CS3102 Theory of Computation
Problem Set 2
Department of Computer Science, University of Virginia

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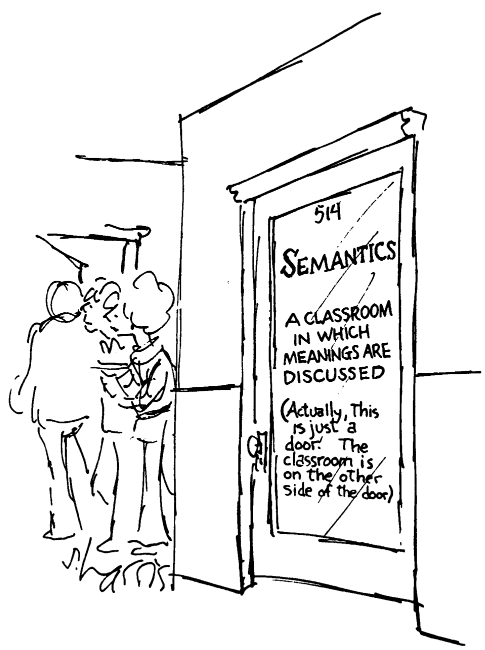
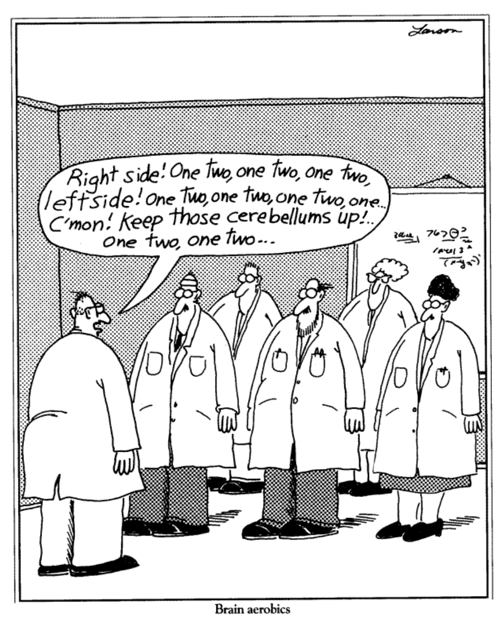
Please start solving these problems immediately, don't procrastinate, and work in study groups. Please prove all your answers; informal arguments are acceptable, but please make them precise / detailed / convincing enough so that they can be easily made rigorous if necessary. To review notation and definitions, please read the "Basic Concepts" summary posted on the class Web site, and also read the corresponding chapters from the Sipser textbook and Polya's "How to Solve It".

Please do not simply copy answers that you do not fully understand; on homeworks and on exams we reserve the right to ask you to explain any of your answers verbally in person (and we have exercised this option in the past). Please familiarize yourself with the UVa Honor Code as well as with the course Cheating Policy summarized on page 3 of the Course Syllabus. To fully understand and master the material of this course typically requires an average effort of at least six to ten hours per week, as well as regular meetings with the TAs and attendance of the weekly problem-solving sessions.

This is not a "due homework", but rather a "pool of problems" meant to calibrate the scope and depth of the knowledge / skills in CS theory that you (eventually) need to have for the course exams, becoming a better problem-solver, be able to think more abstractly, and growing into a more effective computer scientist. You don't necessarily have to completely solve every last question in this problem set (although it would be great if you did!). Rather, please solve as many of these problems as you can, and use this problem set as a resource to improve your problem-solving skills, hone your abstract thinking, and to find out what topics you need to further focus on and learn more deeply. Recall that most of the midterm and final exam questions in this course will come from these problem sets, so your best strategy of studying for the exams in this course is to solve (including in study groups) as many of these problems as possible, and the sooner the better!

Advice: Please try to solve the easier problems first (where the meta-problem here is to figure out which are the easier ones ☺). Don't spend too long on any single problem without also attempting (in parallel) to solve other problems as well. This way, solutions to the easier problems (at least easier for you) will reveal themselves much sooner (think about this as a "hedging strategy" or "dovetailing strategy").





