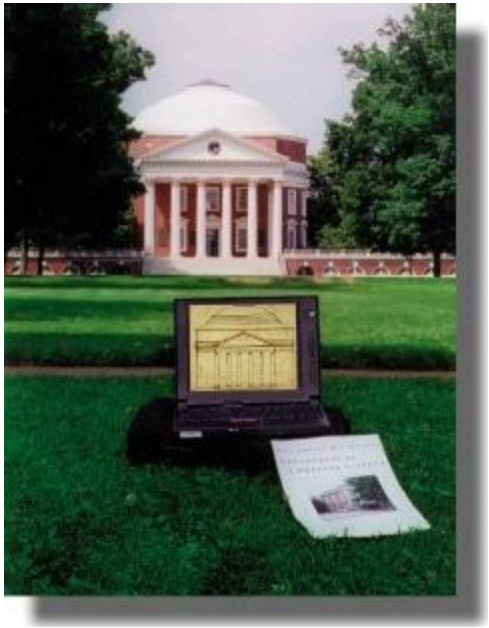


Theory of Computation
CS3102

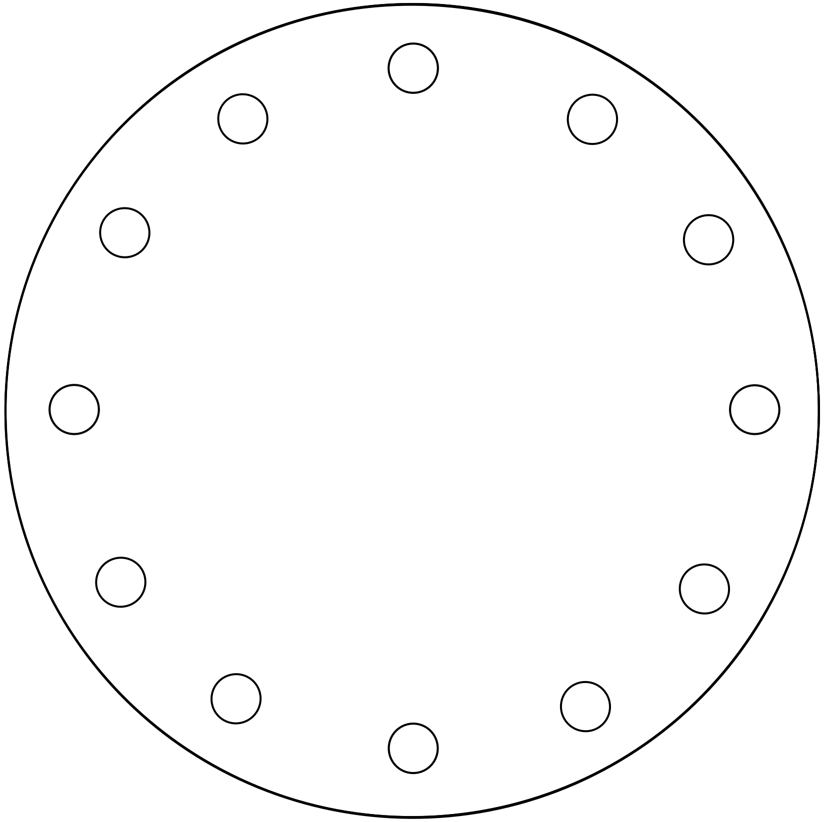
Gabriel Robins

Department of
Computer Science

University of Virginia

www.cs.virginia.edu/robins/theory

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?

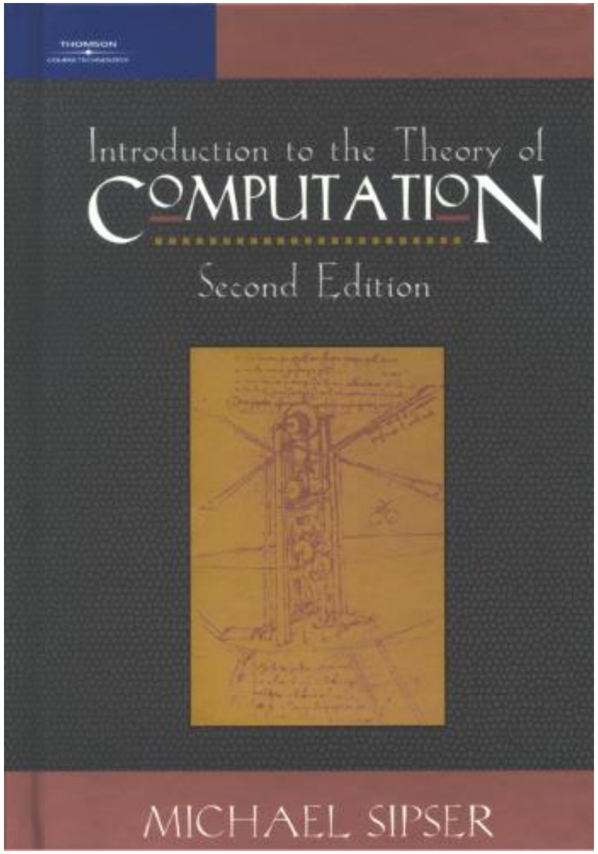


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Theory of Computation (CS3102) - Textbook

Textbook:

Introduction to the Theory of Computation, by Michael Sipser (MIT), 2nd Edition, 2005



Good Articles / videos:

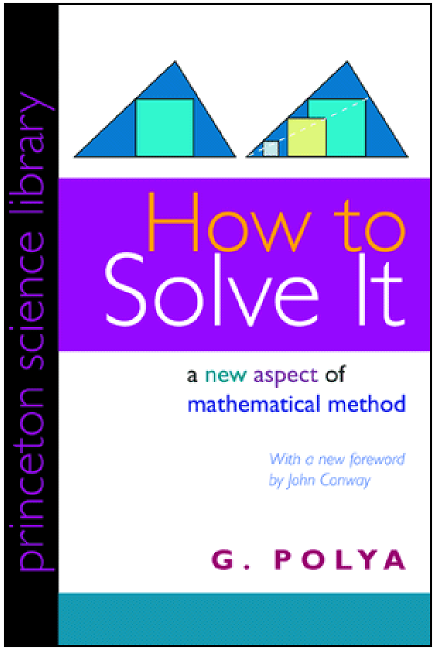
www.cs.virginia.edu/~robins/CS_readings.html

Theory of Computation (CS3102)

Required reading:

How to Solve It, by George Polya (MIT), Princeton University Press, 1945

- A classic on problem solving



George Polya (1887-1985)

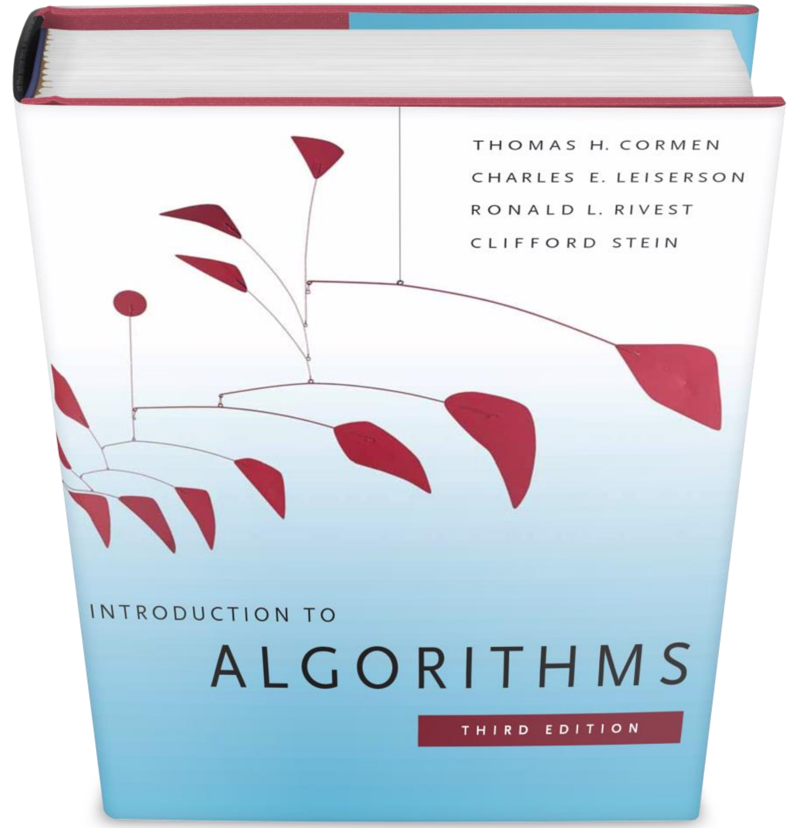
Theory of Computation (CS3102)

Good algorithms textbook:

Introduction to Algorithms

by Cormen et al (MIT)

Third Edition, 2009

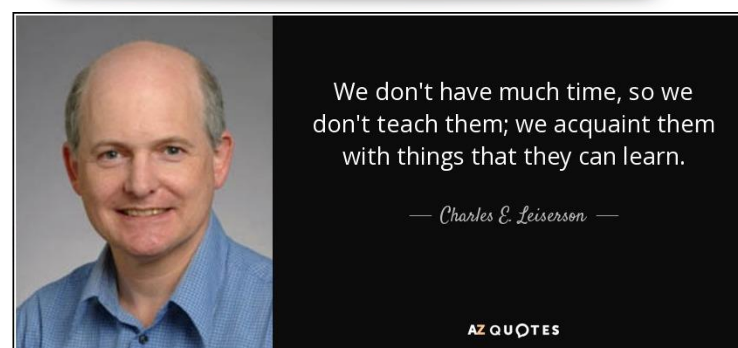



Thomas Cormen


Charles Leiserson


Ronald Rivest


Clifford Stein



computer science

Introduction to Algorithms

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein

Third Edition

Some books on algorithms are rigorous but incomplete; others cover masses of material but lack rigor. *Introduction to Algorithms* uniquely combines rigor and comprehensiveness. The book covers a broad range of algorithms in depth, yet makes their design and analysis accessible to all levels of readers. Each chapter is relatively self-contained and can be used as a unit of study. The algorithms are described in English and in a pseudocode designed to be readable by anyone who has done a little programming. The explanations have been kept elementary without sacrificing depth of coverage or mathematical rigor.

The first edition became a widely used text in universities worldwide as well as the standard reference for professionals. The second edition featured new chapters on the role of algorithms, probabilistic analysis and randomized algorithms, and linear programming. The third edition has been revised and updated throughout. It includes two completely new chapters, on van Emde Boas trees and multithreaded algorithms, and substantial additions to the chapter on recurrences (now called "Divide-and-Conquer"). It features improved treatment of dynamic programming and greedy algorithms and a new notion of edge-based flow in the material on flow networks. Many new exercises and problems have been added for this edition.

As of the third edition, this textbook is published exclusively by the MIT Press.

Thomas H. Cormen is Professor of Computer Science and former Director of the Institute for Writing and Rhetoric at Dartmouth College. Charles E. Leiserson is Professor of Computer Science and Engineering at MIT. Ronald L. Rivest is Andrew and Erna Viterbi Professor of Electrical Engineering and Computer Science at MIT. Clifford Stein is Professor of Industrial Engineering and Operations Research at Columbia University.

"In light of the explosive growth in the amount of data and the diversity of computing applications, efficient algorithms are needed now more than ever. This beautifully written, thoughtfully organized book is the definitive introductory book on the design and analysis of algorithms. The first half offers an effective method to teach and study algorithms; the second half then engages more advanced readers and curious students with compelling material on both the possibilities and the challenges in this fascinating field."
—Shang-Hua Teng, University of Southern California

"*Introduction to Algorithms*, the 'bible' of the field, is a comprehensive textbook covering the full spectrum of modern algorithms: from the fastest algorithms and data structures to polynomial-time algorithms for seemingly intractable problems, from classical algorithms in graph theory to special algorithms for string matching, computational geometry, and number theory. The revised third edition notably adds a chapter on van Emde Boas trees, one of the most useful data structures, and on multithreaded algorithms, a topic of increasing importance."
—Daniel Spielman, Department of Computer Science, Yale University

"As an educator and researcher in the field of algorithms for over two decades, I can unequivocally say that the Cormen book is the best textbook that I have ever seen on this subject. It offers an incisive, encyclopedic, and modern treatment of algorithms, and our department will continue to use it for teaching at both the graduate and undergraduate levels, as well as a reliable research reference."
—Gabriel Robins, Department of Computer Science, University of Virginia

978-0-262-03384-8

The MIT Press
Massachusetts Institute of Technology
Cambridge, Massachusetts 02142
http://mitpress.mit.edu

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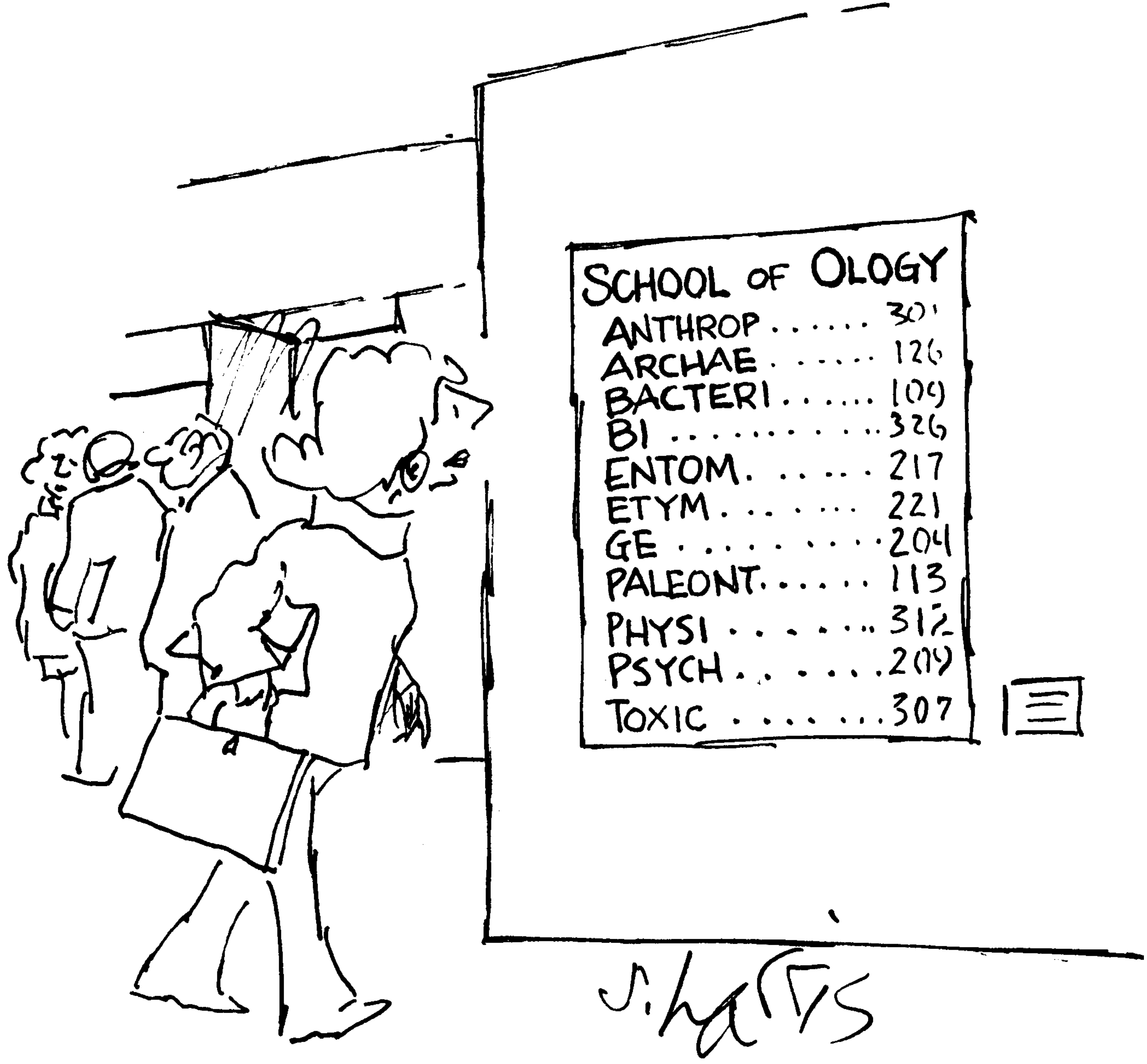
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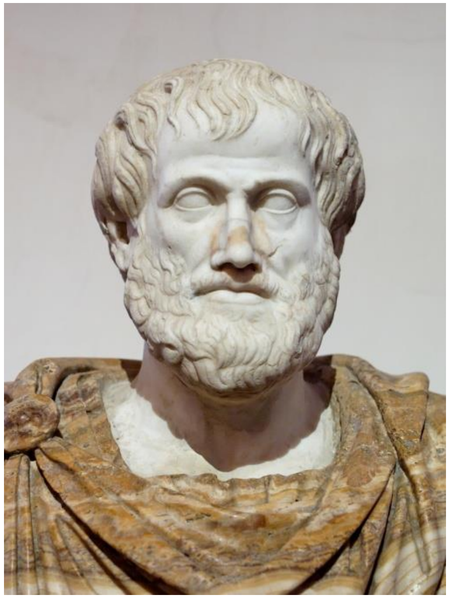
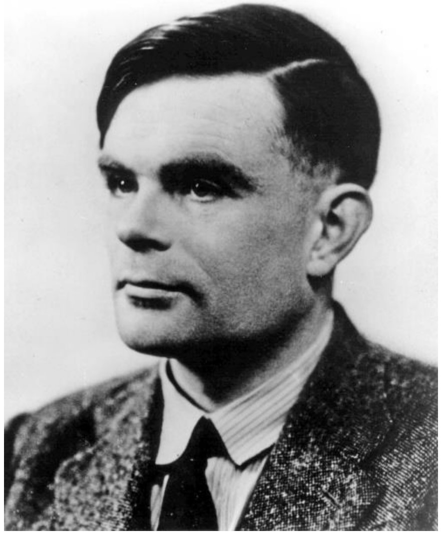
ANTHROP 301
ARCHAE 126
BACTERI 109
BI 326
ENTOM 217
ETYM 221
GE 204
PALEONT 113
PHYSI 312
PSYCH 209
TOXIC 307
J. Harris

Theory of Computation (CS3102) - Syllabus

A brief history of computing:

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan, Babbage, Ada Agusta
- Venn, Carroll, Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Godel, Church, Turing, von Neumann
- Shannon, Kleene, Chomsky

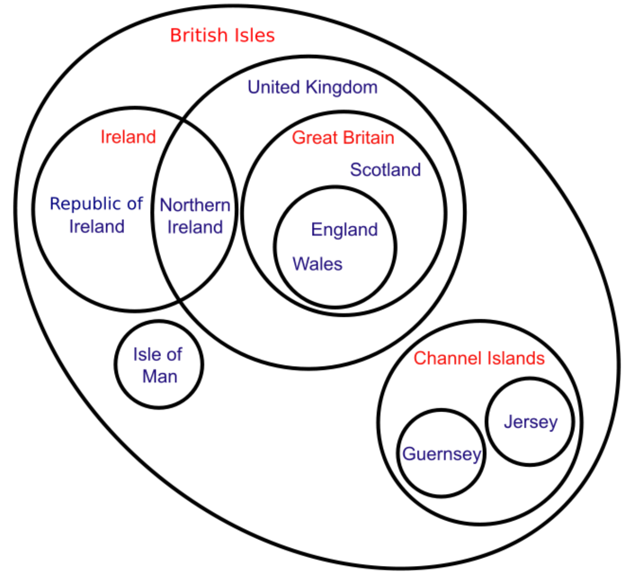


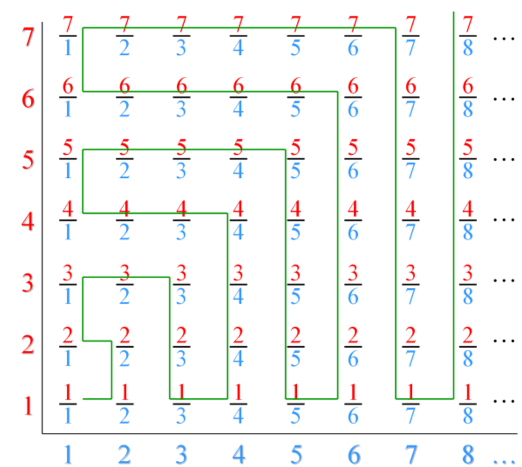


Theory of Computation Syllabus (continued)

Fundamentals:

- Set theory
- Predicate logic
- Formalisms and notation
- Infinities and countability
- Dovetailing / diagonalization
- Proof techniques
- Problem solving
- Asymptotic growth
- Review of graph theory

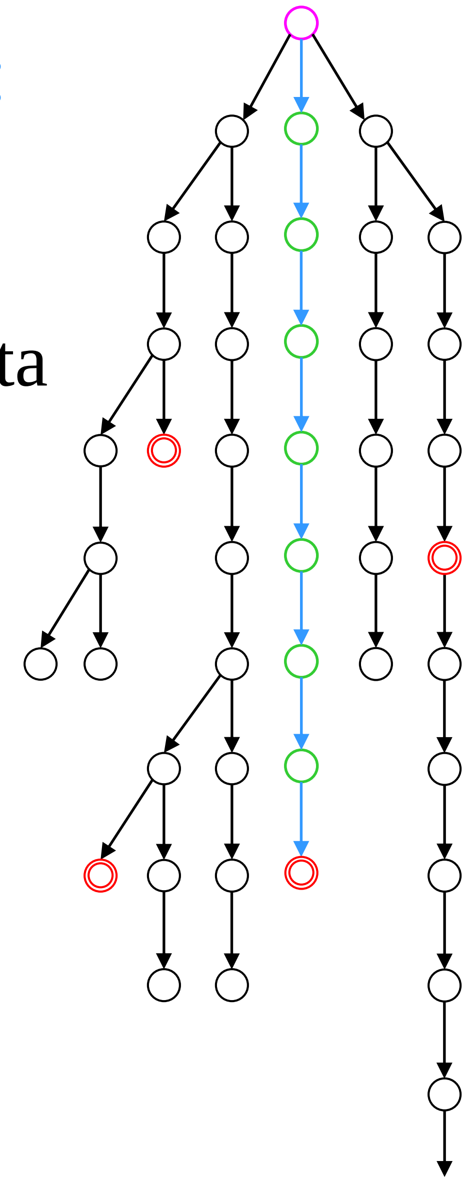




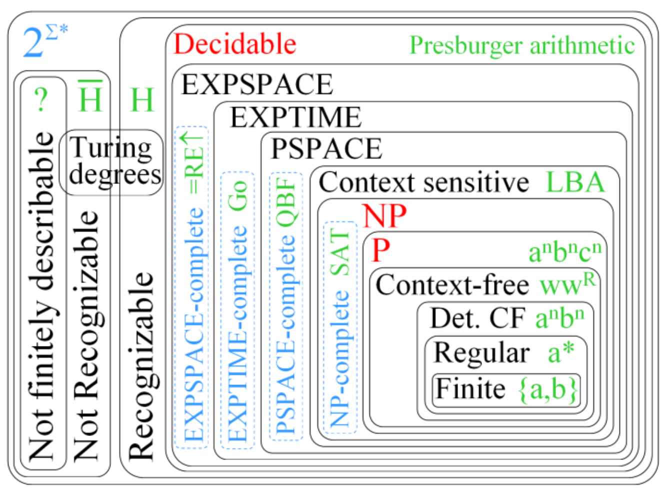
Theory of Computation Syllabus (continued)

