Given that $\log_3 x = 2$, we want to solve for $x$.

Recall that $\log_b a = c$ means $b^c = a$. Using this rule, we can rewrite the equation as $3^2 = x$.

Therefore, $x = 9$.

Next, let's consider the equation $\log_b 27 = 3$.

Similarly, we can use the property of logarithms to rewrite this equation as $b^3 = 27$.

Since $3^3 = 27$, we conclude that $b = 3$.

Finally, let's solve for $x$ in the equation $\log_b (x^2 + 1) = \log_b 9$.

Using the property of logarithms, we can equate the arguments of the logarithms:

$x^2 + 1 = 9$.

Solving for $x$, we get $x^2 = 8$, so $x = \pm \sqrt{8}$.

In summary, we have solved the given logarithmic equations:

1. $\log_3 x = 2$, so $x = 9$.
2. $\log_b 27 = 3$, so $b = 3$.
3. $\log_b (x^2 + 1) = \log_b 9$, so $x = \pm \sqrt{8}$.