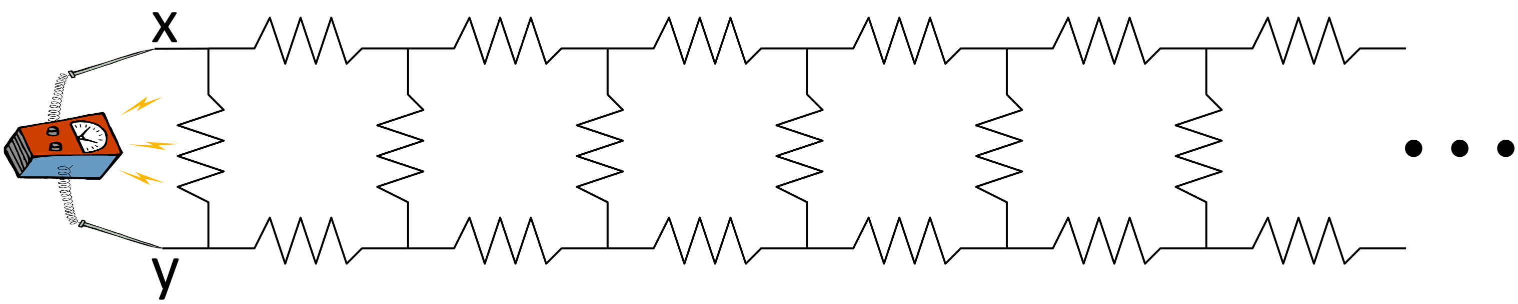
Brain aerobics

1. Solve the following equation for X:

$$X^{X^{X^{X^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

where the stack of exponentiated x's extends forever.

2. For the following infinite ladder of resistors (of resistance R each), what is the resistance measured between points x and y?

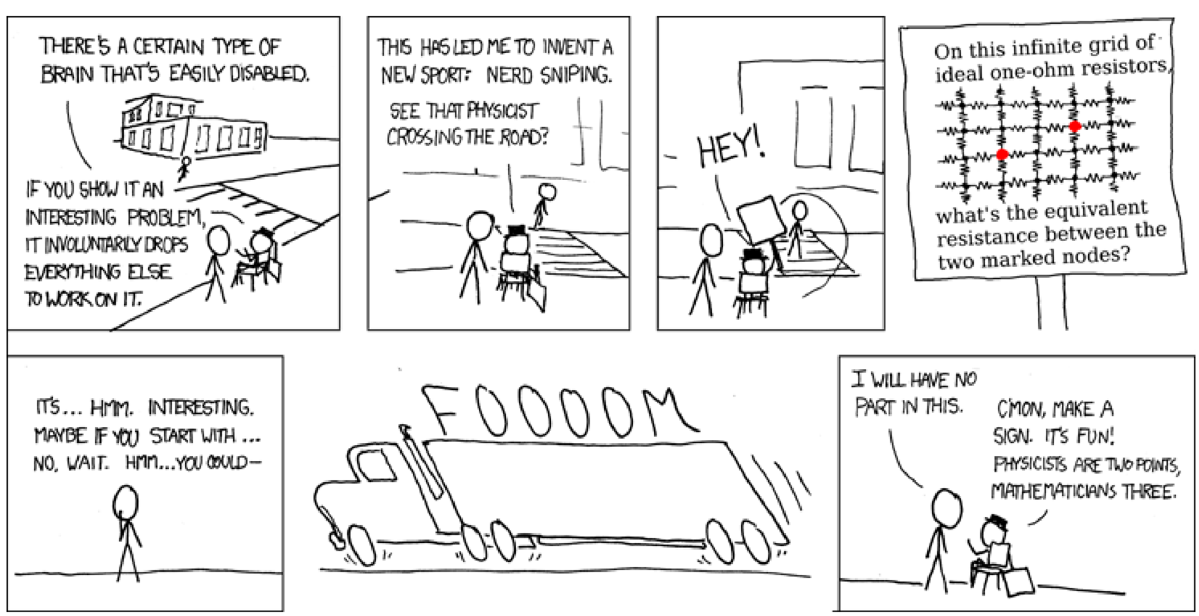


3. How do the above two problems (#1 and #2) relate to Cantor's "infinite hotel" -type scenarios?

4. A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north and ends up back at his house. What color was the bear?

5. Characterize the complete set of locations where the scenario above (#4) can happen.

6. What is the approximate value of [1 + 9^(-(4^(7*6)))] ^ (3^(2^85)) ?



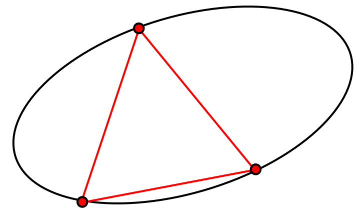
7. Prove or disprove: any set of 5 points in/on the unit (1x1) square must contain a pair with distance between them $\leq 1 / \mathrm{SQRT}(2)$.

8. Find the smallest D for which any set of 10 points in/on the unit square must contains a pair with distance between them $\leq$ D.

9. Prove or disprove: given any 5 points in/on the unit (1x1x1) equilateral triangle must contain a pair with distance between them $\leq 1/2$?

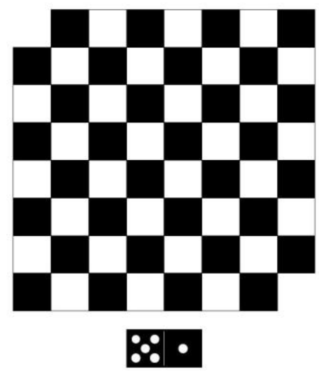
10. Find the smallest D for which any set of 10 points in/on the unit equilateral triangle must contain a pair with distance between them $\leq$ D.

