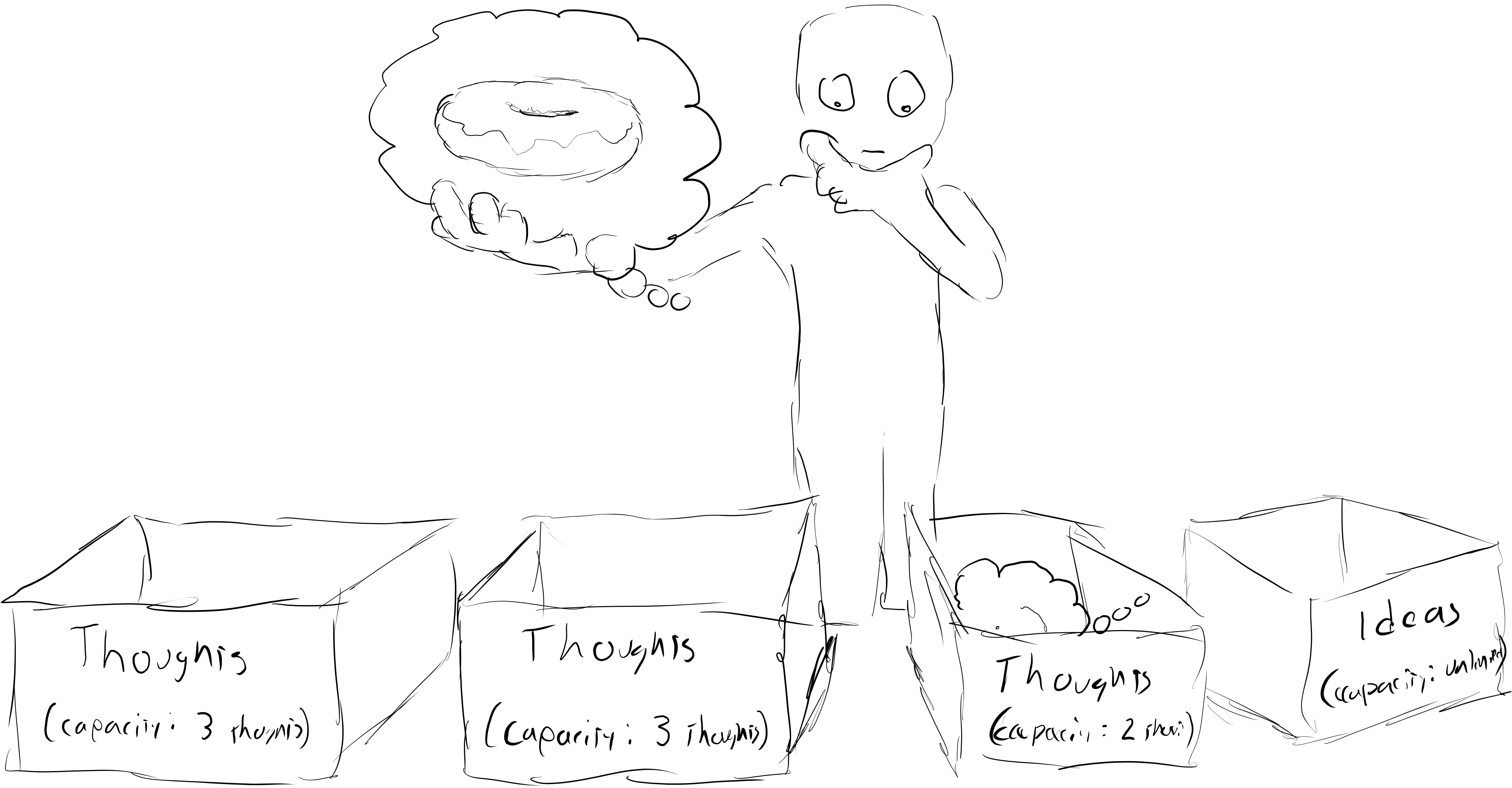
Thoughts
(capacity: 3 thoughts)
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