Formal languages and machine models:

- The Chomsky hierarchy
- Regular languages / finite automata
- Context-free grammars / pushdown automata
- Unrestricted grammars / Turing machines
- Non-determinism
- Closure operators
- Pumping lemmas
- Non-closures
- Decidable properties



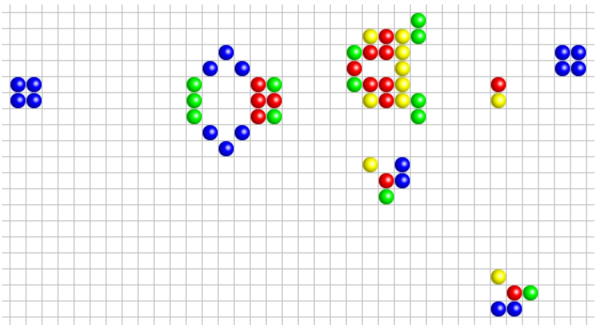
The Extended Chomsky Hierarchy

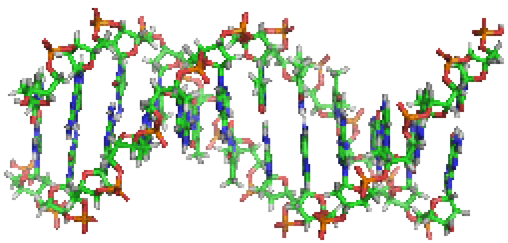


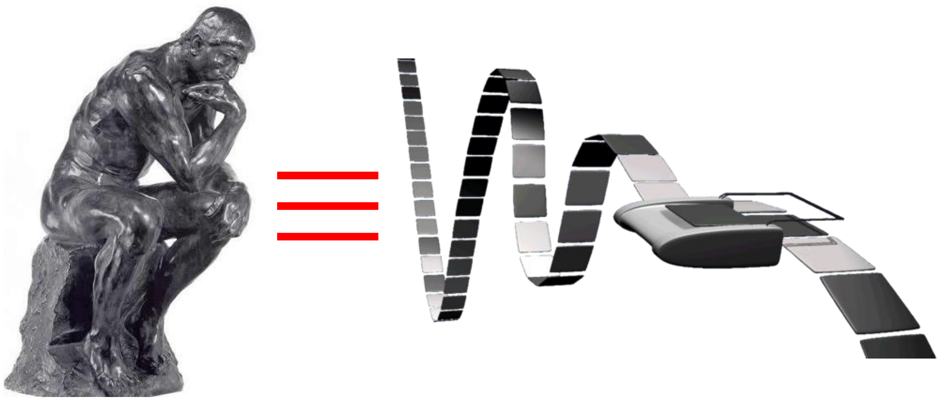
Theory of Computation Syllabus (continued)

Computability and undecidability:

- Basic models
- Modifications and extensions
- Computational universality
- Decidability
- Recognizability
- Undecidability
- Church-Turing thesis
- Rice’s theorem



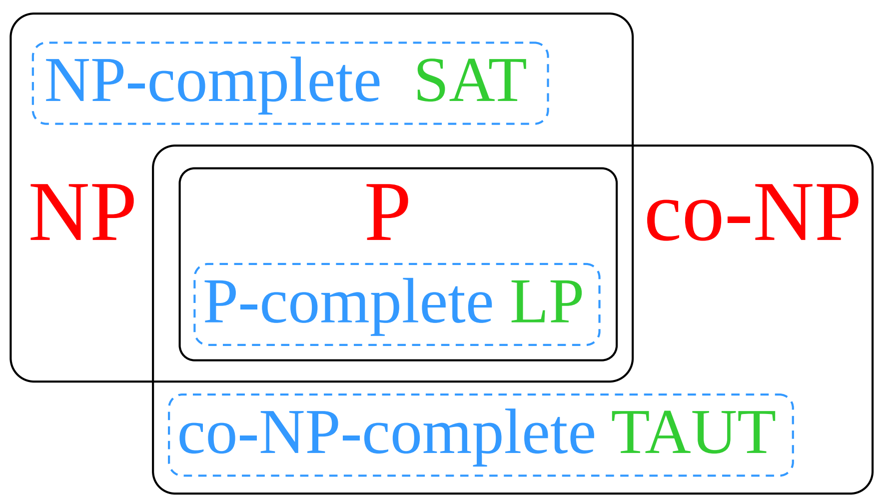




Theory of Computation Syllabus (continued)

NP-completeness:

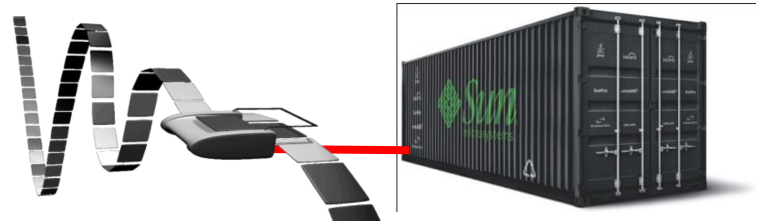
- Resource-constrained computation
- Complexity classes
- Intractability
- Boolean satisfiability
- Cook-Levin theorem
- Transformations
- Graph clique problem
- Independent sets
- Hamiltonian cycles
- Colorability problems
- Heuristics

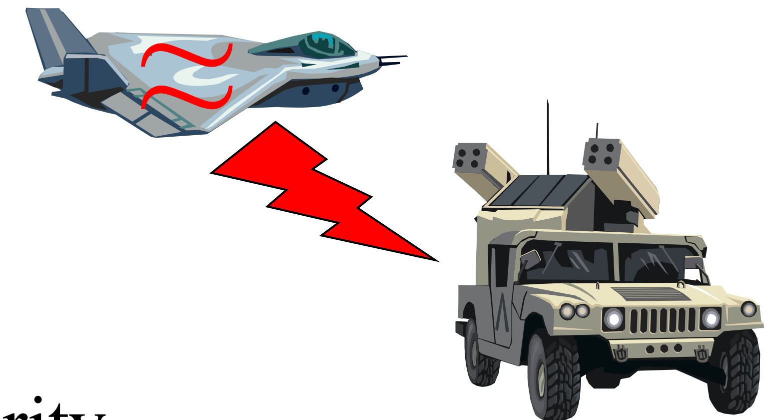


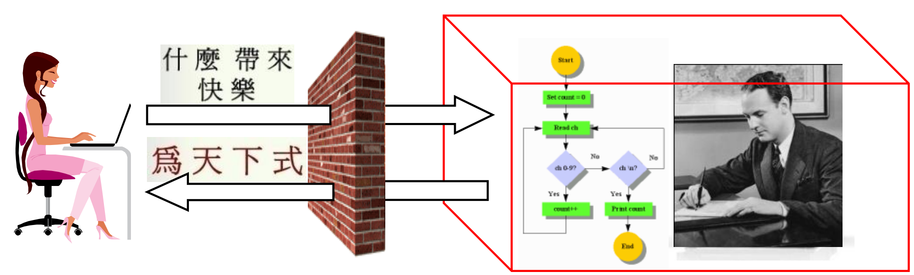
Theory of Computation Syllabus (continued)

Other topics (as time permits):

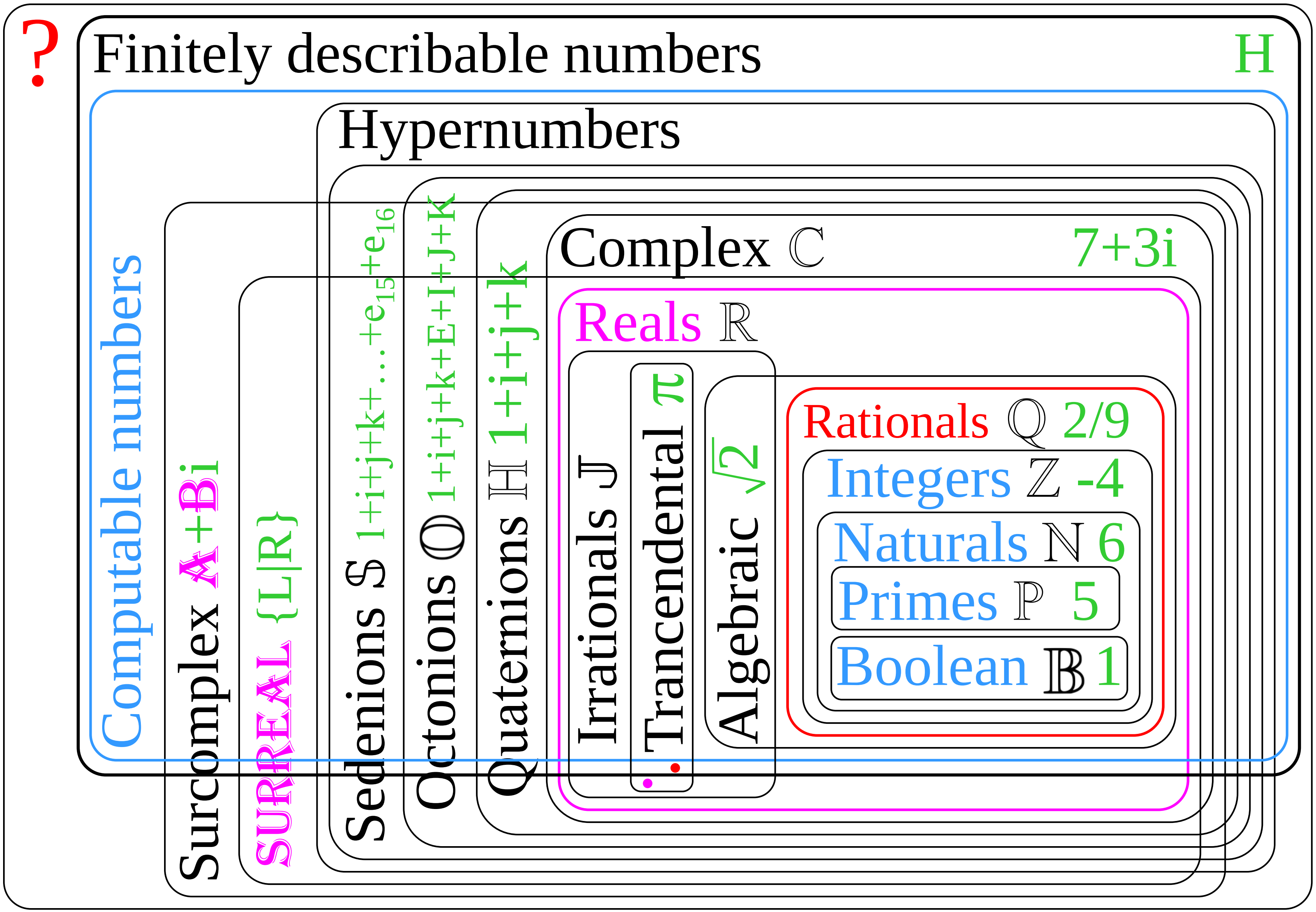
- Generalized number systems
- Oracles and relativization
- Zero-knowledge proofs
- Cryptography & mental poker
- The Busy Beaver problem
- Randomness and compressibility
- The Turing test
- AI and the Technological Singularity







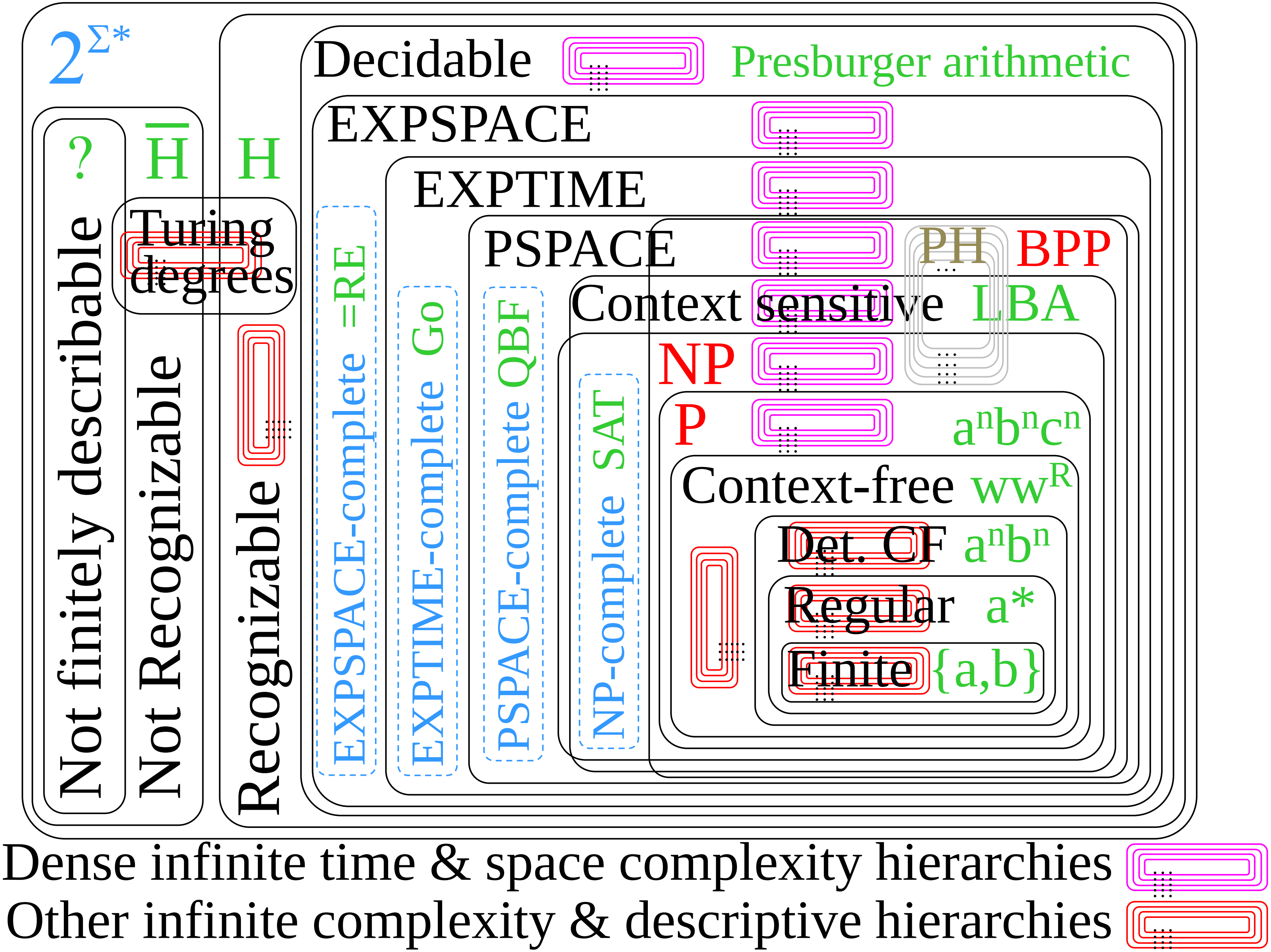
Generalized Numbers



Theorem: some real numbers are not finitely describable!

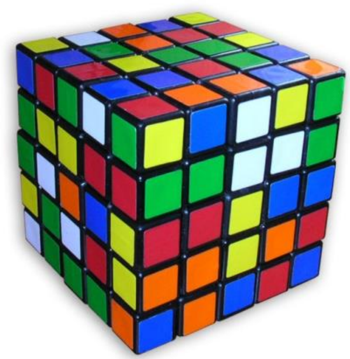
Theorem: some finitely describable real numbers are not computable!

The Extended Chomsky Hierarchy



Overarching Philosophy

- Focus on the "big picture" & "scientific method"
- Emphasis on problem solving & creativity
- Discuss applications & practice
- A primary objective: have fun!

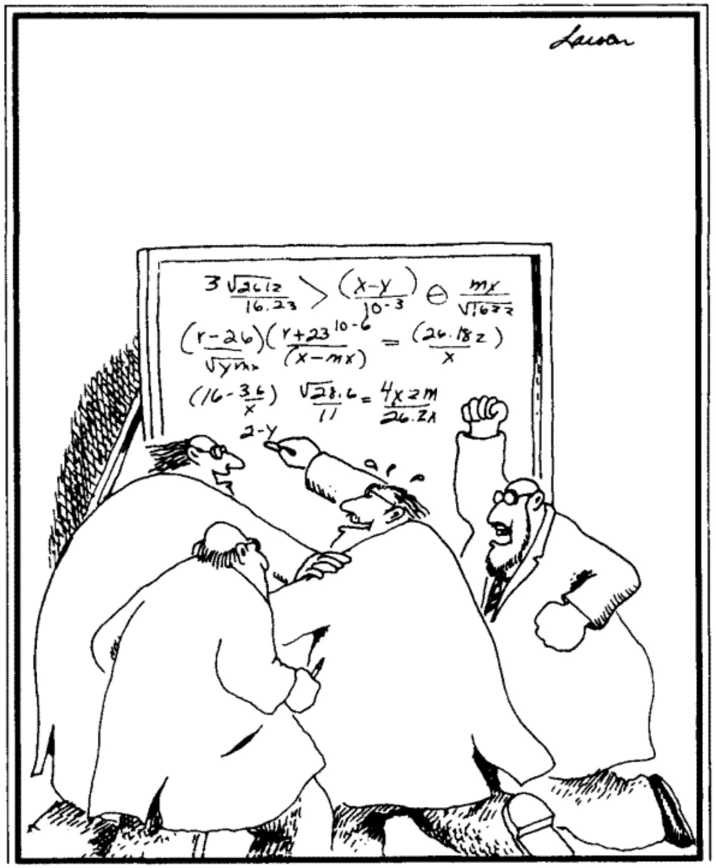


Prerequisites

- Some discrete math & algorithms knowledge
- Ideally, should have taken CS2102
- Course will "bootstrap" (albeit quickly) from first principles
- Critical: Tenacity, patience

Course Organization

- Exams: probably take home
  - Decide by vote
  - Flexible exam schedule
- Problem sets:
  - Lots of problem solving
  - Work in groups! (max size 6 people)
  - Not formally graded
  - Most exam questions will come from these sets!
- Homeworks:
  - Will come from problem sets
  - Formally graded
- Readings: papers / videos / books
- Extra credit problems
  - In class & take-home
  - Find mistakes in slides, handouts, etc.
- Course materials posted on Web site
  www.cs.virginia.edu/robins/theory



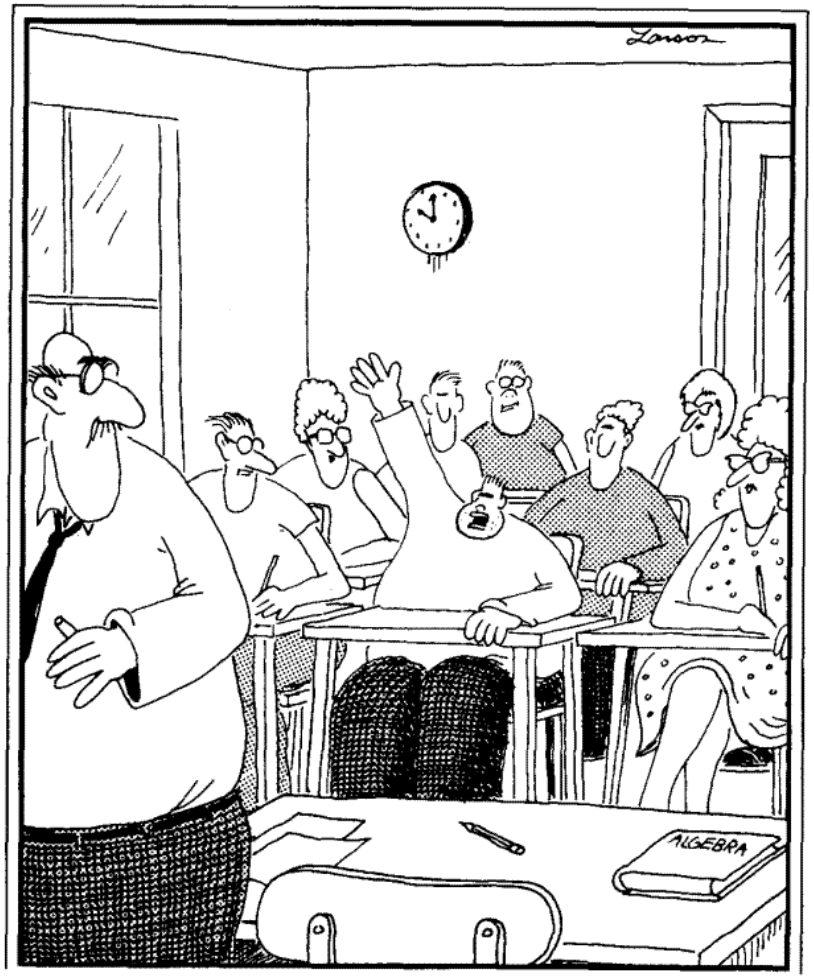
"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

Grading Scheme

• Attendance	10%
• Homeworks	20%
• Readings	20%
• Midterm	25%
• Final	25%
• Extra credit	10%
Total:	110% +

Best strategy:

- Solve lots of problems!
- Do lots of readings / EC!
- "Ninety percent of success is just showing up." – Woody Allen

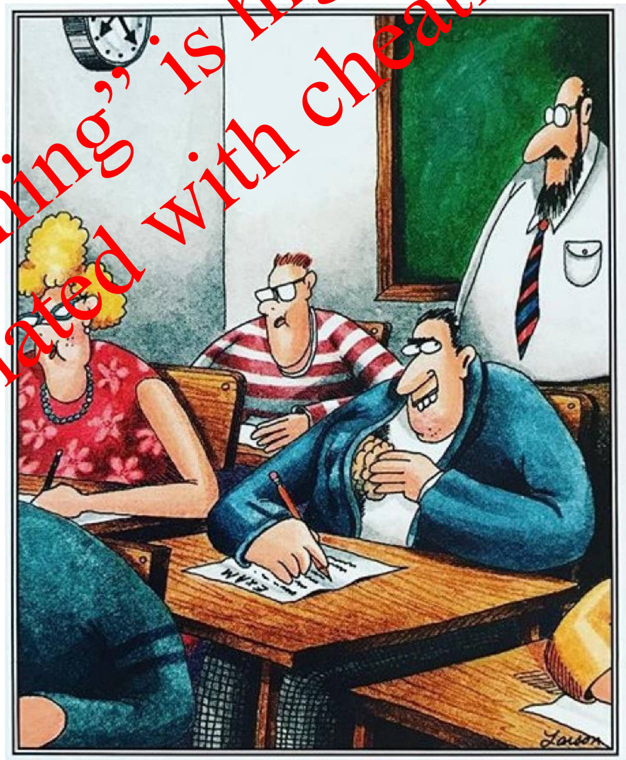


"Mr. Osborne, may I be excused? My brain is full."

