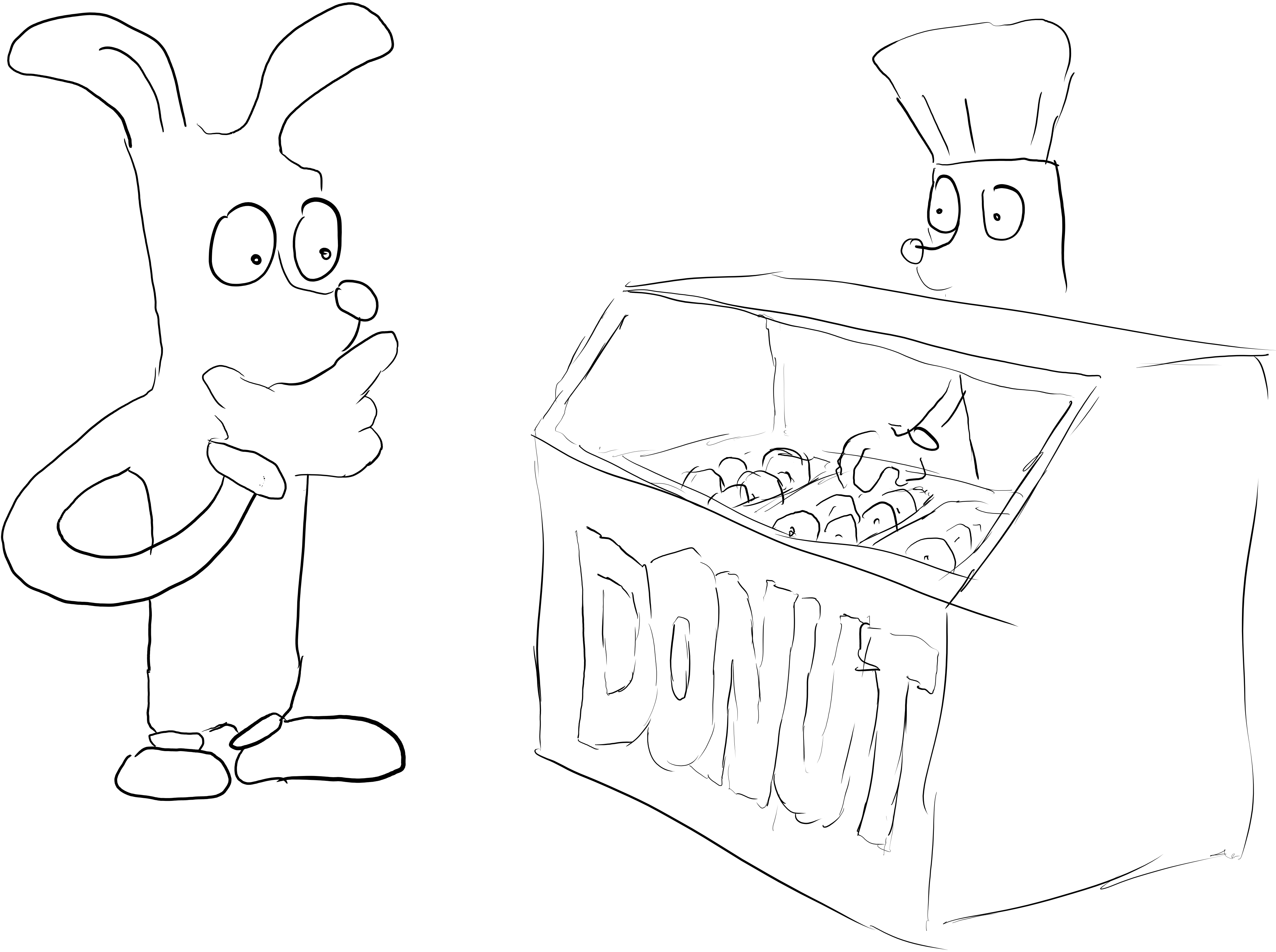
DONUT

```
∃x.P(x) -- optimistic, any true wins

for each x:
    if P(x): return T
return ⊥
```

```
∃x. ∀y. P(x,y) -- some pessimist satisfied

for each x:
    all_ans = T
    for each y:
        if ¬P(x,y): all_ans = ⊥
    if all_ans: return T
return ⊥
```

```
∀x.P(x) -- pessimistic, any false wins

for each x:
    if ¬P(x): return ⊥
return T
```

```
∀x. ∃y. P(x,y) -- all optimists satisfied

for each x:
    exists_ans = ⊥
    for each y:
        if P(x,y): exists_ans = T
    if ¬exists_ans: return ⊥
return T
```

Counting

Or $+$

and $*$

indistinguishable $\div$

not part $-$

# of sets of 2 digits

$\{x, y\}$ (10, 10)

indu $\{0,3\}$ $\{3,0\}$

$\{0,0\}$ - malform

$$\frac{100}{2} - 10 \neq \frac{100 - 10}{2}$$

0, 1

$\{0,0\}$

$\{0,1\}$

$\{1,0\}$

$\{1,1\}$

$$\frac{4}{2} - 2 = 0$$

$$\frac{4-2}{2} = 1$$

$\boxed{\text{\# of subsets of } A} = |\mathcal{P}(A)| = \boxed{2^{|A|}}$

$3! = 3 \cdot 2 \cdot 1 = 6$

Perm $(1,2,3)$

1 2 3
1 3 2
2 1 3
2 3 1
3 1 2
3 2 1

$\boxed{\text{\# of permutations of } S = \text{len}(S)!}$

$\boxed{\text{\# of } \underbrace{k\text{-elem}} \text{ subsets of } n\text{-elem set}}$

shuffle n-elem set as seq → $n!$ $*$
split into k, $n-k$ → 1 $\div$
unshuffle k → $k!$ $\div$
unshuffle $n-k$ → $(n-k)!$

$$= \frac{n!}{k!(n-k)!} = \boxed{\binom{n}{k}}$$ "n choose k"

$\{1,2,3,4,5\}$

2 3 | 1 4 5

2-elem subset of $\{1, 2, 3, 4, 5\}$ — Start 41352 51234

$\{1, 2\}$
1 3
1 4
1 5
2 3
2 4
2 5
3 4
3 5
4 5

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ split

$\{1, 4\}$ $\{1, 5\}$

$$\frac{5!}{2!\;3!} = \frac{120}{2 \cdot 6} = 10$$

$\boxed{1\,4}\,3\,5\,2$

41 532

$z, x, y \in \mathbb{Z}^+$

$\boxed{x + y = 4}$

$1 + 3$

$2 + 2$

$3 + 1$

$x + y + z = 8$

$1 + 1 + 6$

$1 + 2 + 5$

$1 + 3 + 4$

$\vdots$

$2 + 1 + 5$

$2 + 2 + 4$

$\vdots$

analogy

unique representation

o o | o o

o | o o o

o o o | o

o | o o o | o o o o

1 3 4