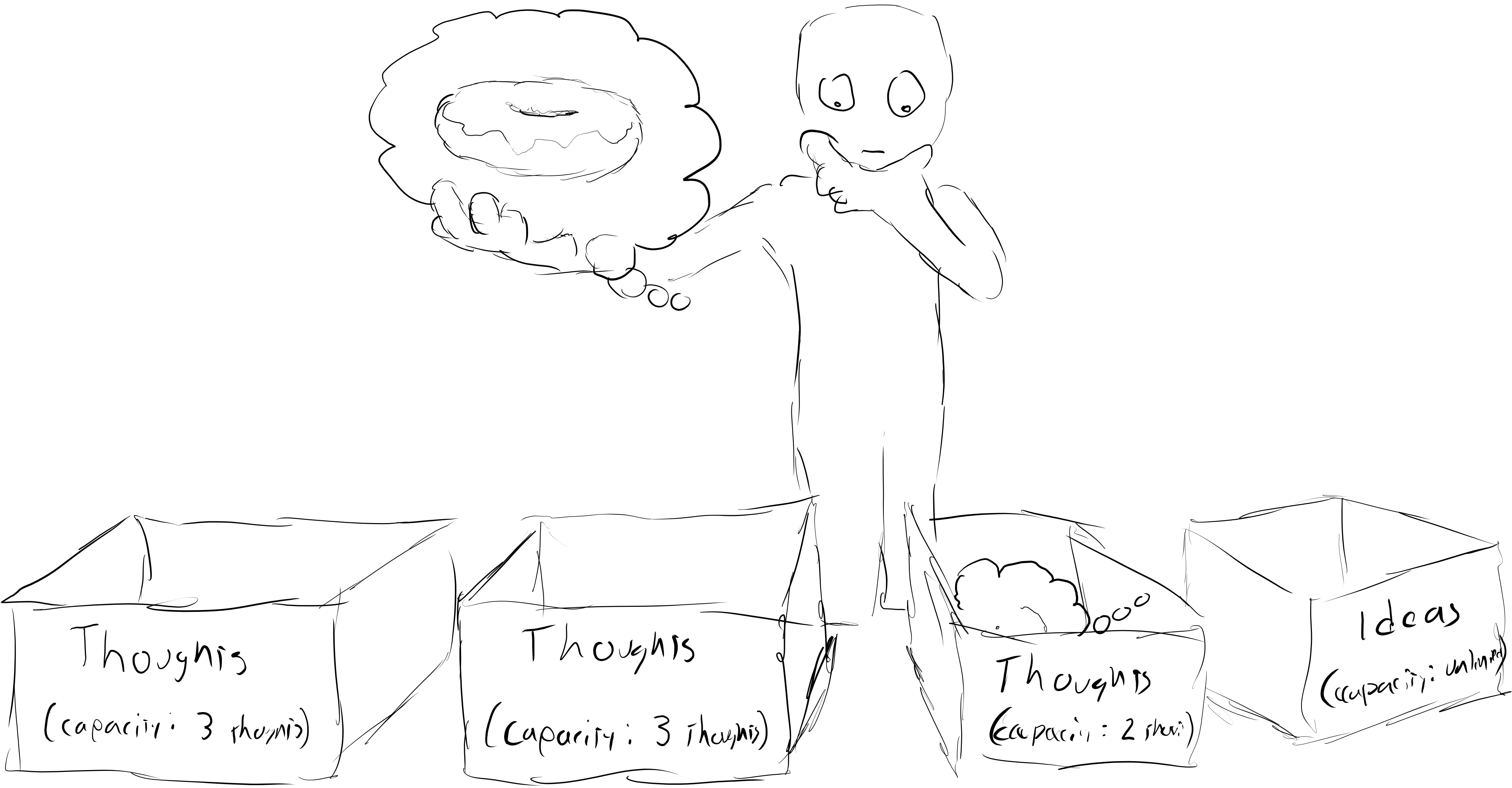
Thoughts
(capacity: 2
Ideas
(capacity: unlimited)