Cheating Policy

- Cheating / plagiarism is strictly prohibited
- Serious penalties for violators
- Please review the UVa Honor Code
- Examples of Cheating / plagiarism:
  - Copying of solutions from others / Web
  - Sharing of solutions with others / Web
  - Cutting-and-pasting from other people / Web
  - Copying article/book/movie reviews from people / Web
  - Other people / Web solving entire problems for you
  - Providing other people / Web with verbatim solutions
  - Submitting answers that you don't understand!
  - This list is not exhaustive!
- We have automated cheating / plagiarism detection tools!
- We encourage collaborations / brainstorming
- Lets keep it positive (and not play "gotcha")

"Cramming" is highly correlated with cheating!



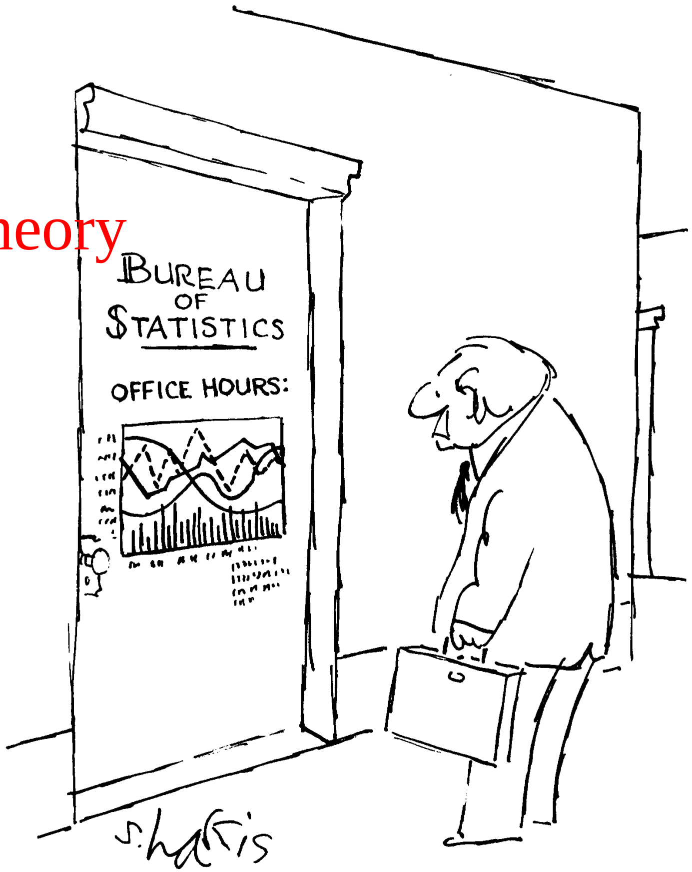
Midway through the exam, Allen pulls out a bigger brain.

Contact Information

Professor	Gabriel Robins
Office:	406 Rice Hall
Phone:	(434) 982-2207
Email:	robins@cs.virginia.edu
Web:	www.cs.virginia.edu/robins
	www.cs.virginia.edu/robins/theory

Office hours: right after class

- Any other time
- By email (preferred)
- By appointment
- Q&A blog posted on class Web site



Course Readings
www.cs.virginia.edu/robins/CS_readings.html

Goal: broad exposure to lots of cool ideas & technologies!

- Required: total of at least 36 items over the semester
- Diversity: minimums in each of 3 categories:
  1. Minimum of 15 videos
  2. Minimum of 15 papers / Web sites
  3. Minimum of 6 books
- More than 36 total is even better! (extra credit)
- Some required items in each category
  - Remaining "elective" items should be a diverse mix
- Email all submissions to: homework.cs3102@gmail.com

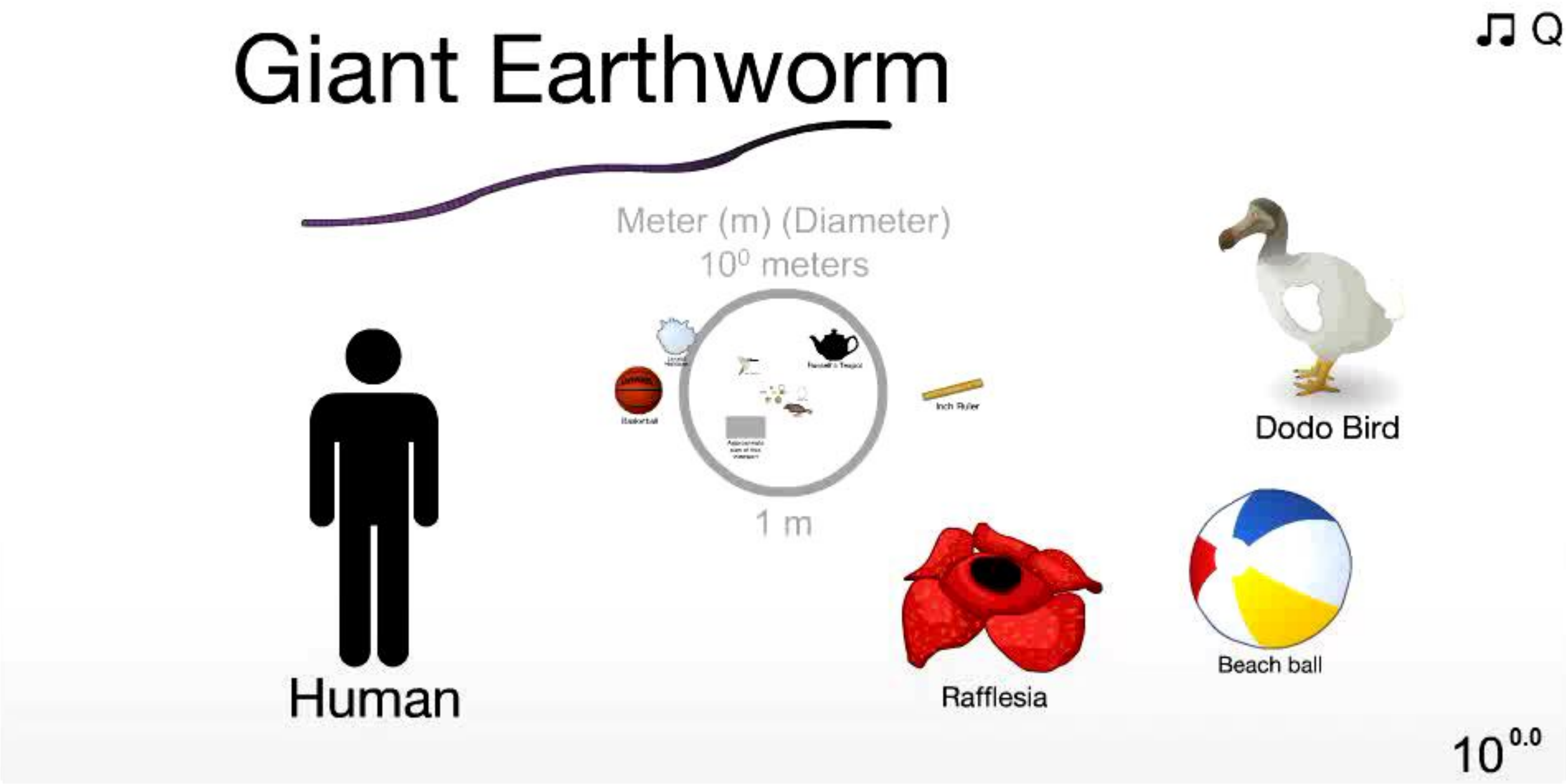
Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Required videos:
  - Last Lecture, Randy Pausch, 2007
  - Time Management, Randy Pausch, 2007
  - Powers of Ten, Charles and Ray Eames, 1977

Required Reading

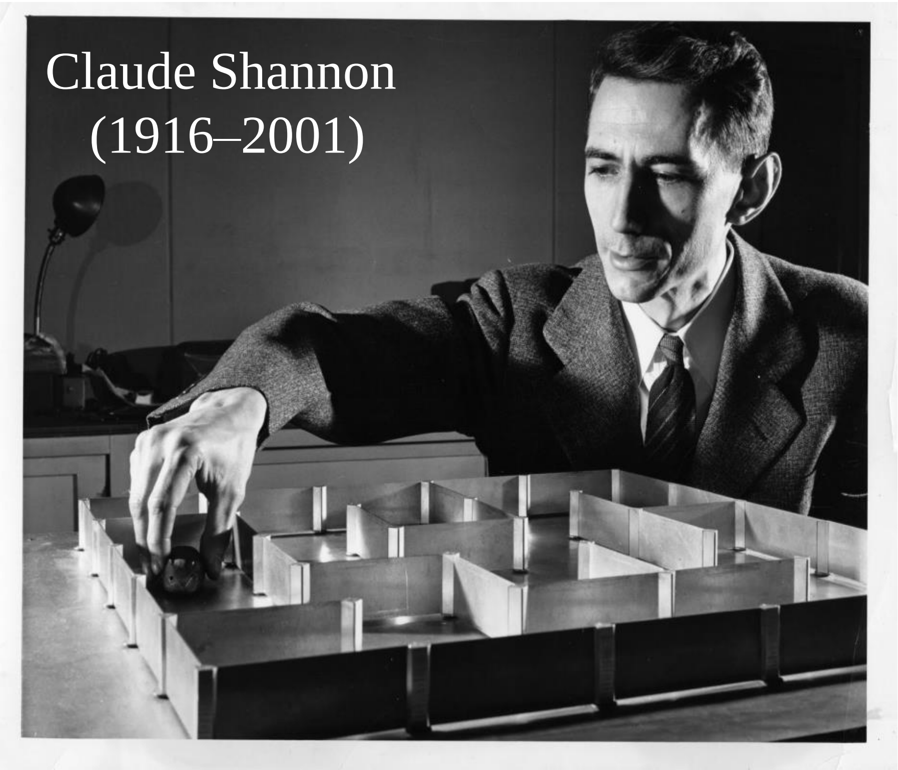
- "Scale of the Universe", Cary and Michael Huang, 2012



- 10^{-24} to 10^{26} meters ⇒ 50 orders of magnitude!

Required Readings
www.cs.virginia.edu/robins/CS_readings.html

- More required videos:
  - Claude Shannon - Father of the Information Age, UCTV
  - The Pattern Behind Self-Deception, Michael Shermer, 2010


Claude Shannon
(1916–2001)

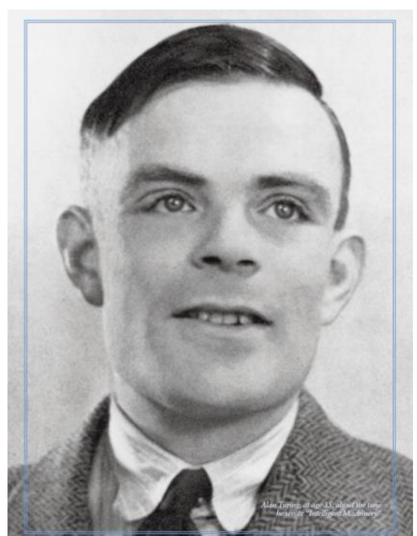
Michael Shermer

Required Readings

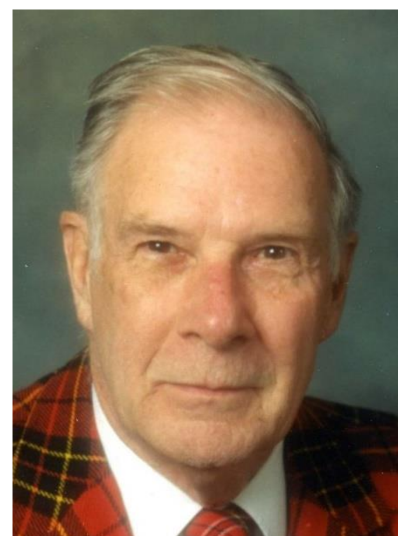
www.cs.virginia.edu/robins/CS_readings.html

- Required articles:
  - Decoding an Ancient Computer, Freeth, 2009
  - Alan Turing's Forgotten Ideas, Copeland and Proudfoot, 1999
  - You and Your Research, Richard Hamming, 1986
  - Who Can Name the Bigger Number, Scott Aaronson, 1999

Antikythera computer, 200BC



Alan Turing



Richard Hamming

Scott Aaronson

"BENEDICT CUMBERBATCH IS OUTSTANDING"
RADIO TIMES
"THE BEST BRITISH FILM OF THE YEAR"
★★★★★
THE INDEPENDENT
"AN INSTANT CLASSIC"
★★★★★
GLAMOUR
"A SUPERB THRILLER"
★★★★
EMPIRE
★★★★
TIME OUT
★★★★
THE TIMES
THE IMITATION GAME
BENEDICT CUMBERBATCH
KEIRA KNIGHTLEY
12A MODERATE SEX REFERENCES
BASED ON THE INCREDIBLE TRUE STORY
/ImitationGameUK
IN CINEMAS NOVEMBER 14
Extra credit!

Basic Concepts and Notation

Gabriel Robins

Required reading

"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean -- neither more nor less."

A **set** is formally an undefined term, but intuitively it is a (possibly empty) collection of arbitrary objects. A set is usually denoted by curly braces and some (optional) restrictions. Examples of sets are {1,2,3}, {hi, there}, and {k | k is a perfect square}. The symbol $\in$ denotes set **membership**, while the symbol $\notin$ denotes set **non-membership**; for example, $7 \in \{p \mid p \text{ prime}\}$ states that 7 is a prime number, while $q \notin \{0,2,4,6,\ldots\}$ states that q is not an even number. Some **common sets** are denoted by special notation:

The **natural numbers**: $\mathbb{N} = \{1,2,3,\ldots\}$

The **integers**: $\mathbb{Z} = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

The **rational numbers**: $\mathbb{Q} = \{\frac{a}{b} \mid a,b \in \mathbb{Z}, b \neq 0\}$

The **real numbers**: $\mathbb{R} = \{x \mid x \text{ is a real number}\}$

The **empty set**: $\varnothing = \{\}$

http://www.cs.virginia.edu/robins/cs3102/basics.pdf

JOHANNES KEPLER'S UPHILL BATTLE

Discrete Math Review Slides

Required reading

Symbolic Logic

Def: *proposition* - statement either true (T) or false (F)

Ex: 1 + 1 = 2

2 + 2 = 3

3 < 7

x + 4 = 5

"today is Monday"

Boolean Functions

- "and" ^
- "or" ∨
- "not" ¬
- "xor" ⊕
- "nand"
- "nor"
- "implication" ⇒
- "equivalence" ⇔

Logical Implication

- "implies" ⇒

Truth table:

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Ex: (x<0) ∧ (x>0) ⇒ (x=0)

$1 < x < y \Rightarrow x^3 < y^3$

"today is Sunday" ⇒ 1+1=3

Logical Equivalence

- "biconditional" ⇔

or "if and only if" ("iff")

or "necessary and sufficient"

or "logically equivalent" ≡

Truth table:

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

Ex: p ⇔ p

[(x=0) ∨ (y=0)] ⇔ (xy=0)

min(x,y)=max(x,y) ⇔ x=y

Predicates

Def: *predicate* - a function or formula involving some variables

Ex: let P(x) = "x > 3"

x is the variable

"x>3" is the predicate

P(5)

P(1)

Ex: Q(x,y,z) = "$x^2+y^2=z^2$"

Q(2,3,4)

Q(3,4,5)

Quantifiers

- Universal: "for all" ∀

∀x P(x)

$\Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \dots$

Ex: ∀x x < x + 1

$\forall x \quad x < x^3$

- Existential: "there exists" ∃

∃x P(x)

$\Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \dots$

Ex: $\exists x \quad x = x^2$

∃x x < x - 1

Combinations:

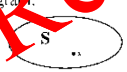
∀x ∃y y>x

Sets

Def: *set* - an unordered collection of elements

Ex: {1, 2, 3} or {hi, there}

Venn Diagram:



Def: two sets are *equal* iff they contain the same elements

Ex: {1, 2, 3} = {2, 3, 1}

{0} ≠ {1}

{3, 5} = {3, 5, 3, 3, 5}

Common Sets

Naturals: N = {1, 2, 3, 4, ...}

Integers: Z = {...,-2, -1, 0, 1, 2,...}

Rationals: Q = {$\frac{a}{b}$ | a,b∈Z, b≠0}

Reals: ℜ = {x | x a real #}

Empty set: Ø = {}

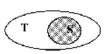
Z^+ = non-negative integers

$ℜ^-$ = non-positive reals, etc.

Subsets

- Subset notation: ⊆

S ⊆ T ⇔ (x∈S ⇒ x∈T)



- Proper subset: ⊂

S ⊂ T ⇔ ((S ⊆ T) ^ (S≠T))

S=T ⇔ ((T ⊆ S) ^ (S ⊆ T))

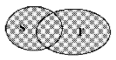
∀S Ø ⊆ S

∀S S ⊆ S

Union / Intersection

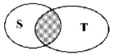
- Union: ∪

S∪T={x | x∈S ∨ x∈T}



- Intersection: ∩

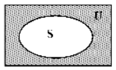
S∩T={x | x∈S ∧ x∈T}



Complements

- Universal set: U (everything)
- Set complement: S' or $\bar{S}$

S' = {x | x∉S} = U - S



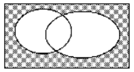
- Disjoint sets: S∩T=Ø

S - T= S ∩ T'

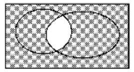
S - S = Ø

DeMorgan's Laws

(S∪T)' = S'∩T'



(S∩T)' = S'∪T'



Boolean logic version:

(X∧Y)'=X'∨Y'

(X∨Y)'=X'∧Y'

Function Types

- One-to-one function: "1-1"

a,b∈S ^ a≠b ⇒ *f*(a)≠*f*(b)

Ex: *f*: ℜ→ℜ, f(x)=2x is 1-1

$g(x)=x^2$ is not 1-1

- Onto function:

∀ t ∈T ∃ s∈S ∍ *f*(s)=t

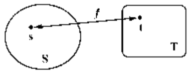
Ex: *f*: Z→Z, f(x)=13-x is onto

$g(x)=x^2$ is not onto

1-to-1 Correspondence

- 1-to-1 correspondence: *f*:S↔T

f is both 1-1 and onto



Ex: *f*: ℜ↔ℜ ∍ *f*(x)=x (identity)

h: N↔Z ∍ h(x)= $\frac{x-1}{2}$, x odd, $\frac{-x}{2}$, x even.

Generalized Cardinality

- S is at least as large as T:

|S ≥ T| ⇒ ∃ *f*:S→T, *f* onto

i.e., "S covers T"

Ex: r:ℜ→Z, r(x)=round(x)

⇒ |ℜ|≥|Z|

- S and T have same cardinality:

|S =|T| ⇒ |S|≥|T| ^ |T|≥|S

or

∃ 1-1 correspondence S↔T

- Generalizes finite cardinality:

{1, 2, 3, 4, 5} ≥ {a, b, c}

Infinite Sets

- Infinite set: |S > k ∀k∈Z

or

∃ 1-1 corres. *f*:S↔T, S⊂T

Ex: {p | p prime}, ℜ

- Countable set: |S ≤ N|

Ex: Ø, {p | p prime}, N, Z

- S is strictly smaller than T:

|S < |T| ⇒ |S|≤|T| ^ |S|≠|T|

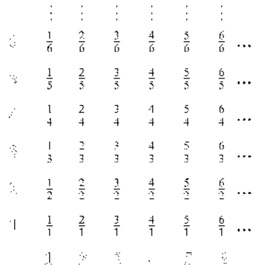
- Uncountable set: |N| < |S|

Ex: |N| < ℜ

|N| < [0,1] = {x | x∈ℜ, 0≤x≤1}

Thm: ∃ 1-1 correspondence Q↔N

Pf (dove-tailing):

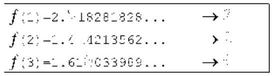


Thm: |ℜ|> N|

Pf (diagonalization):

Assume ∃ 1-1 corres. *f*: ℜ↔N

Construct X ∈ ℜ:



X = 0. ... ≠ *f*(K) ∀K∈N

⇒ *f* not a 1-1 correspondence

→ contradiction

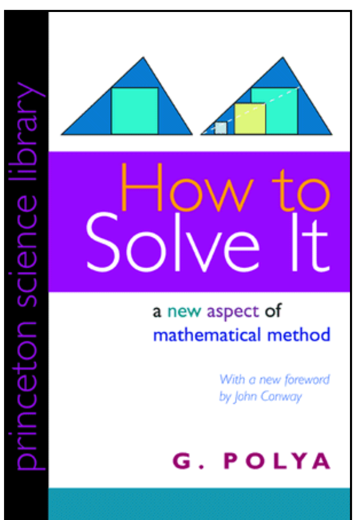
⇒ ℜ is uncountable

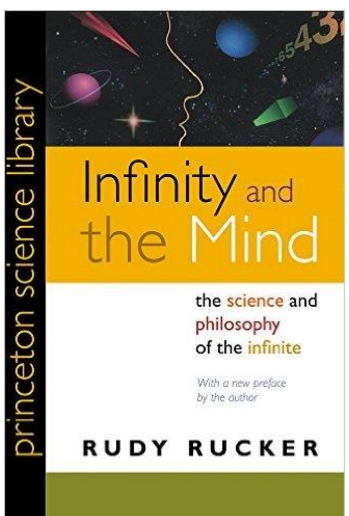
http://www.cs.virginia.edu/robins/cs3102/discrete_math_review_slides.pdf

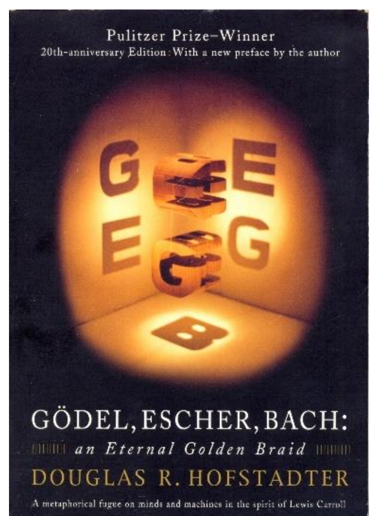
Required Readings

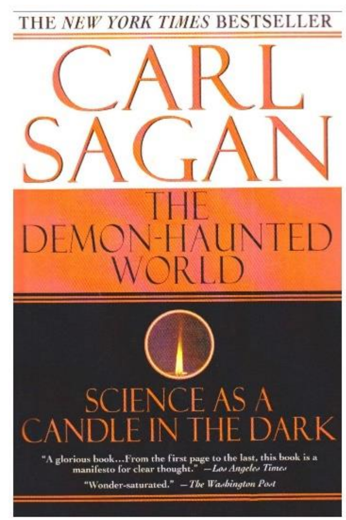
www.cs.virginia.edu/robins/CS_readings.html

