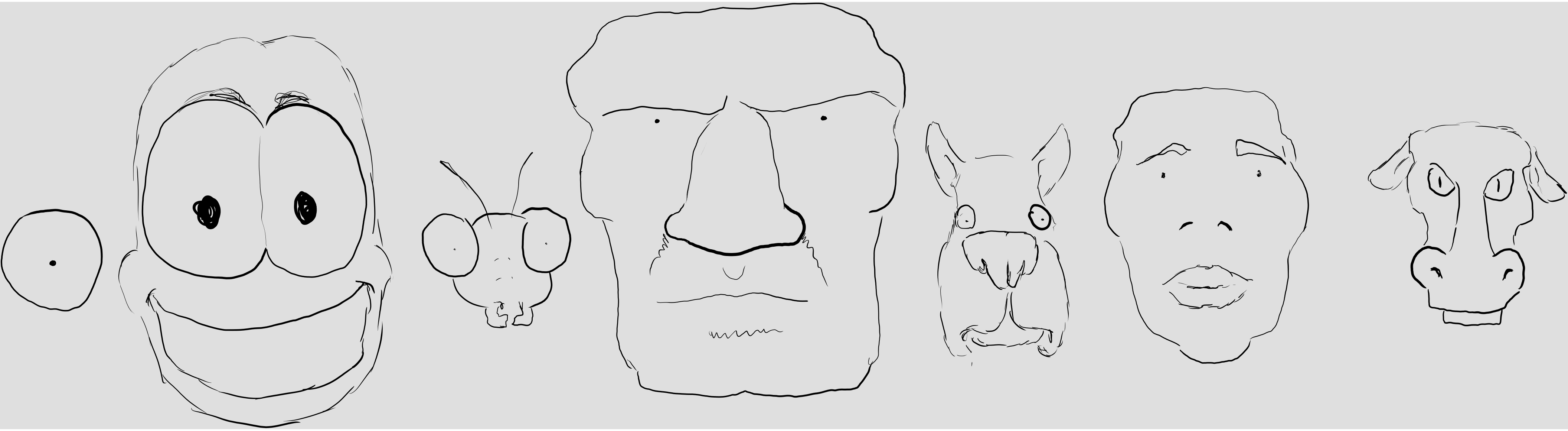

prove $\log_2 3 \notin \mathbb{Q}$

by contradiction

assume $\log_2(3) \in \mathbb{Q}$

$\exists a, b$ ($\mathbb{Z}$, $\mathbb{Z}^+$, co-prime) . assume $\frac{a}{b} = \log_2 3$

$$\frac{a}{b} = \log_2 3$$

$$a = b \log_2 3$$

power

$$a = \log_2 3^b$$

$$2^a = 2^{\log_2 3^b} = 3^b$$

inverse

$x = 2^a = 3^b$

$3 | x$

$3 | 2^a$

by fund thm of arith, must have same prime factors, and $b > 0$

so 3 must be a factor of $2^a$, which it is not $\perp$

because assuming $\log_2(3) \in \mathbb{Q}$ led to a contr., it must be that $\log_2(3) \notin \mathbb{Q}$ ∎

$\log_8(9) \notin \mathbb{Q}$

assum $\log_8(9) \in \mathbb{Q}$

coprm( $\frac{x}{b} = \log_8 9$, $x \in \mathbb{Z}$, $b \in \mathbb{Z}^+$

$$\mathbb{Q} = \left\{ \frac{x}{y} \mid x \in \mathbb{Z}, y \in (\mathbb{Z} \setminus \{0\}) \right\}$$

$$\rightarrow = \left\{ \frac{x}{y} \mid x \in \mathbb{Z}, y \in \mathbb{Z}^+ \right\}$$

$$= \left\{ \frac{x}{y} \mid x \in \mathbb{N}, y \in (\mathbb{Z} \setminus \{0\}) \right\}$$

$$\log_8 9 = \frac{x}{b}$$

$$8^{\log_8 9} = 8^{x/b}$$

$$9 = \sqrt[b]{8^x}$$

$$9^b = 8^x$$

$$(3^2)^b = (2^3)^x$$

$$3^{2b} = 2^{3x}$$

... like last proof

$\log_8(4) \notin \mathbb{Q}$

False thm

$\log_8(4) = \frac{2}{3}$

$\frac{a}{b} \in \mathbb{Q}$, $\frac{a}{b} = \log_8(4)$

$\Downarrow$

$a = b\log_8(4)$

$a = \log_8(4^b)$

$8^a = 4^b$

$2^{3a} = 2^{2b}$

$3a = 2b$

integer co-prim

$a = 2 \quad b = 3$

Proof does not work

$\Rightarrow$ Thm false

thm $\log_8(4) = \frac{2}{3}$

$\Updownarrow$

$3\log_8(4) = 2$

$\Updownarrow$

$\log_8(4^3) = 2$

$4^3 = (2^2)^3 = 2^{2\cdot 3} = 2^6$

$\Updownarrow$

$\log_8(2^6) = 2$

$2^6 = 2^{3\cdot 2} = (2^3)^2 = 8^2$

$\Updownarrow$

$\log_8(8^2) = 2$

$\Updownarrow$

$2 = 2$ ✓

# Simplify

logs: · mult inside of log simple

· same base

$$3 \cdot \log_2(7) + 4 \cdot \log_3\left(\frac{8}{3}\right)$$

simplify

$$3\left(\frac{\log_3(7)}{\log_3(2)}\right) + 4 \cdot \left(\log_3(8) - \underbrace{\log_3(3)}_{1}\right)$$

# Simplify

logs: · mult inside of log simple
· same base

simplify $3 \cdot \log_2(7) + 4 \cdot \log_3(\frac{8}{3})$

$3\left(\frac{\log_3(7)}{\log_3(2)}\right) + 4 \cdot (\log_3(8) - \log_3(3))$ where $\log_3(3) = 1$

$\log(x) \to$ last log

$\log \times \log \to \log(\_^{\log})$

$\frac{3\log_3(7)}{\log_3(2)} + 4\log_3(8) - 4$ ✓

bad: $\log_3(2+8)$

$\frac{3\log_3(7) + 4\log_3(2)\cdot\log_3(8) - 4\log_3(2)}{\log_3(2)}$

$4\log_3(8^{\log_3 2})$

$\log_3(7^3) + \log_3(2^{4\log_3 8})$

$\log_3(7^3 \cdot 2^{4\log_3 8})$

$2^{4\log_3 8} = 2^{\log_3(8^4)} = 2^{\log_3(2^{12})} = 2^{12 \cdot \log_3 2}$

$2^{\left(\frac{\log_2(2^{12})}{\log_2 3}\right)}$

$\sqrt[\log_2 3]{2^{\log_2(2^{12})}} = \sqrt[\log_2 3]{2^{12}} = 2^{12/\log_2 3}$