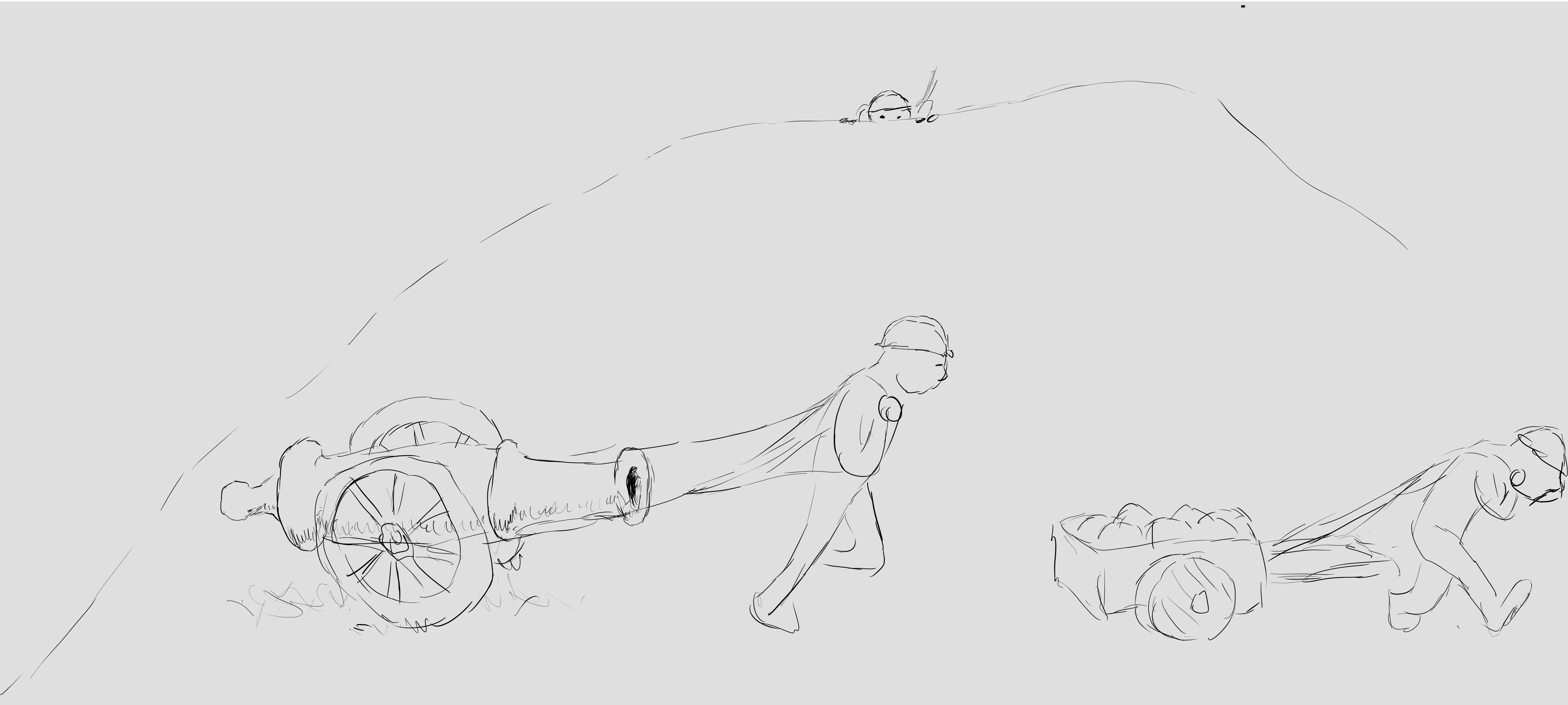


[redacted]

function — math $\qquad \underline{y = f(x)}$

Subroutine — programming $\qquad y = foo(x)$ — func / side effects / state

$$(y, world') = foo(x, \underline{world})$$

function representation

- formula

$$f(x) = x^2 + 3$$

- code

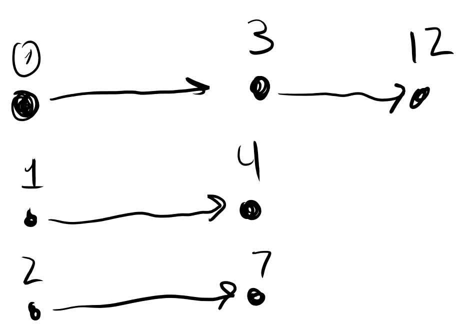
```
f(x) =   y = x*x
         z = y+3
         return z
```

- set of sequences

$$\{(\underbrace{0}_{\text{dom}}, \overbrace{3}^{\text{codom}}), (1,4), (2,7), (3,12), (4,19), \dots\}$$

x	y
0	3
1	4
2	7
3	12
4	19
⋮	⋮

- graph
  - node/vertex
  - edges



Terminology

$y = f(x)$

set of allowable x $\longrightarrow$ <u>domain</u> of f

$f(x,y) = x^2 - y^2$

domain = set of pairs of $\mathbb{R}$

$$\frac{\mathbb{R} \times \mathbb{R}}{\text{range} \quad \mathbb{R}}$$

```
int y = ...
int x = y*y;
```

type(x*x) is int

set of allowable y $\longrightarrow$ <u>codomain</u> of f

set of possible y $\longrightarrow$ <u>range</u> of f

domain: $\mathbb{R}$ not input (0)

$f(x) = \frac{1}{x}$

f is <u>total</u> $\longrightarrow$ defined over whole domain

f is <u>partial</u> $\longrightarrow$ not defined over whole domain

$$f(x) = x^2$$

domain: $\mathbb{N}$

codomain: $\mathbb{N}$

range: $\{0, 1, 4, 9, 16, \ldots\}$

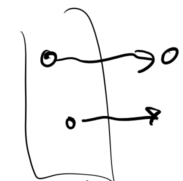
range $\subseteq$ codomain

does foo crash ~ it func computed by foo total

Properties of functions

dom codom

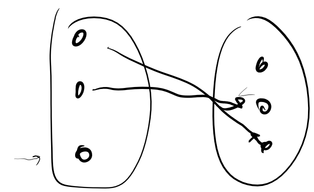
$\forall x \in D . \exists (a,b) \in Pairs . a = x$



all
elem of dom

total — all elem of domain work

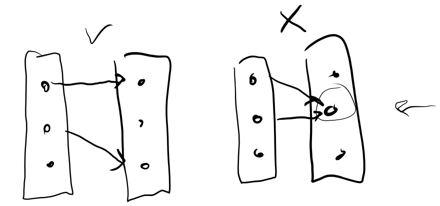
partial — $\neg$ total



D cod

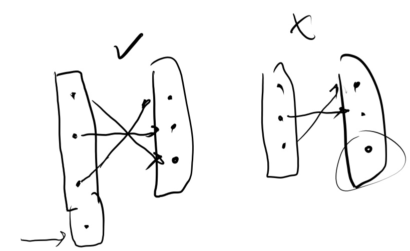
1-to-1 injective — at most 1 elem of domain maps to each elem of codomain

$\forall x \in D, y \in C . x \neq y \to f(x) \neq f(y)$



onto surjective — at least 1 elem of dom maps to each elem of co-domain

codomain = range



invertible

correspondence bijective