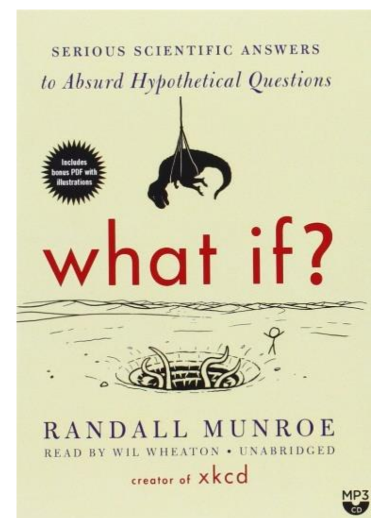
- Required books:
  - "How to Solve It", Polya, 1957
  - "Infinity and the Mind", Rucker, 1995
  - "Godel, Escher, Bach", Hofstadter, 1979
  - "The Demon-Haunted World", Sagan, 2009
  - "What If", Munroe, 2014











Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Remaining videos / articles / books are "electives"
- At least 2 submissions per week (due 11:59pm Mon)
- At most 2 submissions per day
- This policy is intended to help you avoid "cramming"
- "Cramming" is highly correlated with cheating!
- Length: 1-2 paragraphs per article / video
  1-2 pages per book
- Books are worth more credit than articles / videos
- Additional readings beyond 36 are welcome! (extra credit)
- Email all submissions to: homework.cs3102@gmail.com

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Theory and Algorithms:
  - Who Can Name the Bigger Number, Scott Aaronson, 1999
  - The Limits of Reason, Gregory Chaitin, Scientific American, March 2006, pp. 74-81.
  - Breaking Intractability, Joseph Traub and Henryk Wozniakowski, Scientific American, January 1994, pp. 102-107.
  - Confronting Science's Logical Limits, John Casti, Scientific American, October 1996, pp. 102-105.
  - Go Forth and Replicate, Moshe Sipper and James Reggia, Scientific American, August 2001, pp. 34-43.
  - The Science Behind Sudoku, Jean-Paul Delahaye, Scientific American, June 2006, pp. 80-87.
  - The Traveler's Dilemma, Kaushik Basu, Scientific American, June 2007, pp. 90-95.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- Biological Computing:
  - Computing with DNA, Leonard Adleman, Scientific American, August 1998, pp. 54-61.
  - Bringing DNA Computing to Life, Ehud Shapiro and Yaakov Benenson, Scientific American, May 2006, pp. 44-51.
  - Engineering Life: Building a FAB for Biology, David Baker et al., Scientific American, June 2006, pp. 44-51.
  - Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
  - DNA Computers for Work and Play, Macdonald et al, Scientific American, November 2007, pp. 84-91.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- Quantum Computing:
  - Quantum Mechanical Computers, Seth Lloyd, Scientific American, 1997, pp. 98-104.
  - Quantum Computing with Molecules, Gershenfeld and Chuang, Scientific American, June 1998, pp. 66-71.
  - Black Hole Computers, Seth Lloyd and Jack Ng, Scientific American, November 2004, pp. 52-61.
  - Computing with Quantum Knots, Graham Collins, Scientific American, April 2006, pp. 56-63.
  - The Limits of Quantum Computers, Scott Aaronson, Scientific American, March 2008, pp. 62-69.
  - Quantum Computing with Ions, Monroe and Wineland, Scientific American, August 2008, pp. 64-71.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- History of Computing:
  - The Origins of Computing, Campbell-Kelly, Scientific American, September 2009, pp. 62-69.
  - Ada and the First Computer, Eugene Kim and Betty Toole, Scientific American, April 1999, pp. 76-81.
- Security and Privacy:
  - Malware Goes Mobile, Mikko Hypponen, Scientific American, November 2006, pp. 70-77.
  - RFID Powder, Tim Hornyak, Scientific American, February 2008, pp. 68-71.
  - Can Phishing be Foiled, Lorrie Cranor, Scientific American, December 2008, pp. 104-110.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- Future of Computing:
  - Microprocessors in 2020, David Patterson, Scientific American, September 1995, pp. 62-67.
  - Computing Without Clocks, Ivan Sutherland and Jo Ebergen, Scientific American, August 2002, pp. 62-69.
  - Making Silicon Lase, Bahram Jalali, Scientific American, February 2007, pp. 58-65.
  - A Robot in Every Home, Bill Gates, Scientific Am, January 2007, pp. 58-65.
  - Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
  - Dependable Software by Design, Daniel Jackson, Scientific American, June 2006, pp. 68-75.
  - Not Tonight Dear - I Have to Reboot, Charles Choi, Scientific American, March 2008, pp. 94-97.
  - Self-Powered Nanotech, Zhong Lin Wang, Scientific American, January 2008, pp. 82-87.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

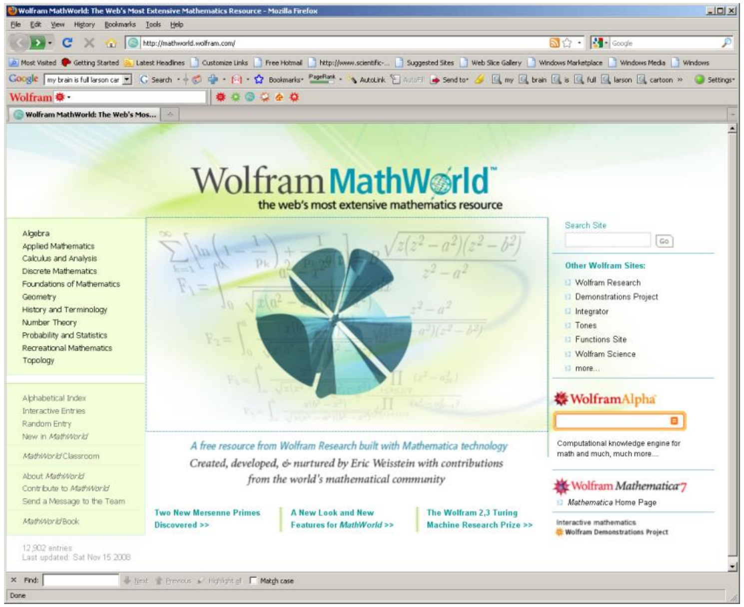
- The Web:
  - The Semantic Web in Action, Lee Feigenbaum et al., Scientific American, December 2007, pp. 90-97.
  - Web Science Emerges, Nigel Shadbolt and Tim Berners-Lee, Scientific American, October 2008, pp. 76-81.
- The Wikipedia Computer Science Portal:
  - Theory of computation and Automata theory
  - Formal languages and grammars
  - Chomsky hierarchy and the Complexity Zoo
  - Regular, context-free &Turing-decidable languages
  - Finite & pushdown automata; Turing machines
  - Computational complexity
  - List of data structures and algorithms

Email all submissions to: homework.cs3102@gmail.com

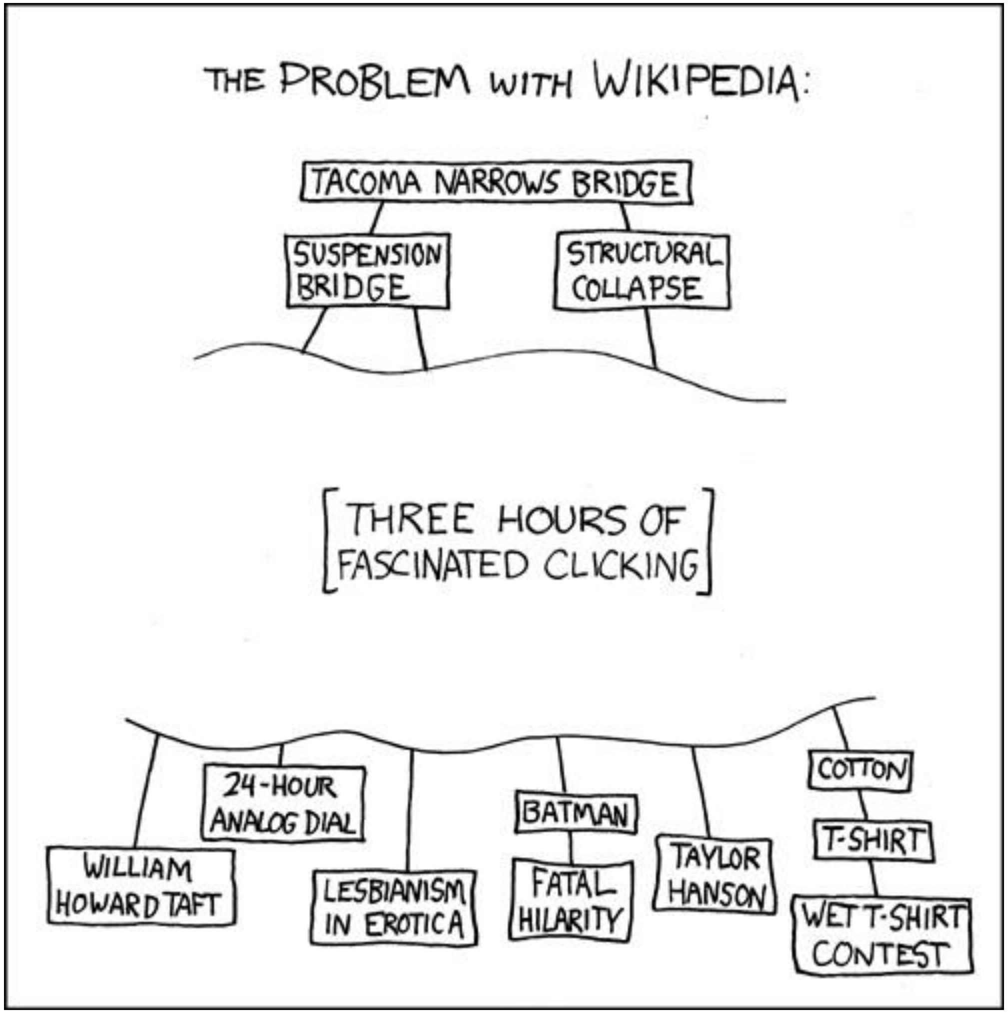
Other "Elective" Readings

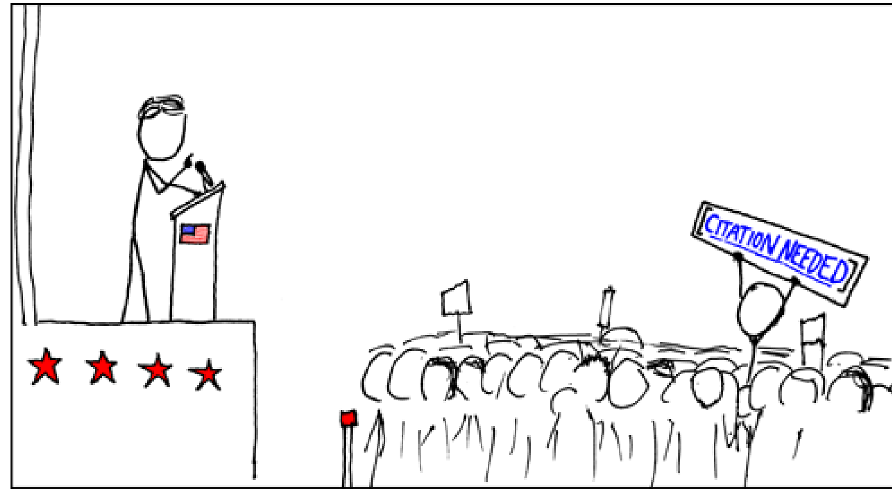
www.cs.virginia.edu/robins/CS_readings.html

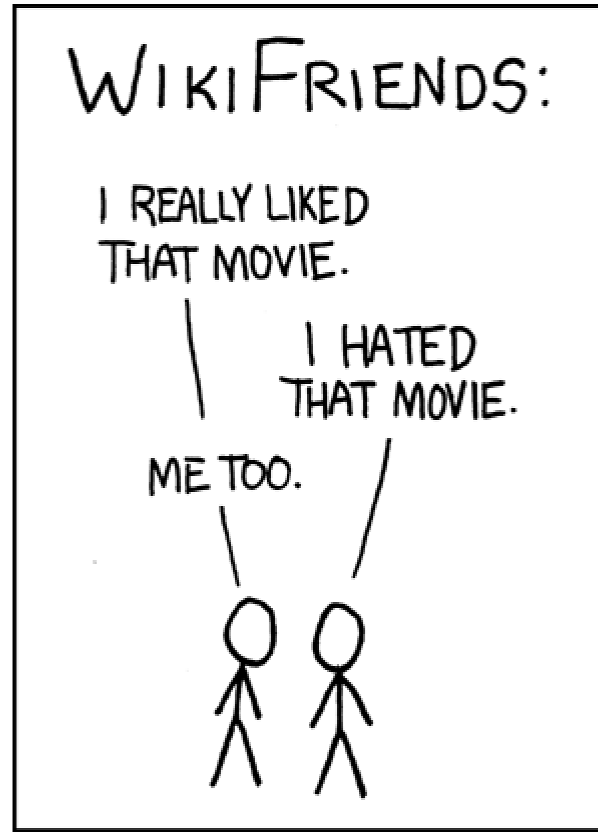
- The Wikipedia Math Portal:
  - Problem solving
  - List of Mathematical lists
  - Sets and Infinity
  - Discrete mathematics
  - Proof techniques and list of proofs
  - Information theory & randomness
  - Game theory
- Mathematica's "Math World"



Email all submissions to: homework.cs3102@gmail.com

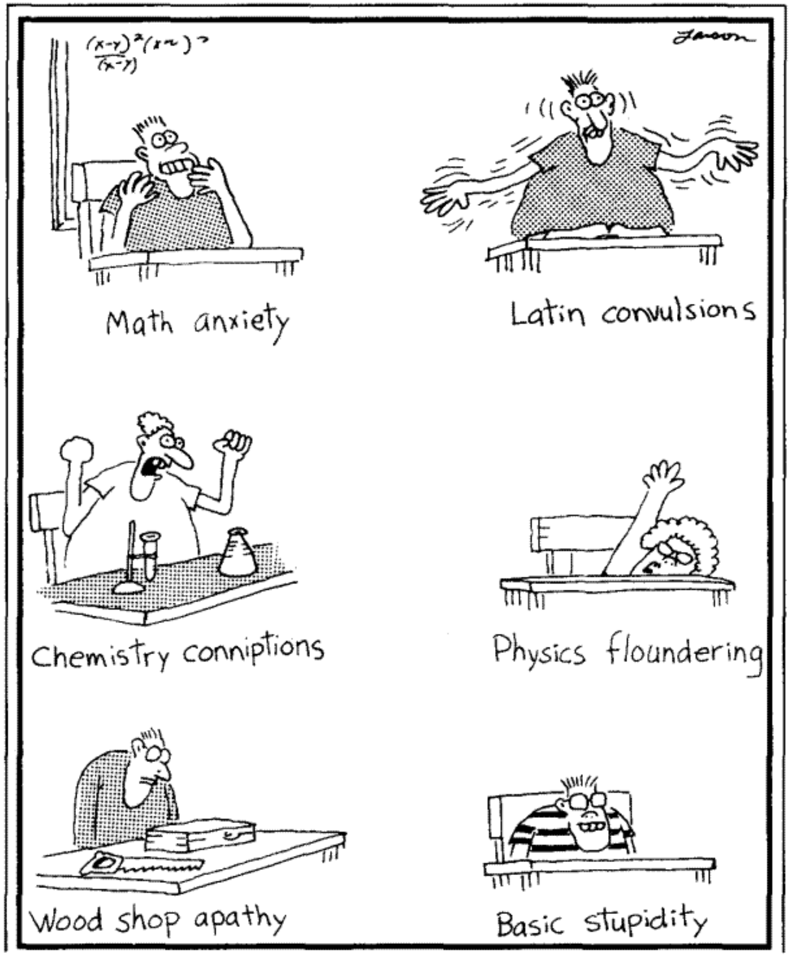

THE PROBLEM WITH WIKIPEDIA:
TACOMA NARROWS BRIDGE
SUSPENSION BRIDGE
STRUCTURAL COLLAPSE
[THREE HOURS OF FASCINATED CLICKING]
WILLIAM HOWARD TAFT
24-HOUR ANALOG DIAL
LESBIANISM IN EROTICA
BATMAN
FATAL HILARITY
TAYLOR HANSON
COTTON
T-SHIRT
WET T-SHIRT CONTEST


CITATION NEEDED


WIKIFRIENDS:
I REALLY LIKED THAT MOVIE.
I HATED THAT MOVIE.
ME TOO.

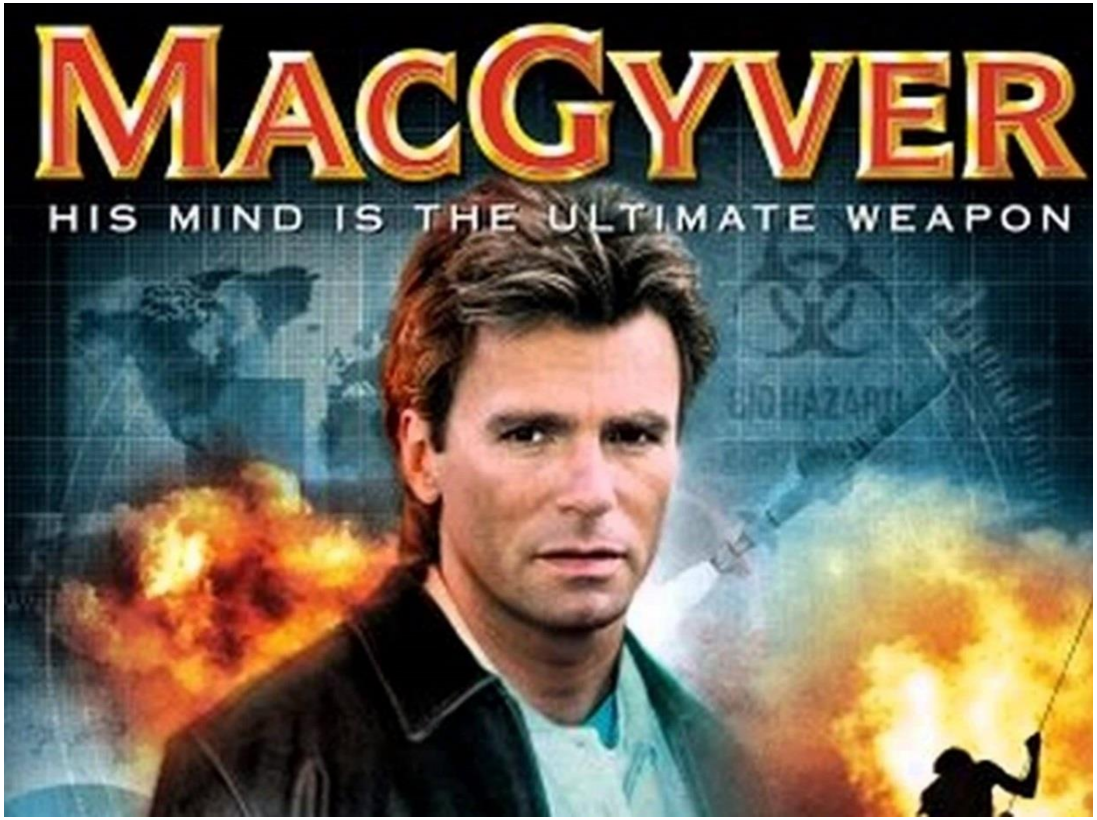
Good Advice

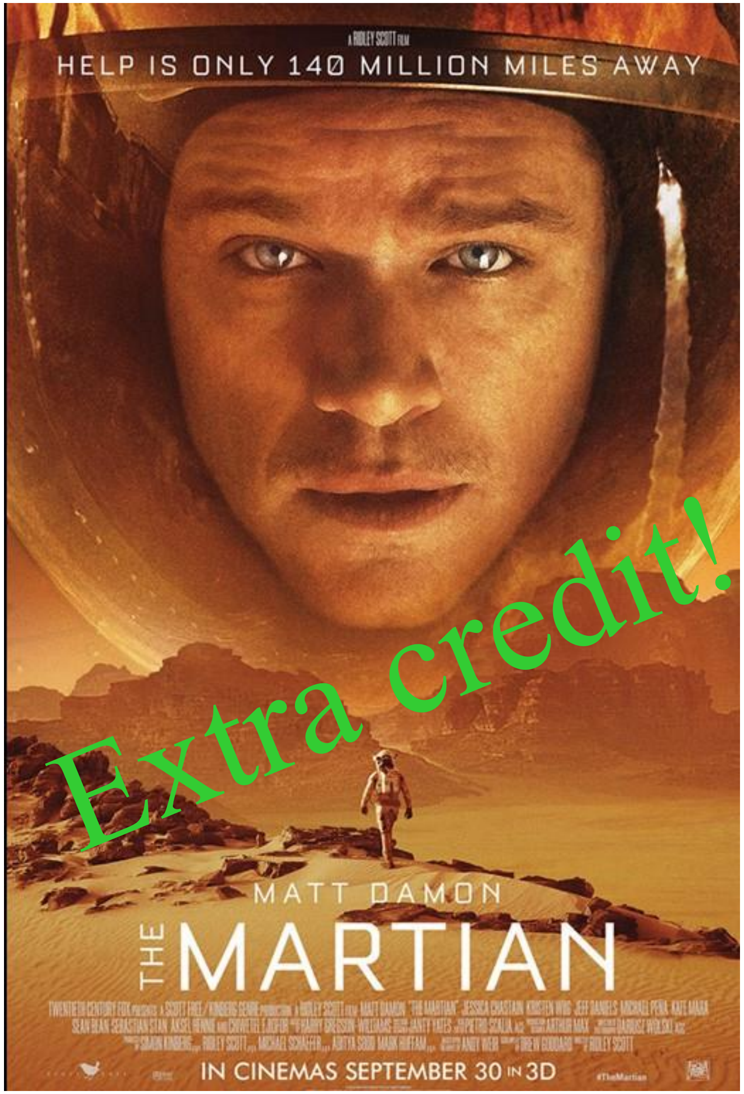
- Ask questions ASAP
- Solve problems ASAP
- Work in study groups
- Do not fall behind
- “Cramming” won’t work
- Do lots of extra credit
- Attend every lecture
- Visit class Website often
- Solve lots of problems



Classroom afflictions

Goal: Become a more effective problem solver!