Thoughts
(capacity: 3 thoughts)
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(capacity: unlimited)

and $*$

or $+$

indistinguishable $\div$

but not $-$

$$|\mathcal{P}(A)| = 2^{|A|}$$

subsets of size k of A $= \binom{|A|}{k} = \dfrac{|A|!}{k!(|A|-k)!}$

Permutations $n!$

dozen = 12

donut   doughnut

4 flavors

$\boxed{\text{distribute } n \text{ things b/w } k \text{ buckets: } \binom{n+k-1}{k-1}}$

$12 - f_1$

$11 - f_1 \qquad 1 f_2$

$\vdots$

plain	glazed	pwd su	maple
f_1	f_2	f_3	f_4
9	1	0	2

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & & & & & 12 & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & | & \circ & | & | & \circ & \circ \\
1 & & & & & & & & 9 & 10 & 11 & & 13 & &
\end{array}
\rightarrow \begin{array}{l} 15 \text{ symbols} \\ 3 \text{ dividers} \end{array}
$$

set of 3 elem $\subseteq$ 15-elem set

$\{10, 12, 13\}$

$$\binom{12 + (4-1)}{(4-1)}$$

$$\binom{15}{3} = \frac{15!}{3!\ 12!} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1}$$

$$= 5 \cdot 7 \cdot 13$$

k n

4 positive integers that sum to 12

$2+3+1+6 \overset{?}{=} 3+2+1+6$

1

$1 \mid 1$

1 1 | 1 1 1 | 1 | 1 1 1 1 1 1

1 2 3 4 5 6 7 8 9 10 11

11 spots

$\binom{11}{3}$

$\boxed{\binom{n-1}{k-1}}$

$\{2, 5, 6\}$

3-elem subset of 11-elem set

order matters $(2,3,1,6) \neq (3,2,1,6)$

$= \boxed{\binom{11}{3}} = \frac{11!}{3!\,8!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1}$

$= \frac{11!}{3!(11-3)!}$

Order does not matter $(2,3,1,6) = (3,2,1,6)$

$\binom{11}{3} \div 4!$ ... almost - too small

$(3,3,3,3)$

| | | \| | | | \| | | | \| | | |

1 2 3 4 5 6 7 8 9 10 11

$\{3, 6, 9\}$

k-elem seq from n-elem set

- rep: $n^k = \underbrace{n \cdot n \cdot \cdots \cdot n}_{k}$

- no rep: $\dfrac{n!}{(n-k)!} = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1)$

$$\text{Prob} = \frac{\text{desired possiblities}}{\text{all possiblities}} \qquad \frac{1}{\binom{5}{3}}$$

x x x s s

pick s-spots: $\binom{5}{2}$

pick x-spots: $\binom{5}{3}$

$$\binom{5}{2} = \frac{5!}{2!\,3!} = \binom{5}{3}$$

$$\boxed{\binom{7}{3}\cdot\binom{4}{2}} = \frac{7\cdot 6\cdot 5}{\cancel{3}\cdot 2}\cdot\frac{4\cdot\cancel{3}}{\cancel{2}\cdot 1} = 7\cdot 6\cdot 5$$

place the as

place the bs

shuffle: $\underbrace{a\,a\,a}\,\underbrace{b\,b}\,\underbrace{c\,c}$

$$\boxed{\frac{7!}{3!\,2!\,2!}} = \frac{7\cdot 6\cdot 5\cdot 4}{2\cdot 1\cdot 2\cdot 1} = 7\cdot 6\cdot 5 = \boxed{210}$$

$$\underbrace{7\cdot 6\cdot 5}\cdot\underbrace{4\cdot 3\cdot 2}\cdot 1$$

$= 24$

$\boxed{\text{multinomial}}$ $\binom{7}{3,2}$

$r_1 \quad \frac{1}{10}$ or $r_2 \quad \frac{1}{100}$

both

$\boxed{\text{double counts}}$

$$\frac{1}{10} + \frac{1}{100} = \cancel{\frac{11}{100}}$$

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at least 1: $\underbrace{\frac{1}{10}}_{\text{win } r_1} + \underbrace{\left(\frac{9}{10}\right)}_{\text{not } r_1}\underbrace{\left(\frac{1}{100}\right)}_{r_2} = \frac{109}{1000}$ 1 or 2

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both $\left(\frac{1}{10}\right)\left(\frac{1}{100}\right) = \frac{1}{1000}$ 2 wins

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$$\frac{1}{10}\left(\frac{99}{100}\right) + \frac{1}{100}\left(\frac{9}{10}\right) = \frac{99+9}{1000} = \frac{108}{1000}$$