11. Prove or disprove: every closed simple (non self-intersecting) curve contain the vertices of an equilateral triangle?



12. Prove or disprove: 0.999999… is equal to 1.

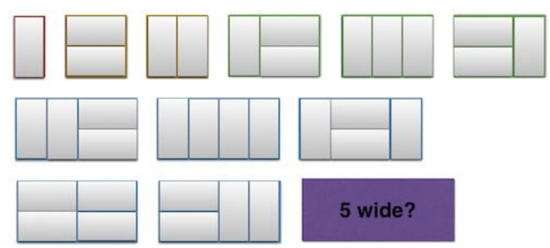
13. Prove or disprove: it is possible to tile (completely cover) a chessboard with two opposite corner squares missing, using 32 dominoes (see diagram on the right).

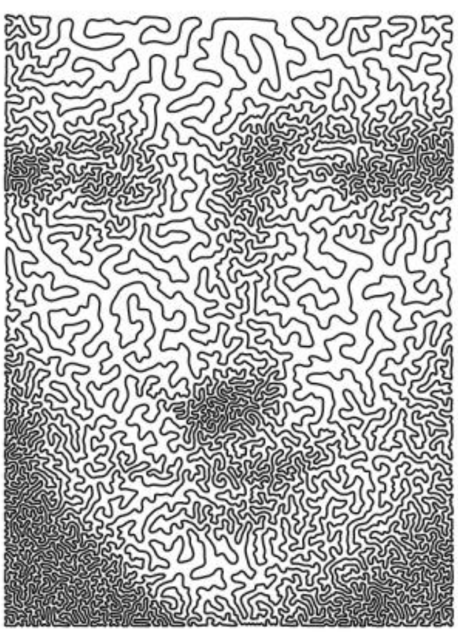


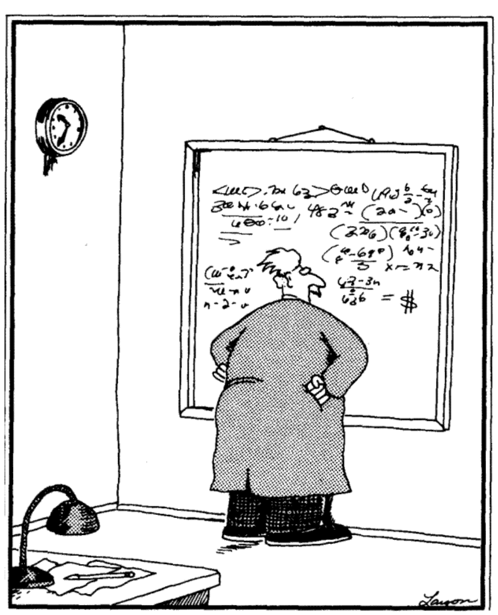
14. Prove or disprove: it is never possible to tile (completely cover) a chessboard with any two same-color squares missing, using 32 dominoes.

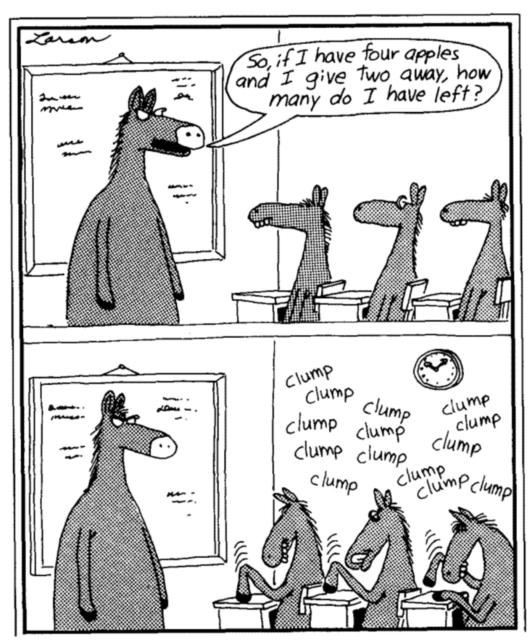
15. Prove or disprove: it is always possible to tile (completely cover) a chessboard with any two different-color squares missing using 32 dominoes.

16. In how many ways (as a function of N) can a 2xN rectangle be tiled with domonies? (see examples in the diagram on the right).






Einstein discovers that time is actually money.



17. Does the Pythagorean Theorem generalize to arbitrary similar figures on the sides of a right triangle? In other words, does the sum of the areas of the two smaller (similar) figures always equal to the area of the larger one, not only for squares, but for other arbitrary figures similar to one another? See below for examples of this assertion.