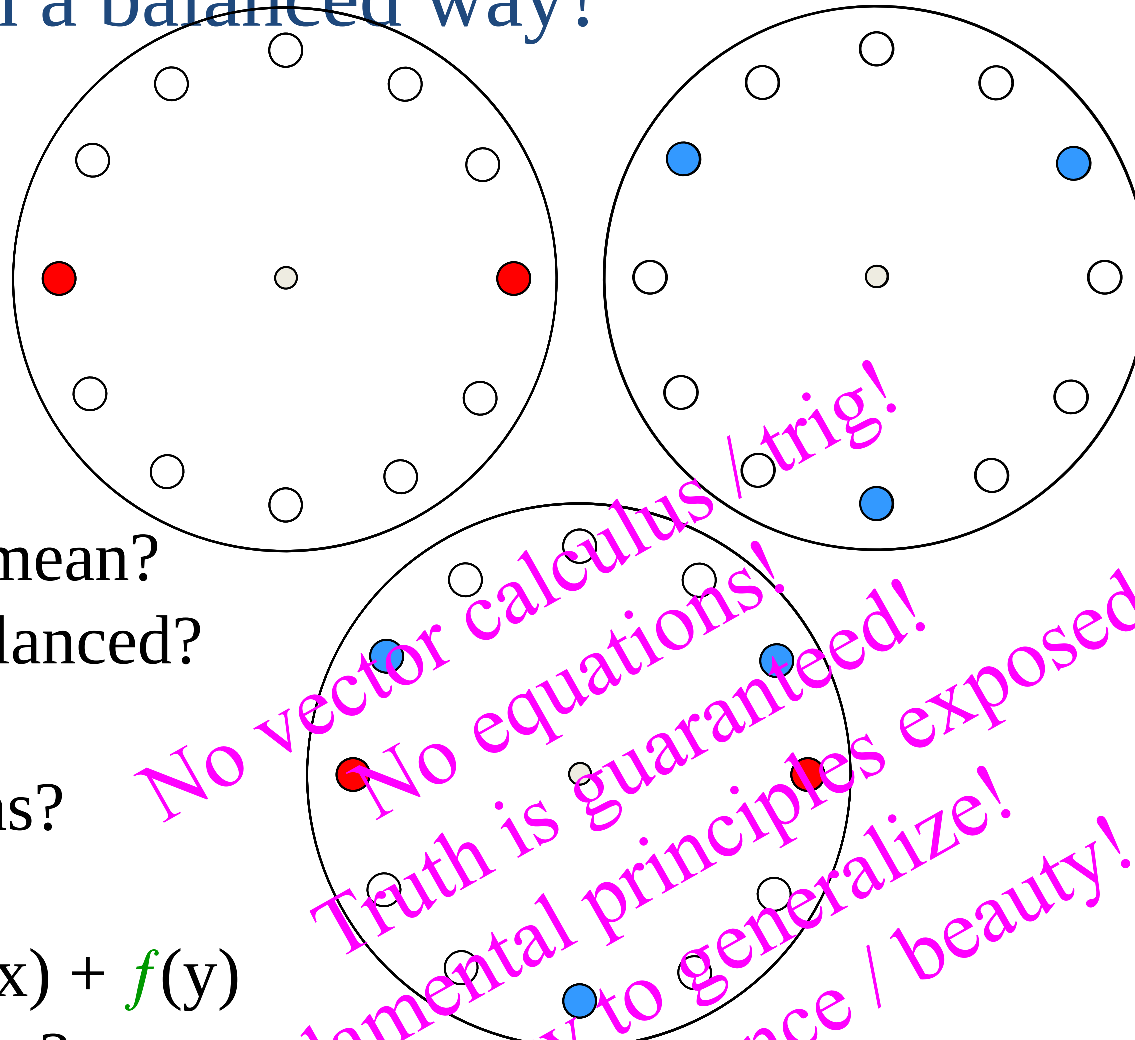




Extra credit!

Email all submissions to: homework.cs3102@gmail.com

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?



- What does "balanced" mean?
- Why are 3 test tubes balanced?
- Symmetry!
- Can you merge solutions?
- Superposition!
- Linearity! $f(x + y) = f(x) + f(y)$
- Can you spin 7 test tubes?
- Complementarity!
- Empirical testing…

No vector calculus / trig!
No equations!
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!

Problem: 1 + 2 + 3 + 4 + …+ 100 = ?

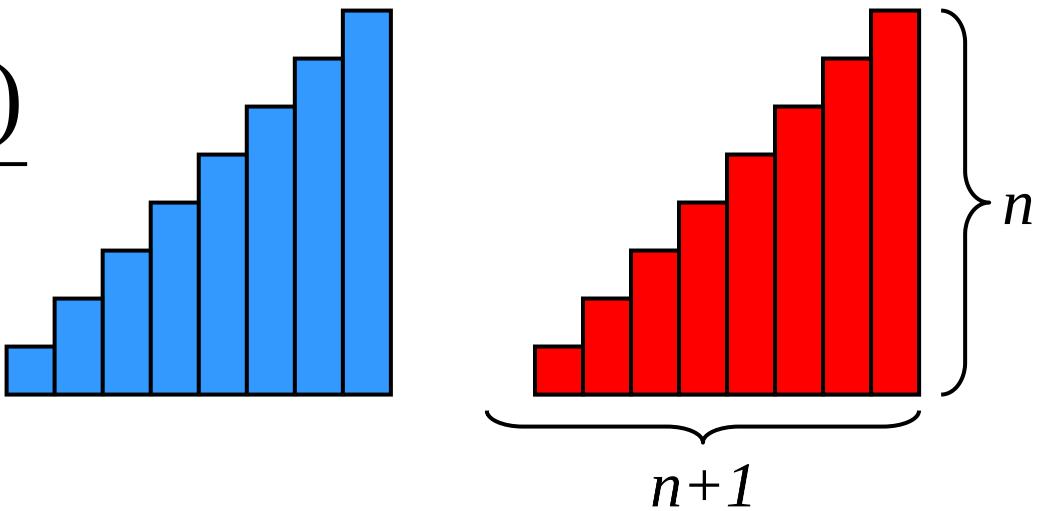
= (100*101)/2

= 5050

Proof: Induction…

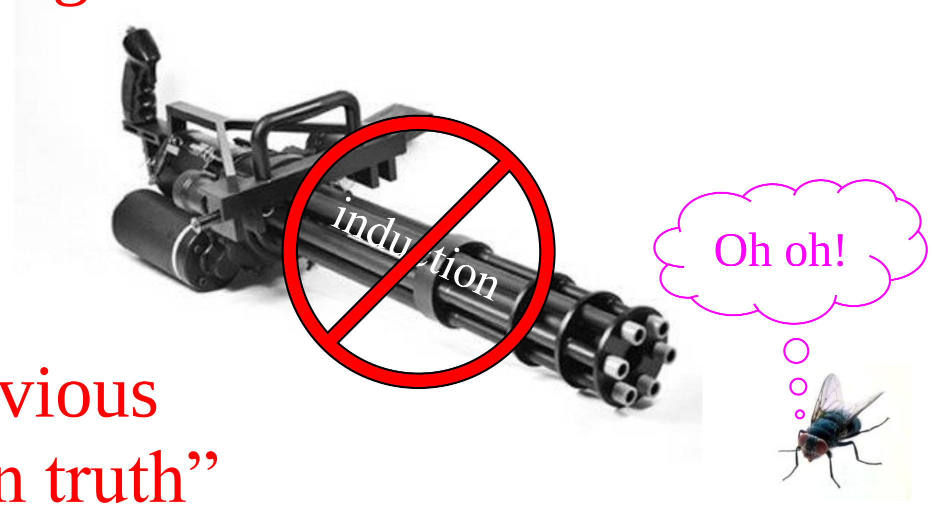
1	+	2	+	3	+	…	+	99	+	100	
100	+	99	+	98	+	…	+	2	+	1	
101	+	101	+	101	+	…	+	101	+	101	= 100*101

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$



Drawbacks of Induction

- You must a priori know the formula / result
- Easy to make mistakes in inductive proof
- Mostly “mechanical” – ignores intuitions
- Tedious to construct
- Difficult to check
- Hard to understand
- Not very convincing
- Generalizations not obvious
- Does not “shed light on truth”
- Obfuscates connections



Conclusion: only use induction as a last resort! (i.e., rarely)

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

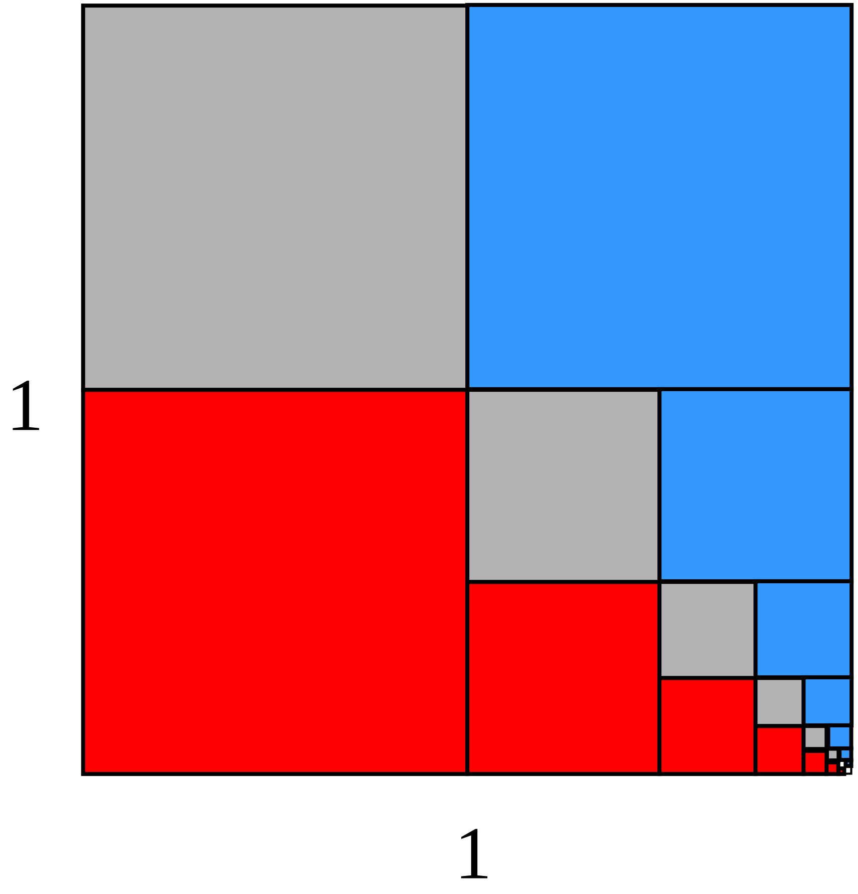
$$\sum_{i=1}^{\infty} \frac{1}{4^i} = ?$$

Extra Credit:

Find a short, geometric, induction-free proof.

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.



$$\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}$$

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = ?$$

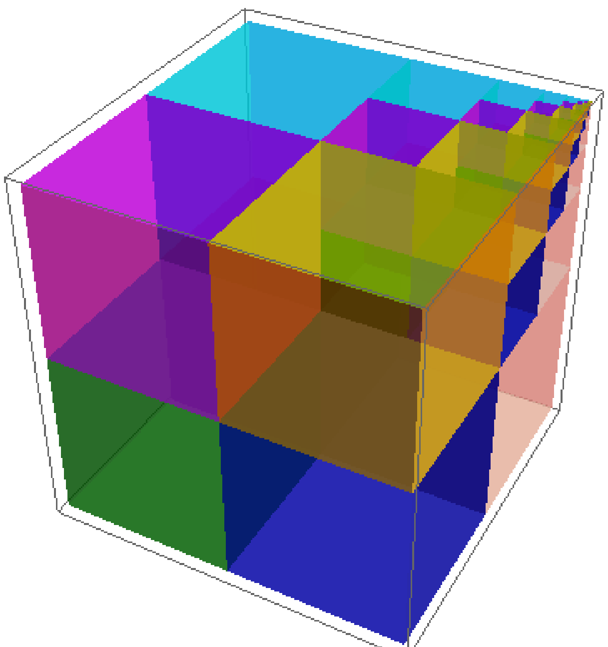
Extra Credit:

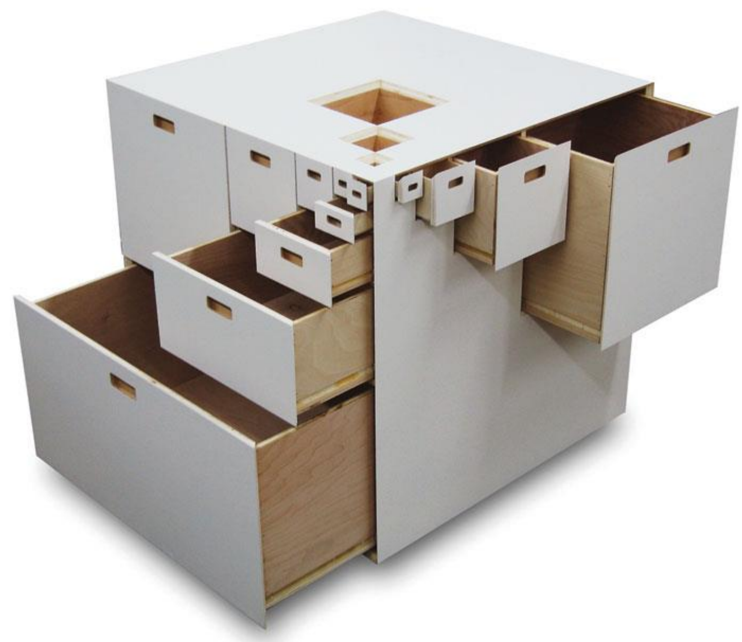
Find a short, geometric, induction-free proof.

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}$$



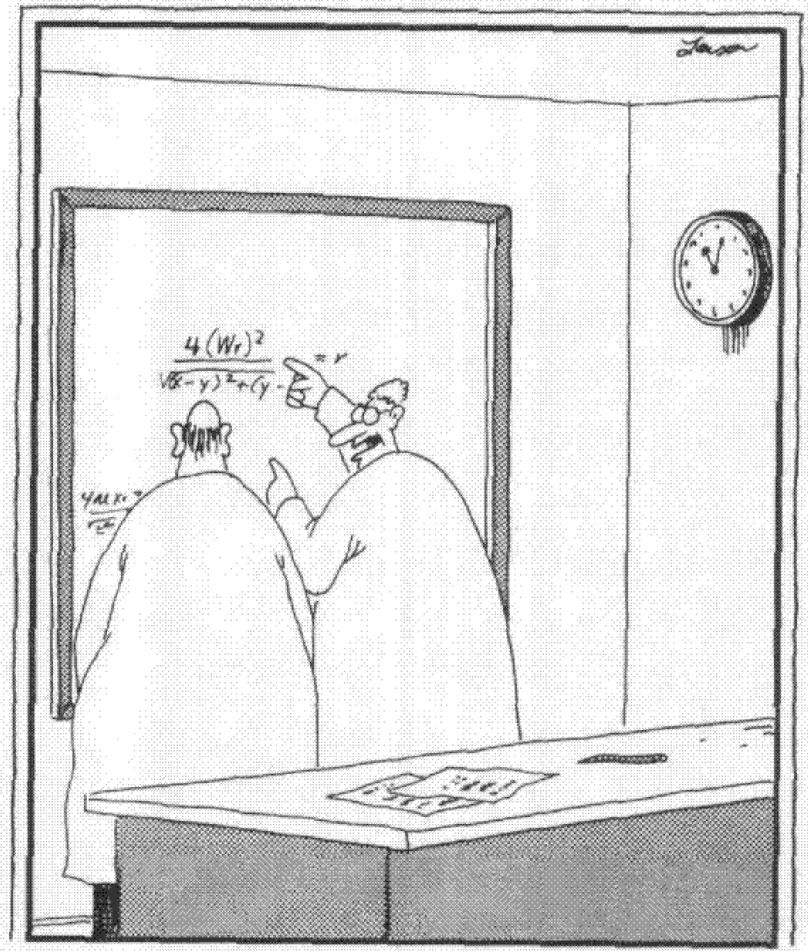


Problem: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = ?$

$$\sum_{i=1}^{n} i^3 = ?$$

Extra Credit:

find a short, geometric, induction-free proof.



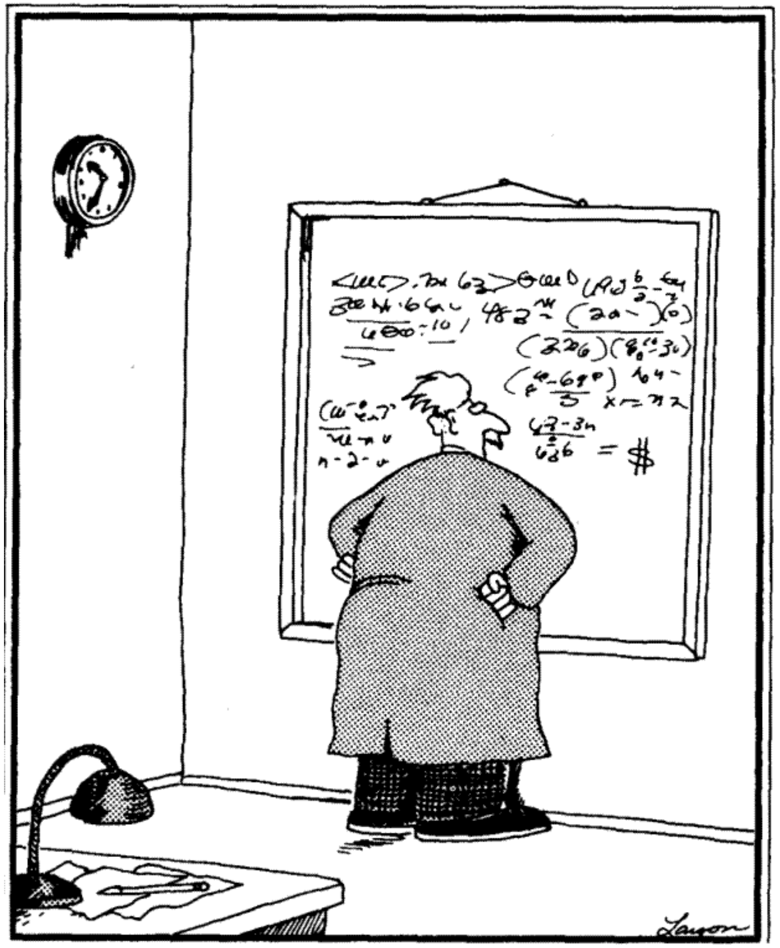
"Yes, yes, I *know* that, Sidney ... everybody knows *that*! ... But look: Four wrongs *squared*, minus two wrongs to the fourth power, divided by this formula, *do* make a right."

Email all submissions to: homework.cs3102@gmail.com

Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Einstein discovers that time is actually money.

Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

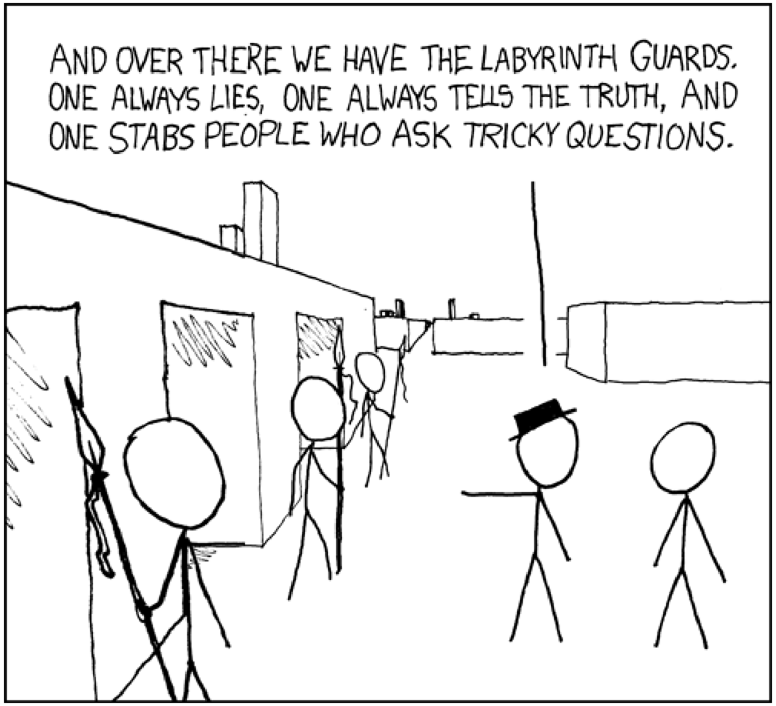
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Email all submissions to: homework.cs3102@gmail.com

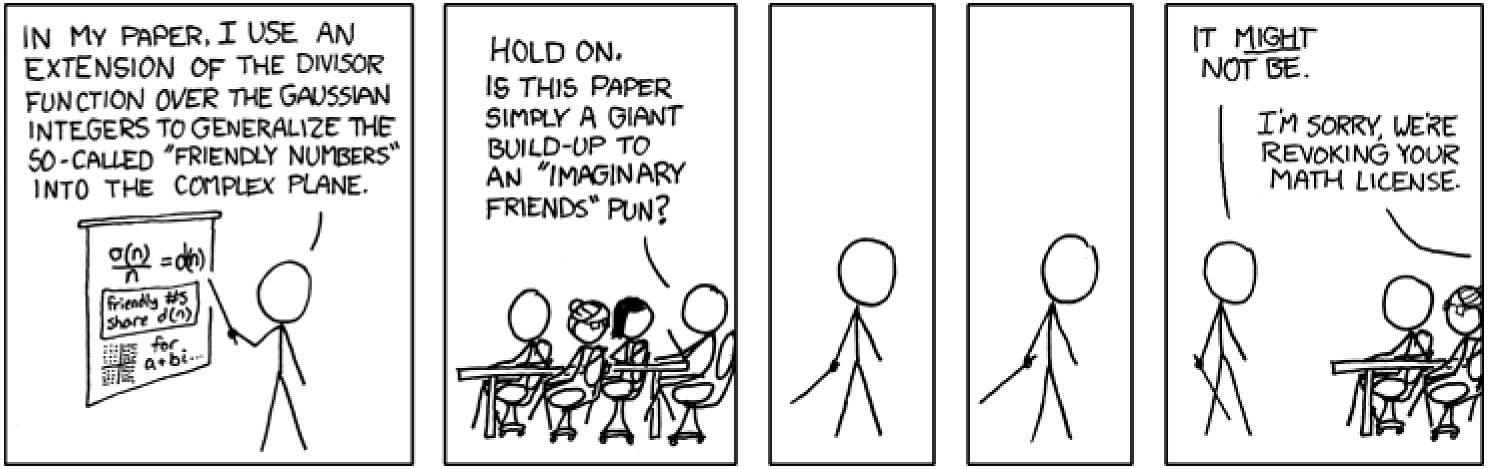
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



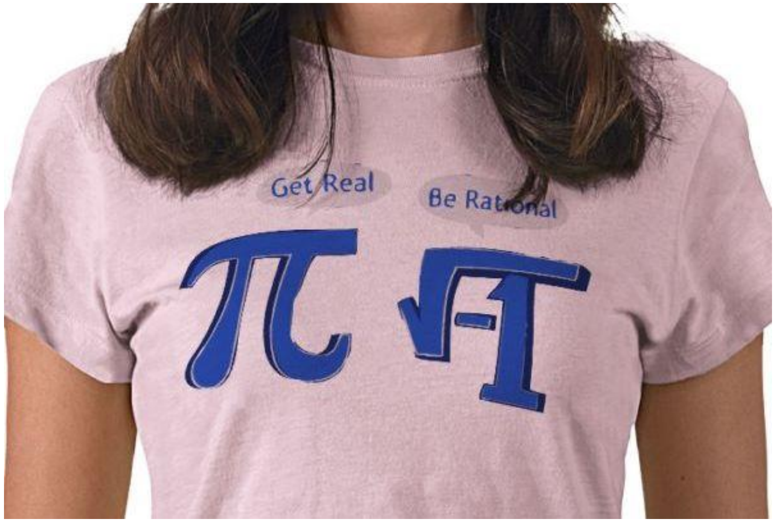
Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?



Problem: Does exponentiation preserve irrationality? i.e., are there two irrational numbers x and y such that x^y is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

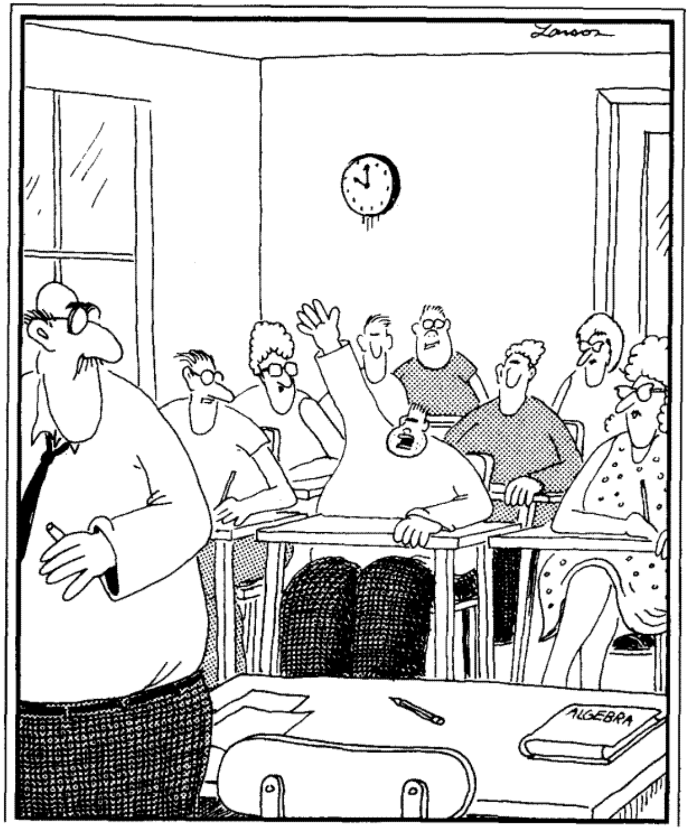


Problem: Solve the following equation for X:

$$X^{X^{X^{X^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

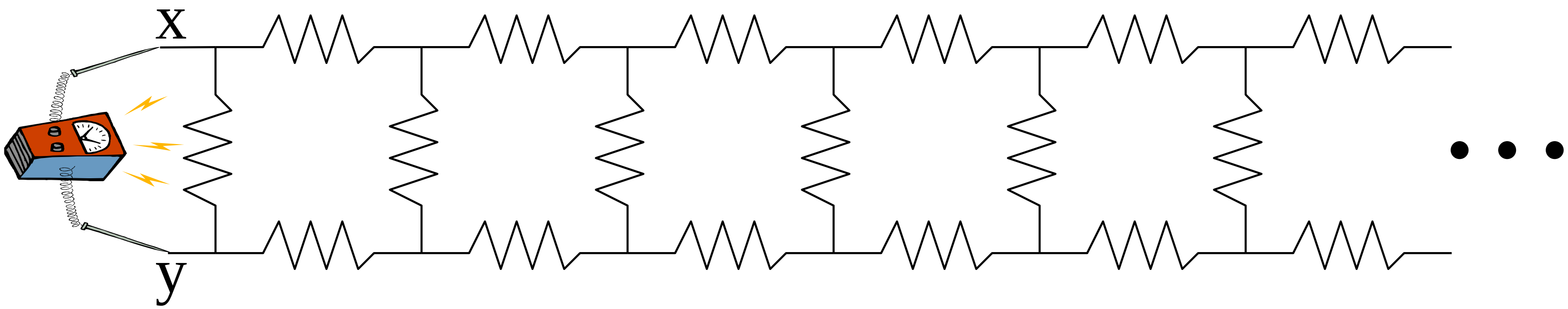
where the stack of exponentiated x's extends forever.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

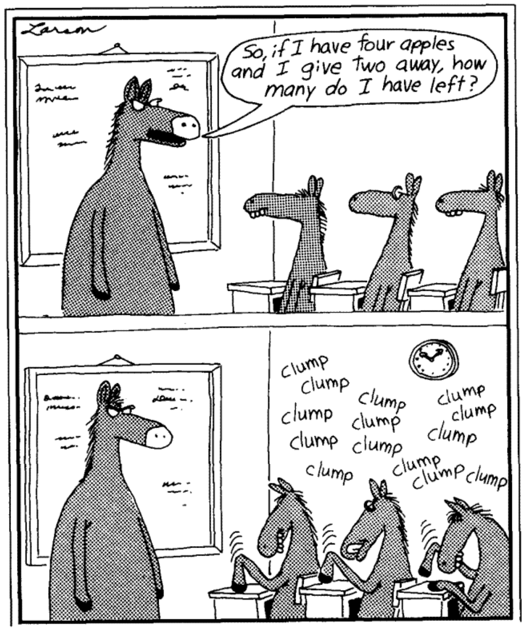


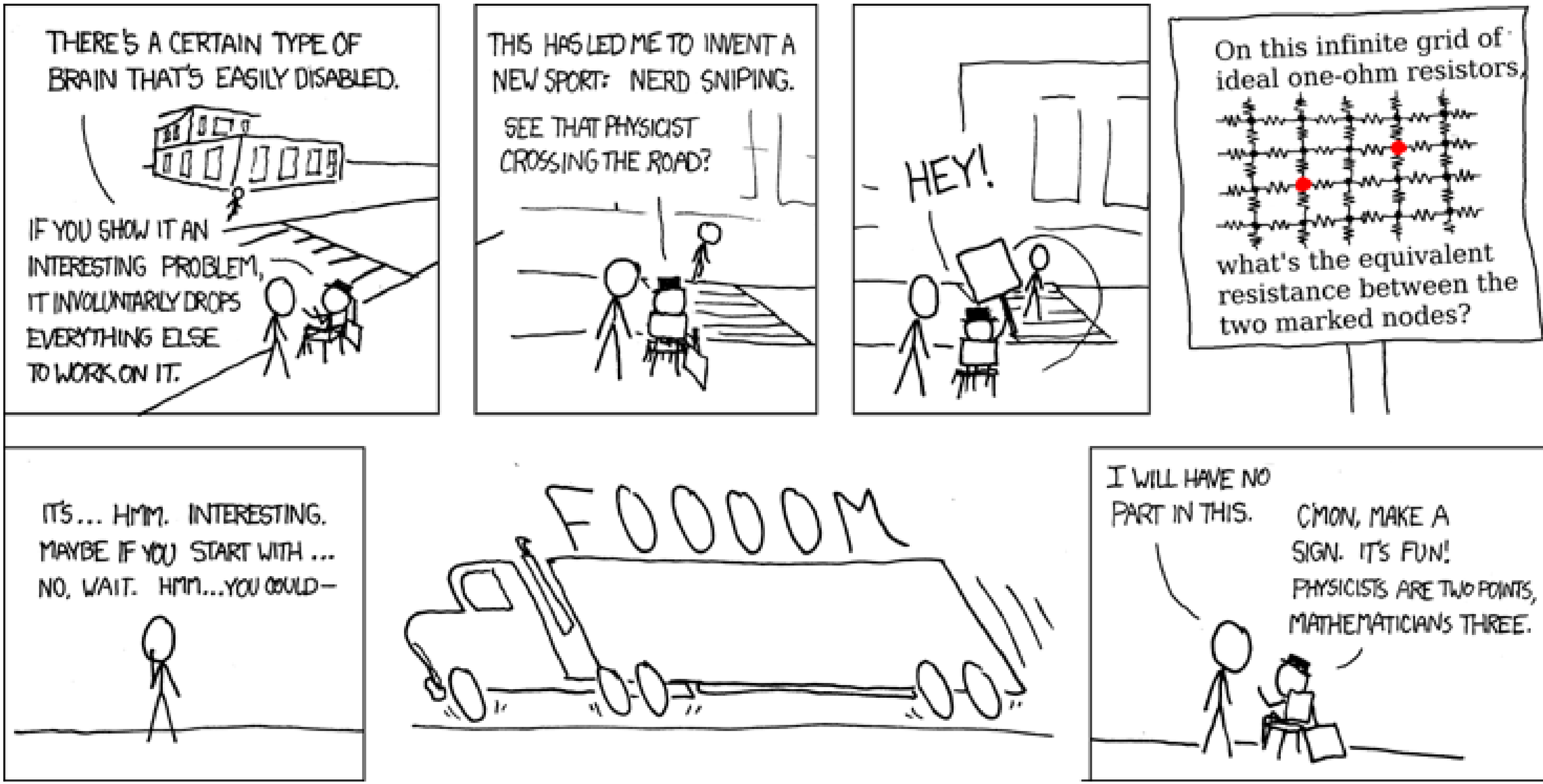
"Mr. Osborne, may I be excused? My brain is full."

Problem: For the given infinite ladder of resistors of resistance R each, what is the resistance measured between points x and y?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations




THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.
IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.
THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.
SEE THAT PHYSICIST CROSSING THE ROAD?
HEY!
On this infinite grid of ideal one-ohm resistors, what's the equivalent resistance between the two marked nodes?
IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH ... NO, WAIT. HMM...YOU COULD—
FOOOOM
I WILL HAVE NO PART IN THIS.
C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.

Historical Perspectives

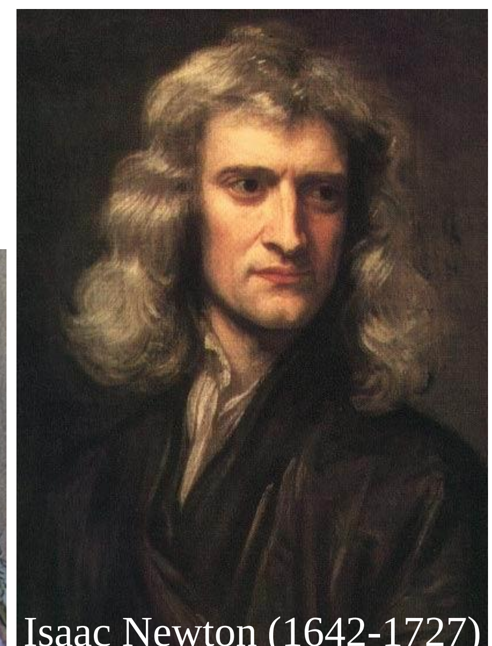
Historical Perspectives

- Knowing the "big picture" is empowering
- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many "parents"
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!

“Standing on the Shoulders of Giants”

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan
- Babbage, Ada Lovelace
- Venn, Carroll


Ada Lovelace (1815-1852)

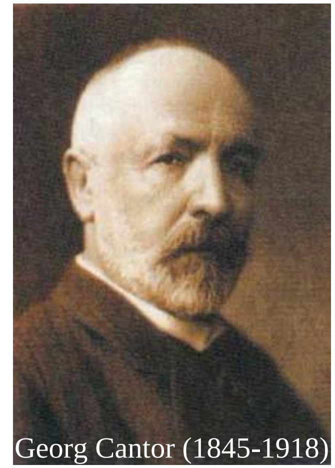

Isaac Newton (1642-1727)


Euclid (300 BC)

“Standing on the Shoulders of Giants”

- Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Gödel, Church, Turing
- von Neumann, Shannon
- Kleene, Chomsky
- Hoare, McCarthy, Erdos
- Knuth, Backus, Dijkstra

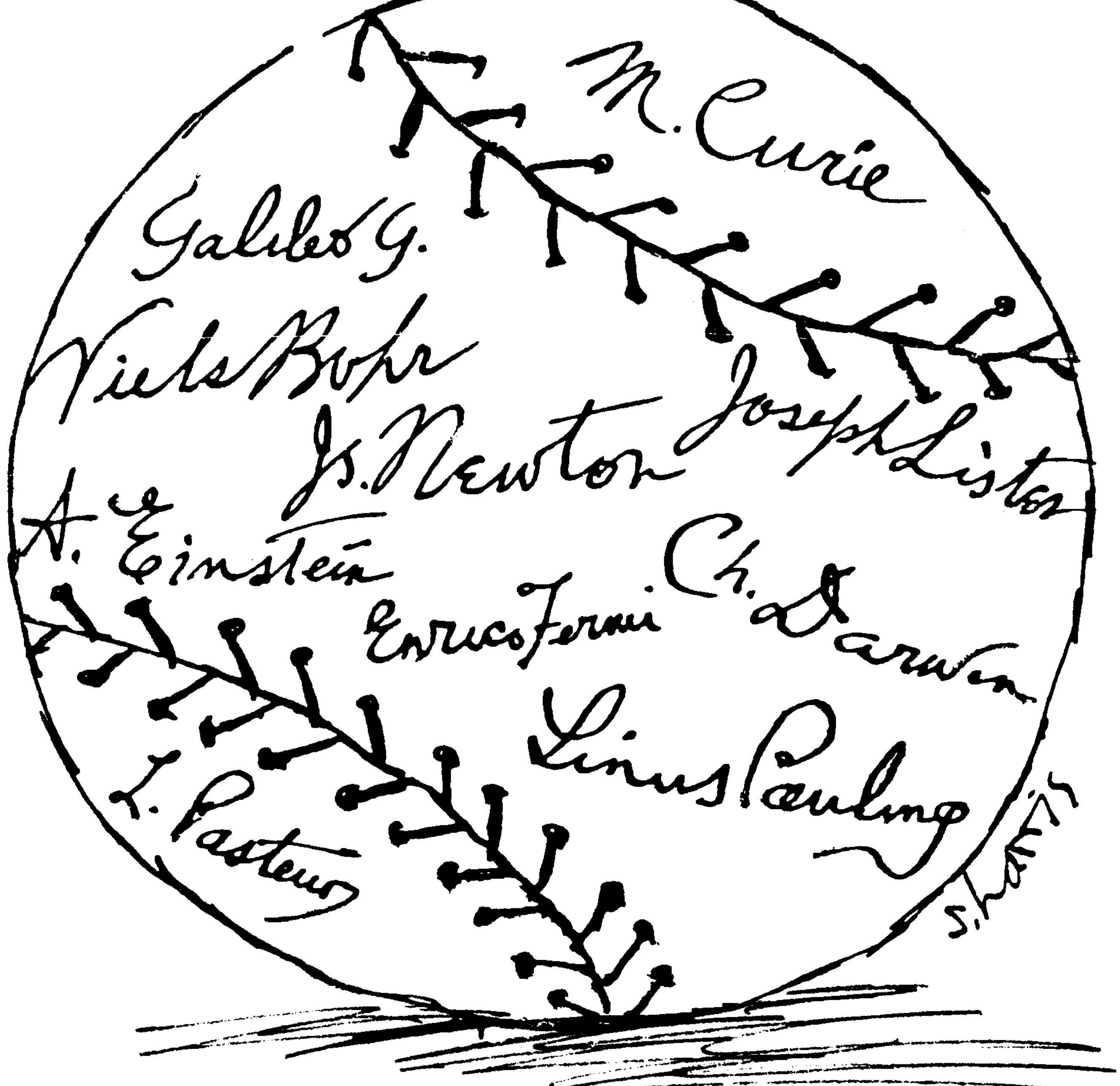
Many others…


Georg Cantor (1845-1918)


Bertrand Russell (1872-1970)


David Hilbert (1862-1943)

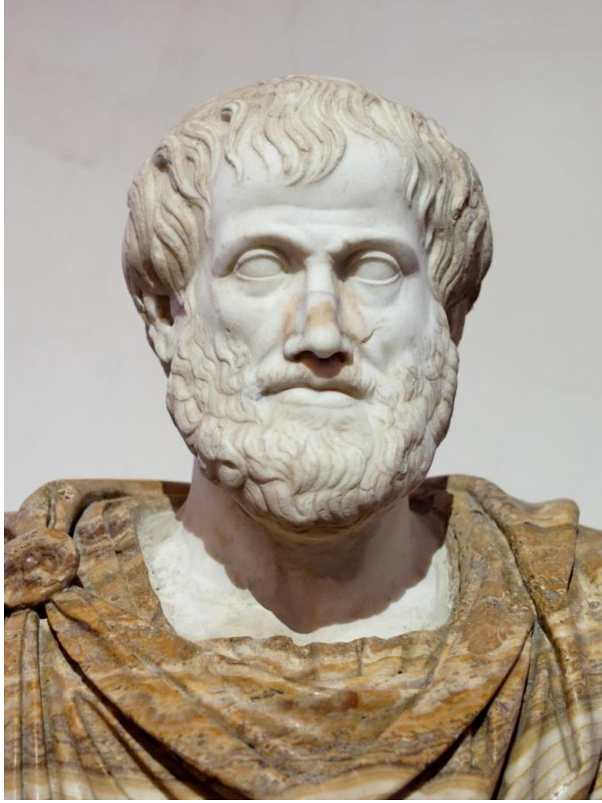

Kurt Gödel (1906-1978)


M. Curie
Galileo G.
Niels Bohr
Is. Newton
Joseph Lister
A. Einstein
Ch. Darwin
Enrico Fermi
Linus Pauling
L. Pasteur

Gauss
Newton
Archimedes
Euler
Cauchy
Poincare
Riemann
Cantor
Cayley
Hamilton
Eisenstein
Pascal
Abel
Hilbert
Klein
Leibniz
Descartes
Galois
Mobius
Jacob
Johann Bernoulli
Daniel Bernoulli
Dirichlet
Fermat
Pythagoras
Laplace
Lagrange
Kronecker
Jacobi
Bolyai
Lobatchewsky
Noether
Germain
Euclid
Legendre

MAKING PHILOSOPHY ACCESSIBLE: POP-UP PLATO

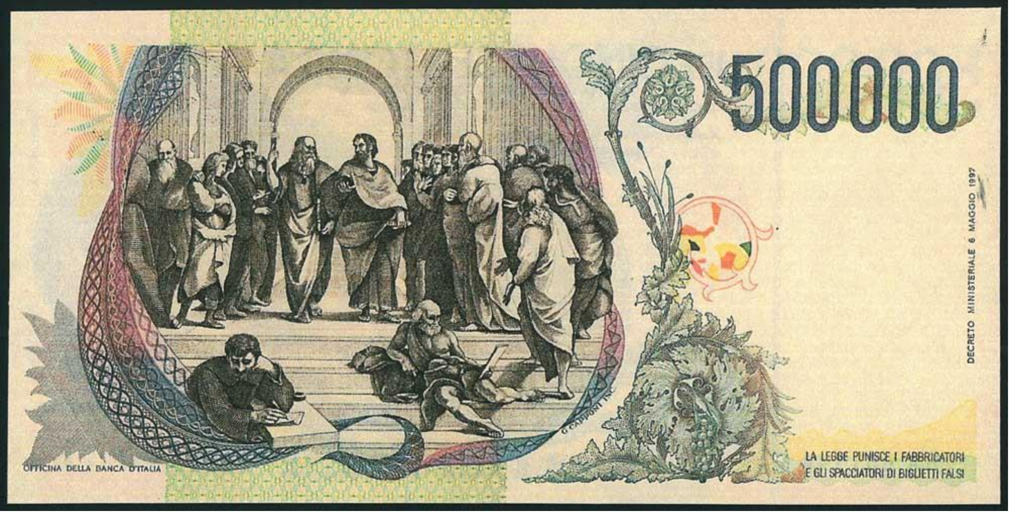
Historical Perspectives

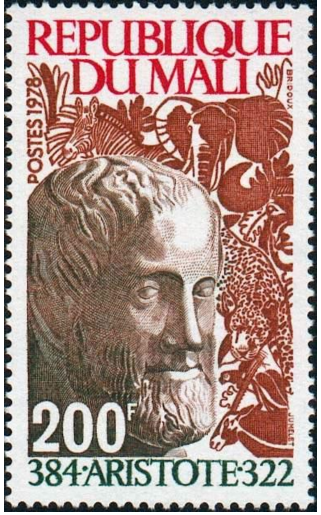


Aristotle (384BC-322BC)

- Founded Western philosophy
- Student of Plato
- Taught Alexander the Great
- "Aristotelianism"
- Developed the "scientific method"
- One of the most influential people ever
- Wrote on physics, theatre, poetry, music, logic, rhetoric, politics, government, ethics, biology, zoology, morality, optics, science, aesthetics, psychology, metaphysics, …
- Last person to know everything known in his own time!

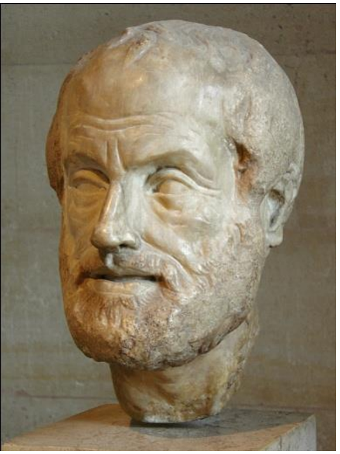
"Almost every serious intellectual advance has had to begin with an attack on some Aristotelian doctrine." – Bertrand Russell

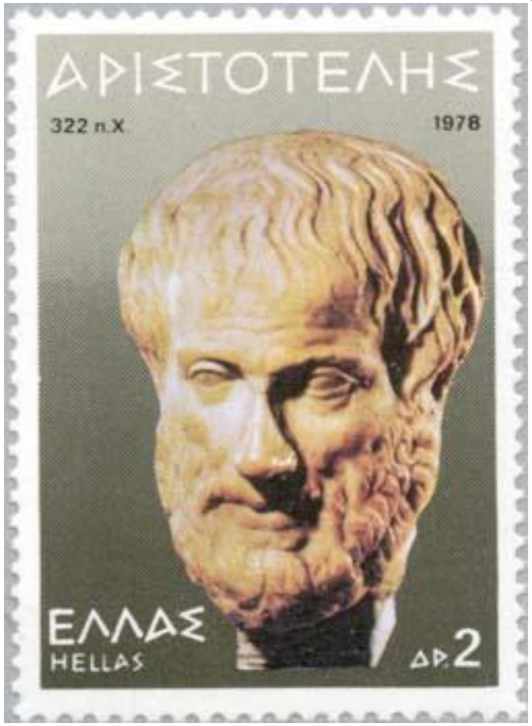


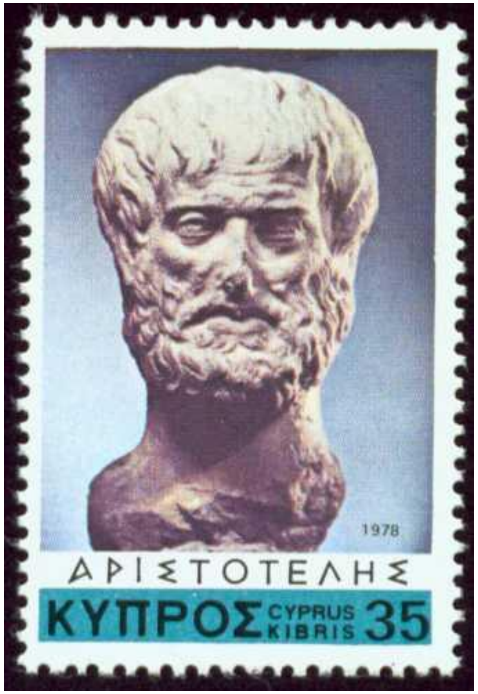


“Wit is educated insolence.”

- Aristotle (384-322 B.C.)







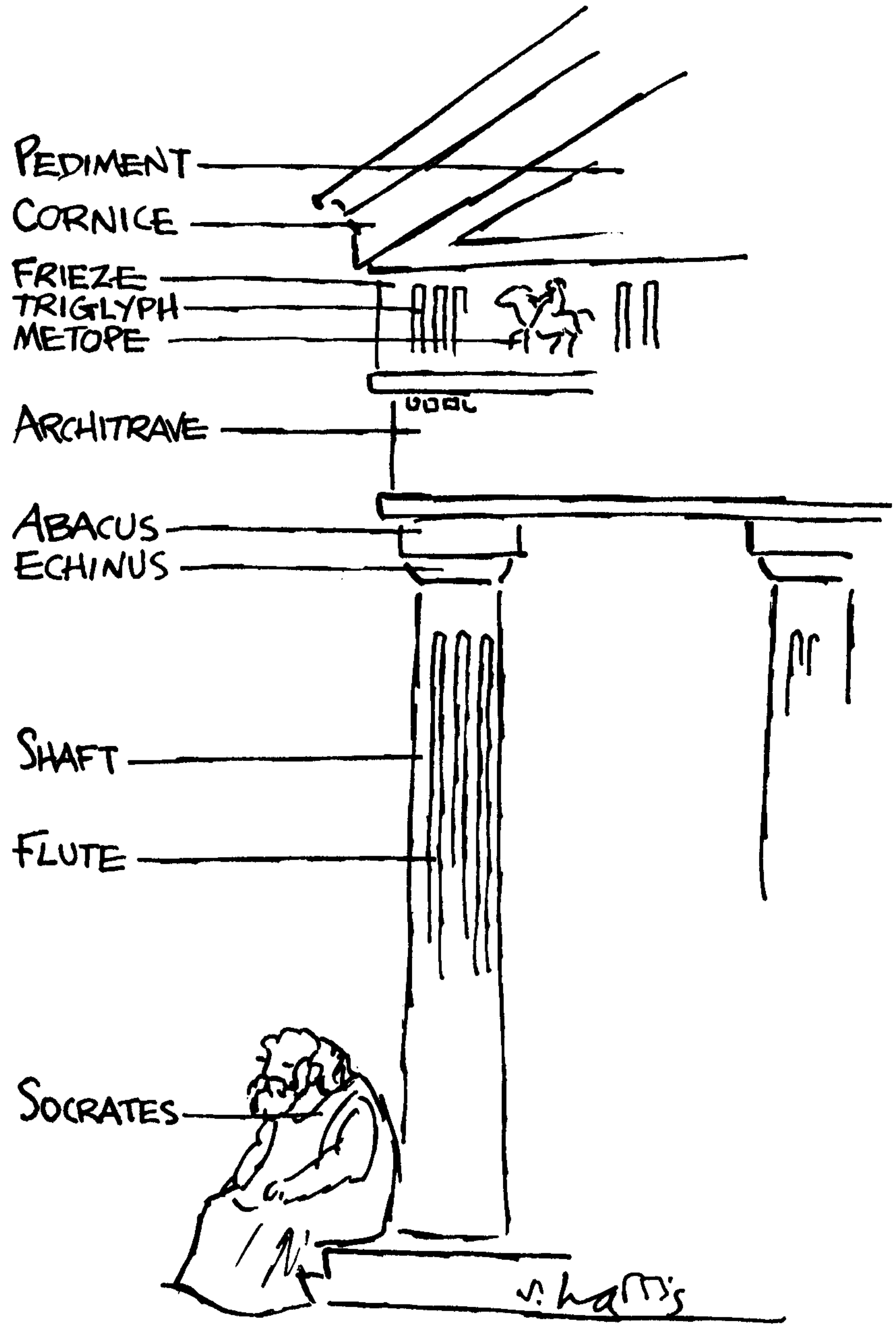
“The School of Athens” (by Raphael, 1483-1520)


ARISTOTELES
87 km
97 / 10 / 09
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
8
B/W QuickCam
a.cidadao@mail.telepac.pt


THE UNKNOWN PHILOSOPHER
WHO FIRST REALIZED LIFE IS NO PICNIC


Birds fly because they're lighter than air.
Some trees have different fruits each year.
At night, clouds rest on the ground.
Are you sure he's Aristotle?
S.Harris

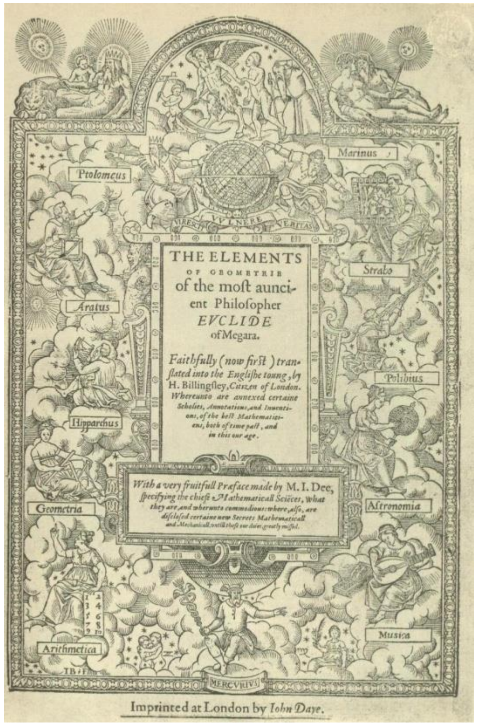
"What I especially like about being a philosopher-scientist is that I don't have to get my hands dirty."

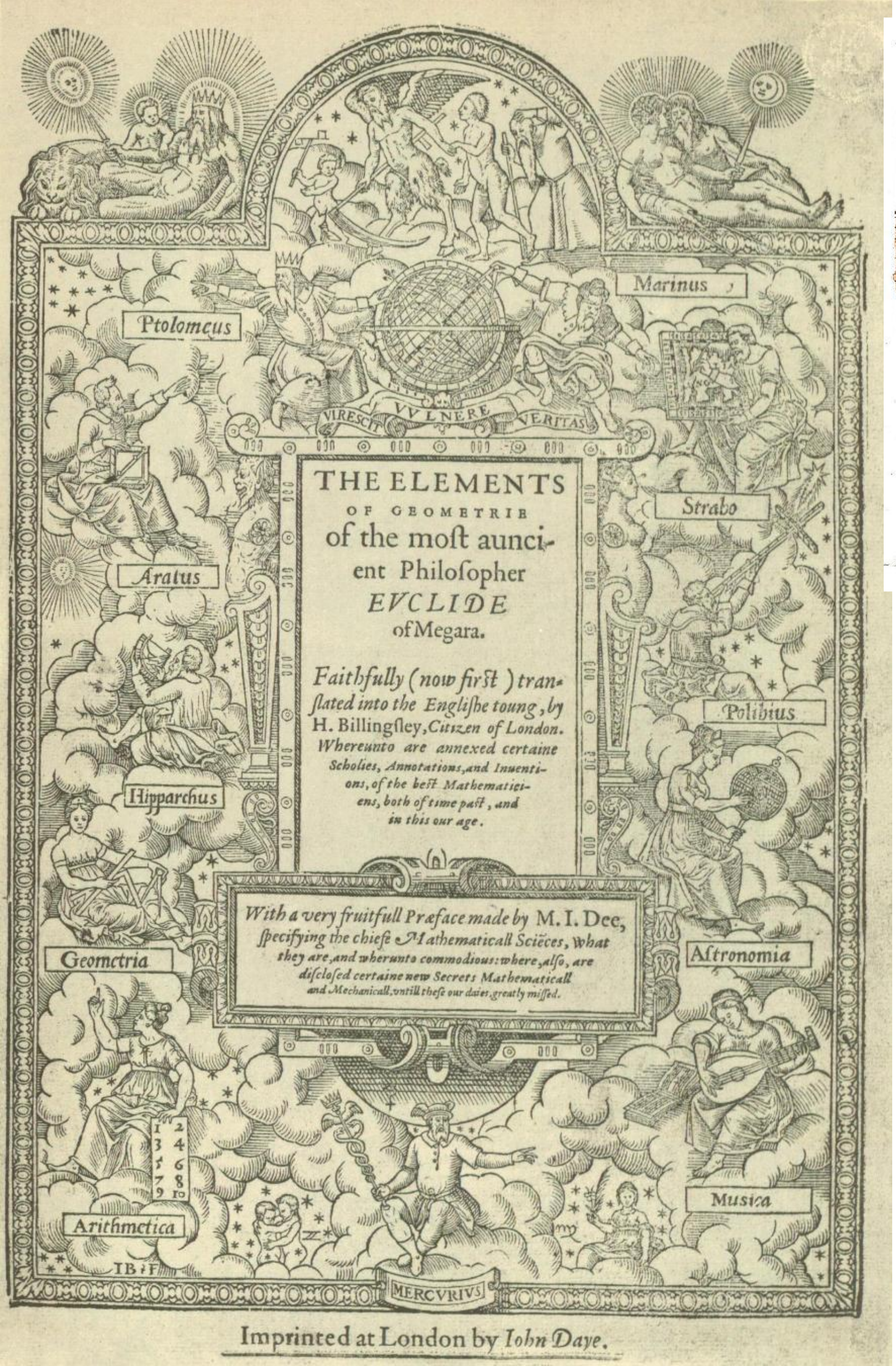
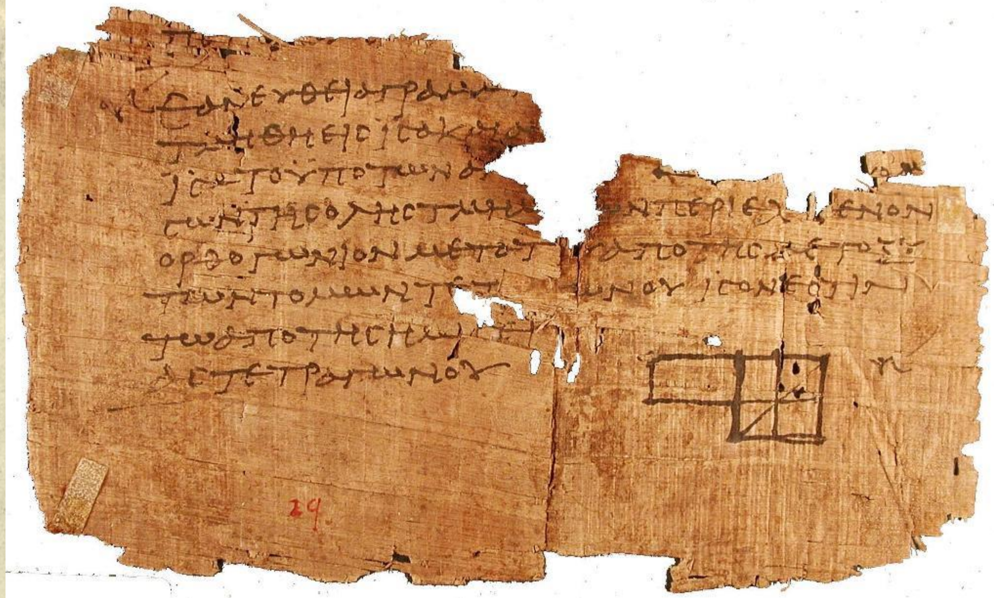

PEDIMENT
CORNICE
FRIEZE
TRIGLYPH
METOPE
ARCHITRAVE
ABACUS
ECHINUS
SHAFT
FLUTE
SOCRATES
S. Harris

Historical Perspectives

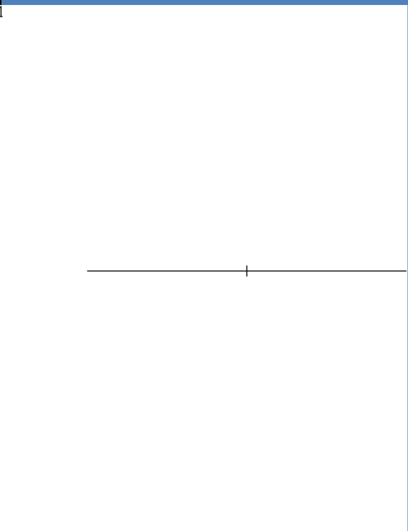
Euclid (325BC-265BC)

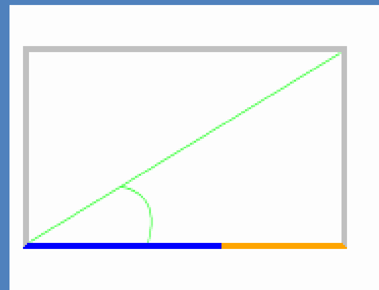
- Founder of geometry & the axiomatic method
- "Elements" – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and "Euclidean" geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others

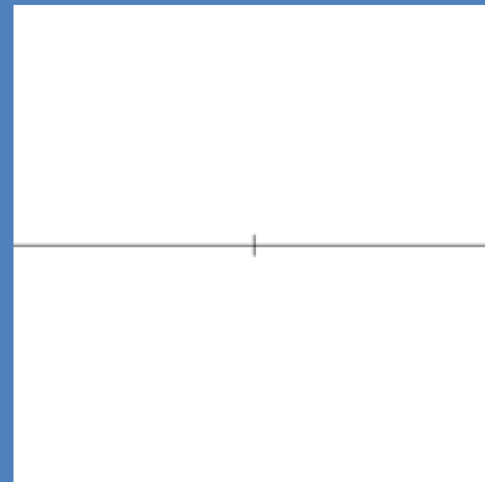


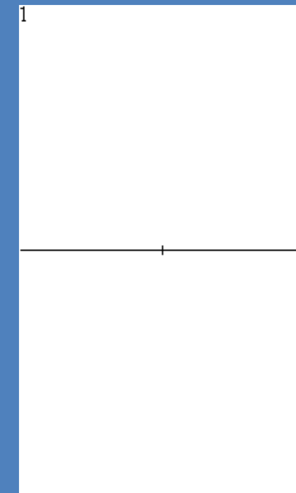

THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.
Faithfully (now first) tran-
slated into the Englishe toung, by
H. Billingsley, Citizen of London.
Whereunto are annexed certaine
Scholies, Annotations, and Inventi-
ons, of the best Mathematici-
ens, both of time past, and
in this our age.
With a very fruitfull Præface made by M. I. Dee,
specifying the chiefe Mathematicall Sciẽces, what
they are, and wherunto commodious: where, also, are
disclosed certaine new Secrets Mathematicall
and Mechanicall, untill these our daies, greatly missed.
Imprinted at London by Iohn Daye.
Ptolomeus
Marinus
VIRESCIT VVLNERE VERITAS
Strabo
Aratus
Polibius
Hipparchus
Geometria
Astronomia
Arithmetica
Musica
MERCVRIVS


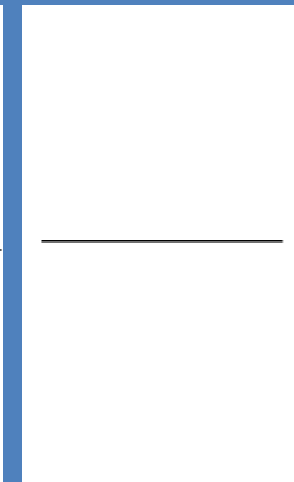

Euclid's Straight-Edge and Compass Geometric Constructions

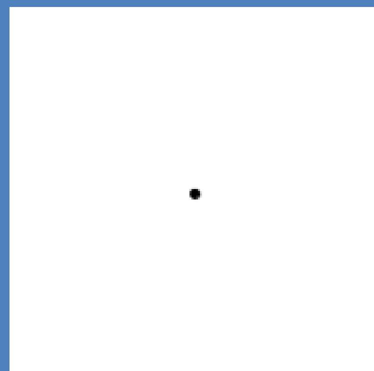


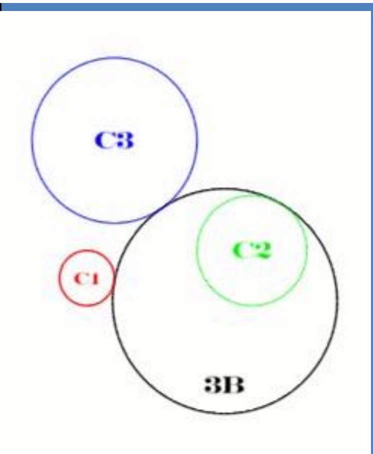


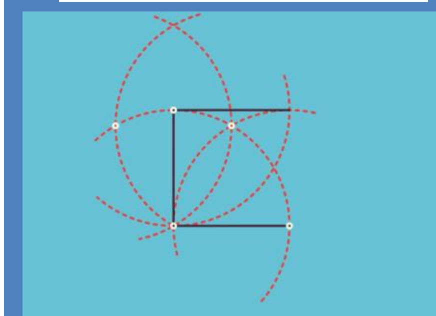


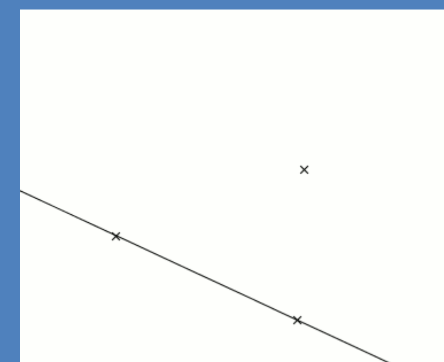




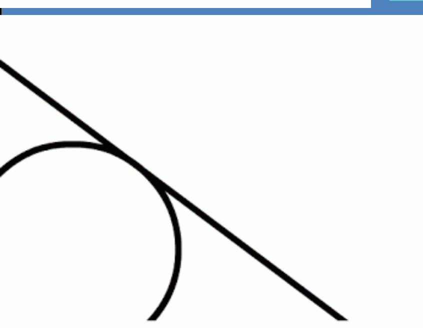


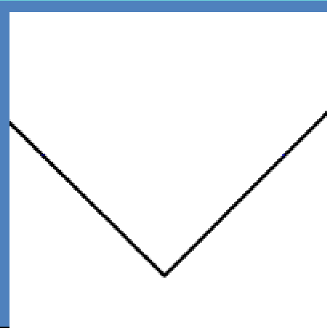


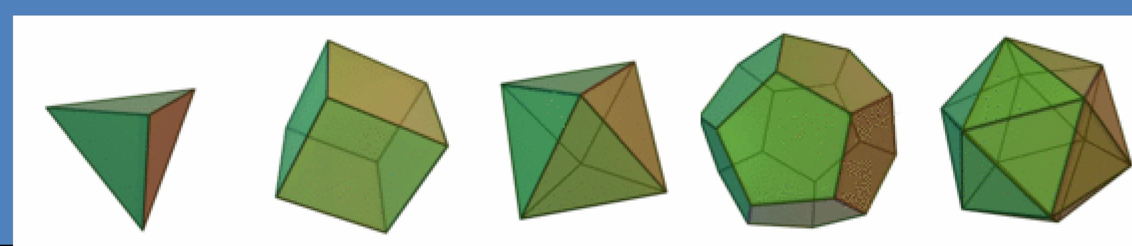




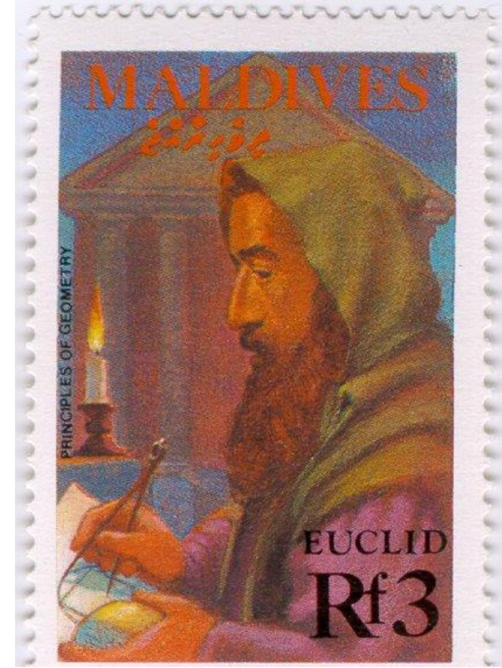
Extra credit!





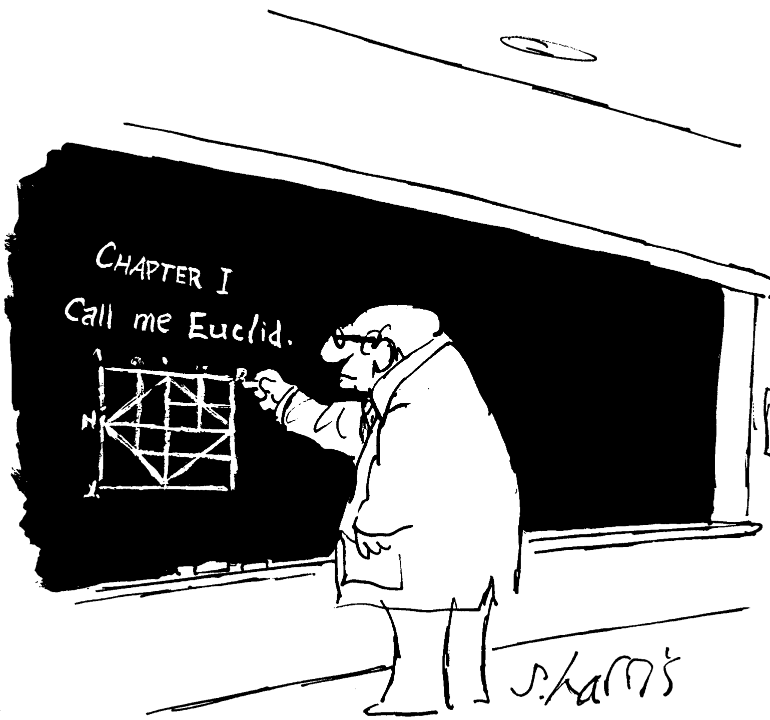



SCHOOL OF ATHENS
50c
SIERRA LEONE
500th ANNIVERSARY
BIRTH OF RAPHAEL
(detail) Euclid and his pupils
1983
SCALA NEW YORK/FLORENCE


MALDIVES
PRINCIPLES OF GEOMETRY
EUCLID
Rf3


POSTE VATICANE
PONTIFICIA
ACADEMIA
SCIENTIARVM
2500
I.P.Z.S.-ROMA-1986
P.N. ARGHITTU INC.


EUCLIDES
12 km
98 / 04 / 07
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
11
B/W QuickCam
a.cidadao@mail.telepac.pt


CHAPTER I
Call me Euclid.
S. Harris

Euclid's Axioms

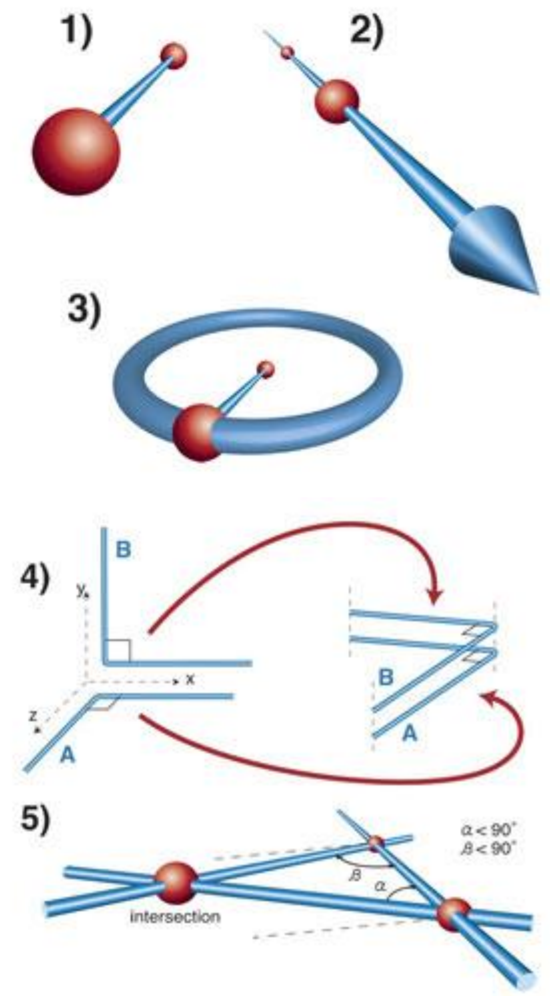
1: Any two points can be connected by exactly one straight line.

2: Any segment can be extended indefinitely into a straight line.

3: A circle exists for any given center and radius.

4: All right angles are equal to each other.

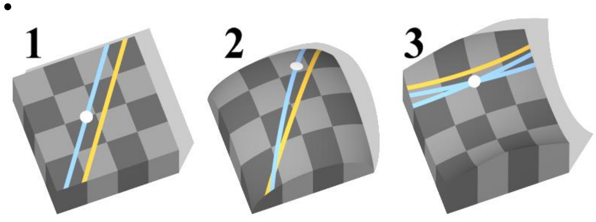
5: The parallel postulate: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

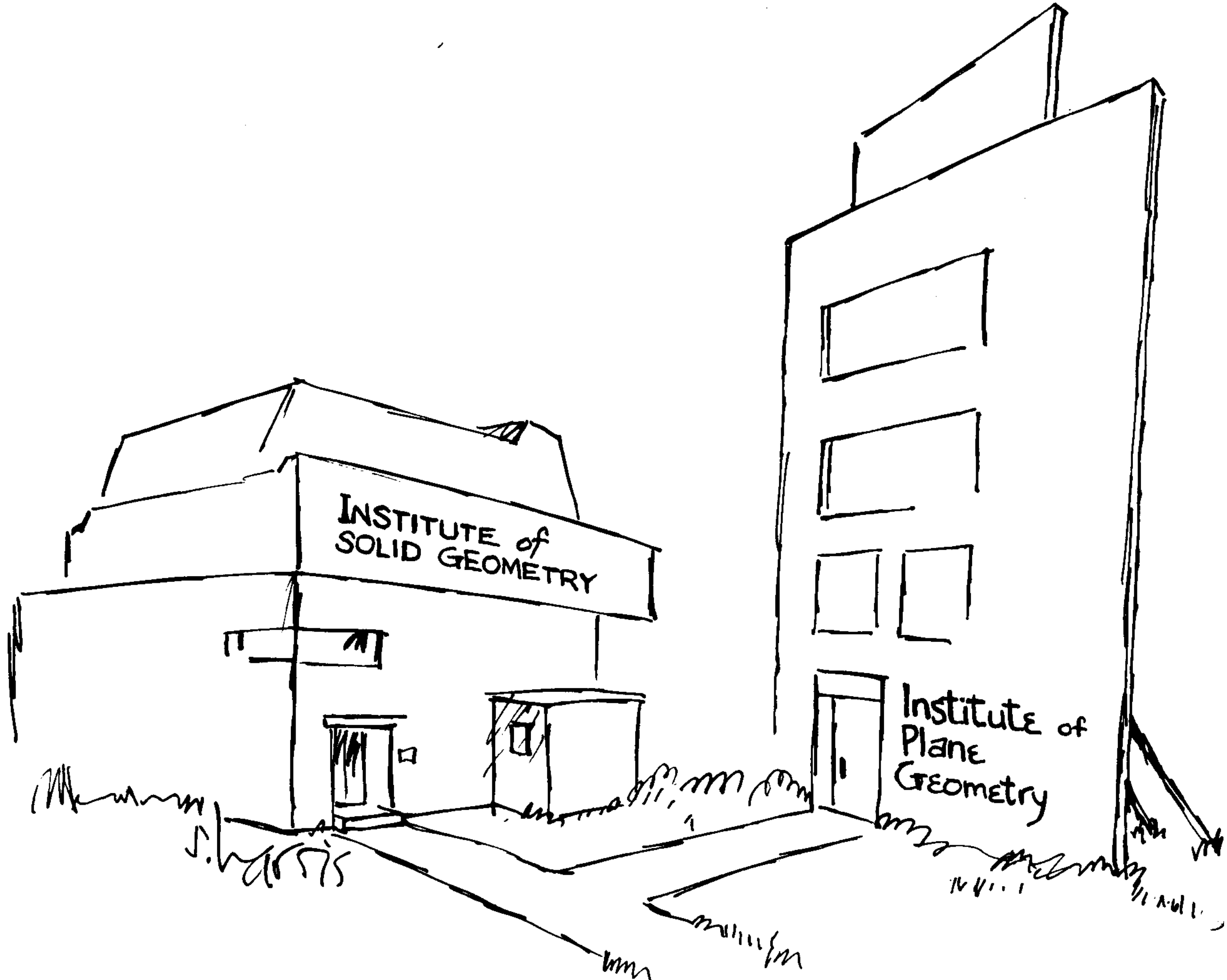


The first 28 propositions of Euclid's Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is independent of the other axioms of Euclidean geometry.

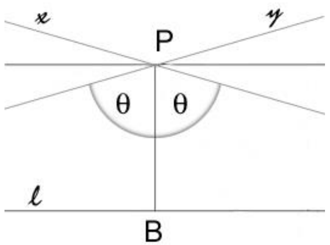
The parallel postulate can be modified to yield non-Euclidean geometries!



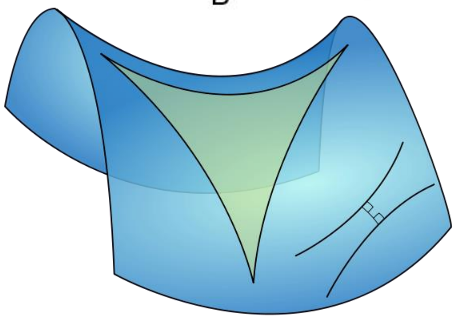

INSTITUTE of SOLID GEOMETRY
Institute of Plane Geometry

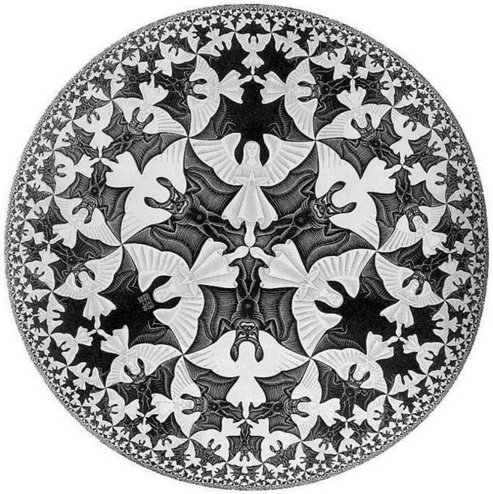
Non-Euclidean Geometries

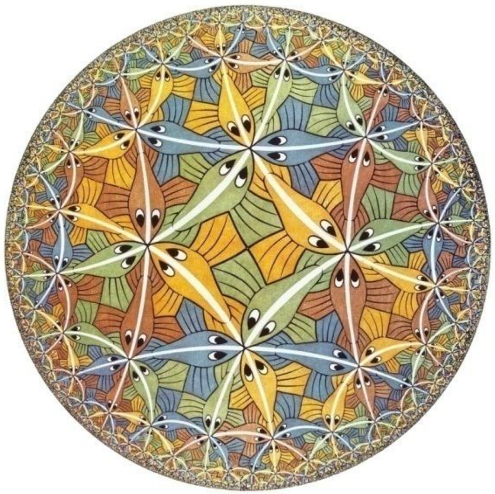
Hyperbolic geometry: Given a line and a point off that line, there are an infinity of lines passing through that point that do not intersect the first line.

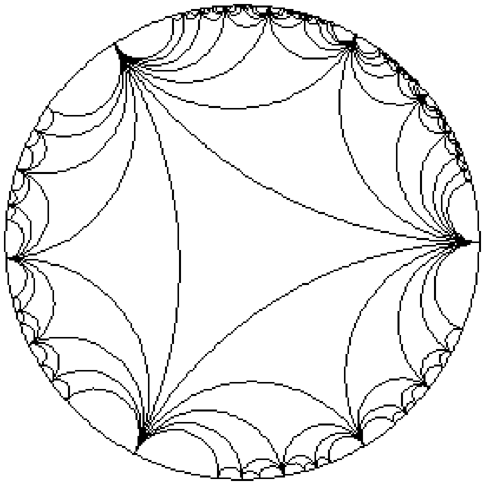


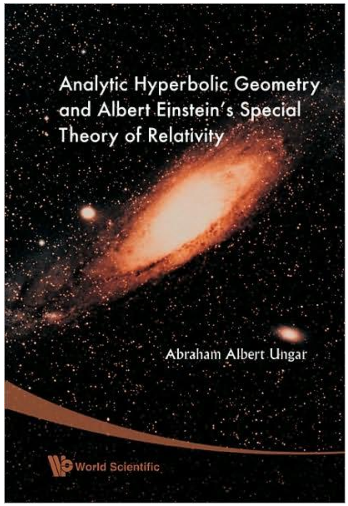
- Sum of triangle angles is less than 180°
- Different triangles have different angle sum
- Triangles with same angles have same area
- There are no similar triangles
- Used in relativity theory







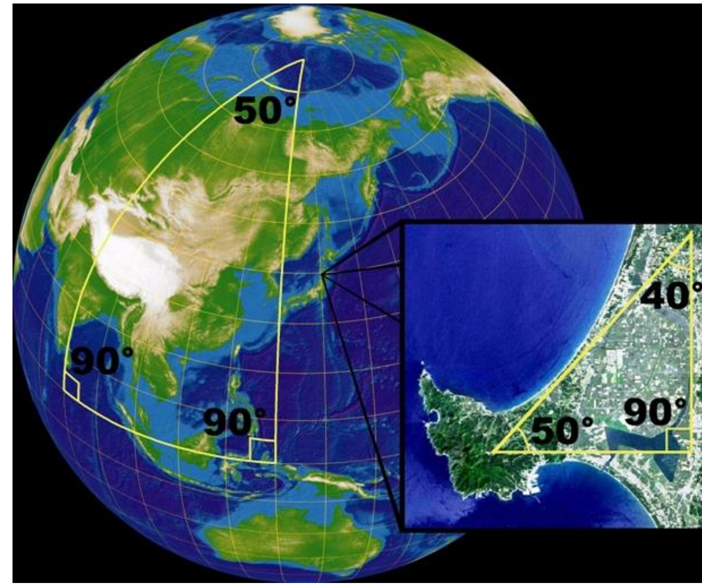


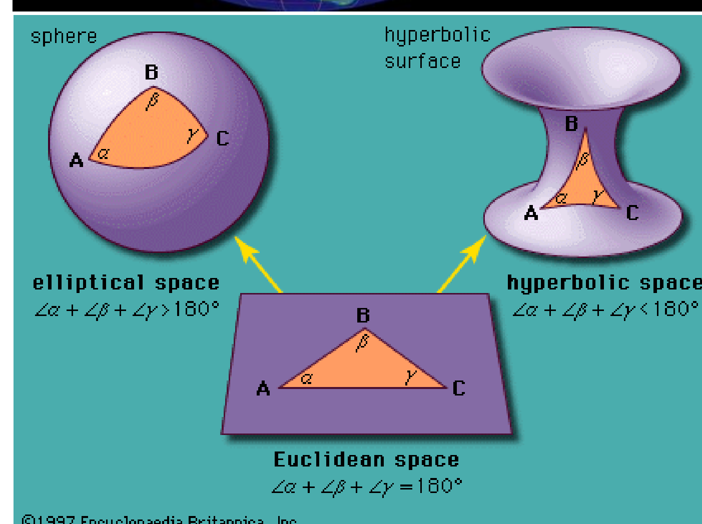


Non-Euclidean Geometries

Spherical / Elliptic geometry: Given a line and a point off that line, there are no lines passing through that point that do not intersect the first line.

- Lines are geodesics - “great circles”
- Sum of triangle angles is > 180°
- Not all triangles have same angle sum
- Figures can not scale up indefinitely
- Area does not scale as the square
- Volume does not scale as the cube
- The Pythagorean theorem fails
- Self-consistent, and complete



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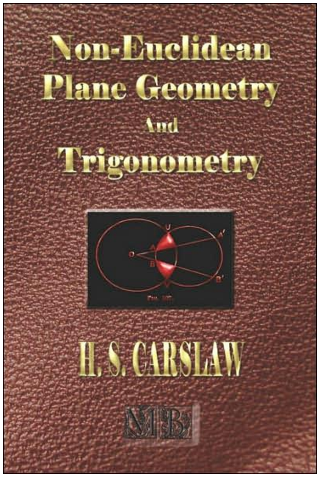
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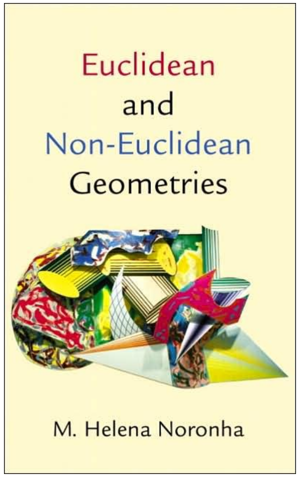
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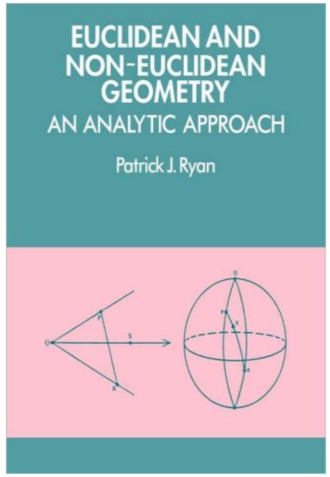
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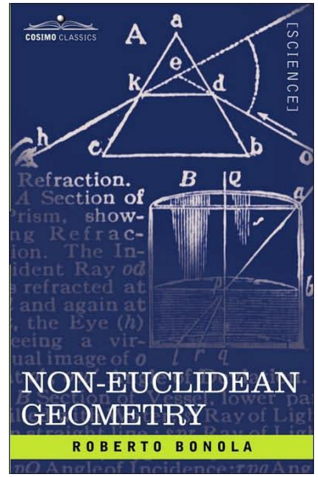
AN ANALYTIC APPROACH
Patrick J. Ryan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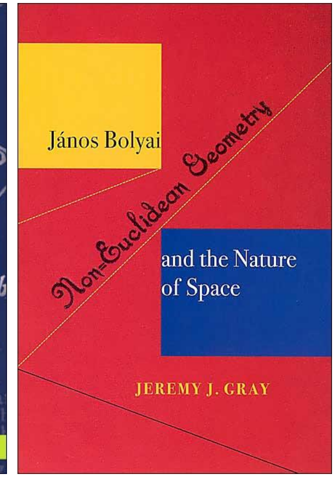
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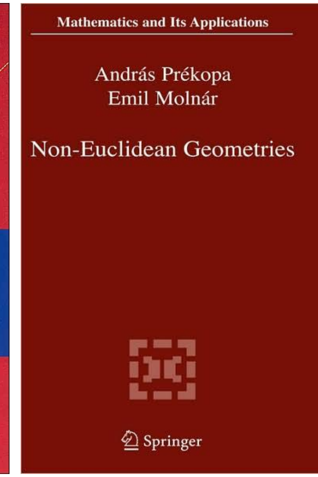
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Emil Molnár
Non-Euclidean Geometries
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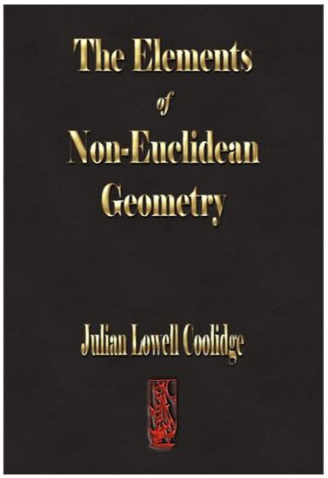
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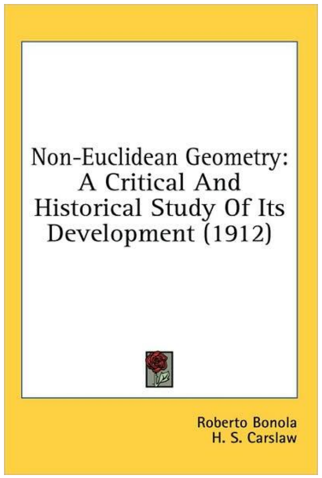
Julian Lowell Coolidge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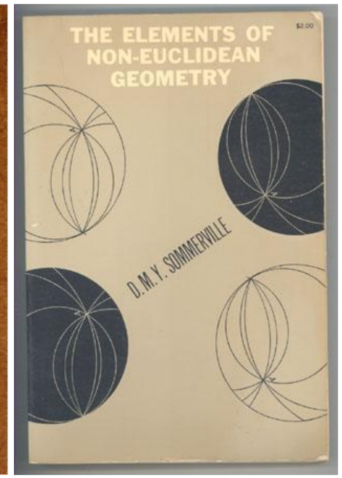
Harold E. Wolfe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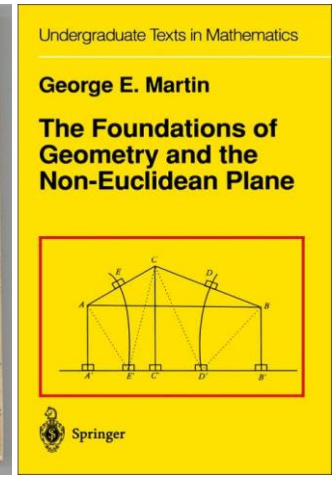
D. M. Y. SOMMERVILLE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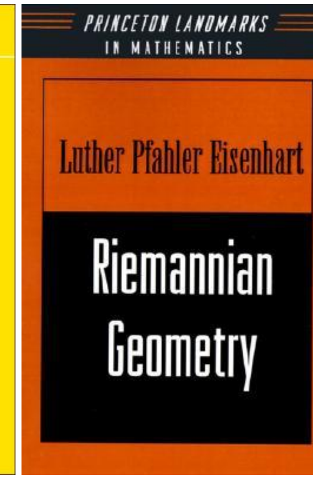
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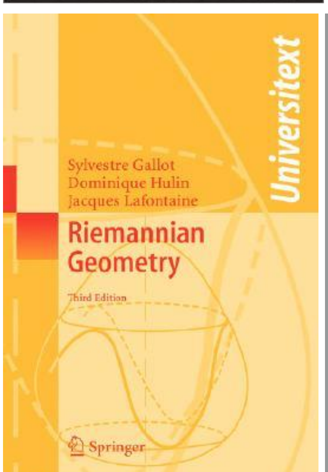
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Dominique Hulin
Jacques Lafontaine
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Third Edition
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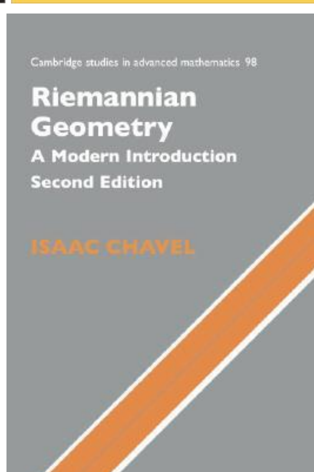
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A Modern Introduction
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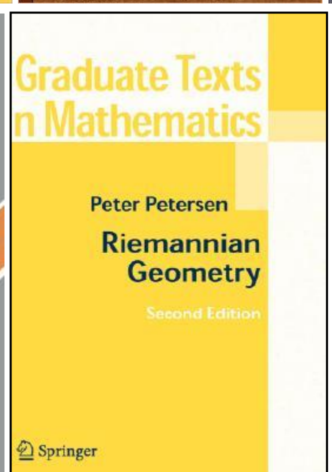
ISAAC CHAVEL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Peter Petersen
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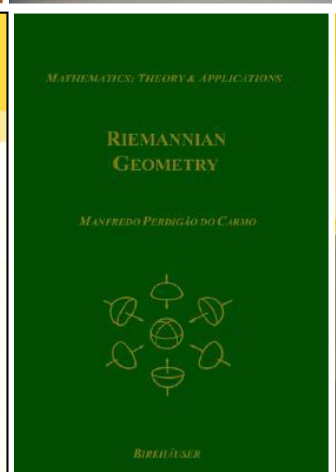
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RIEMANNIAN
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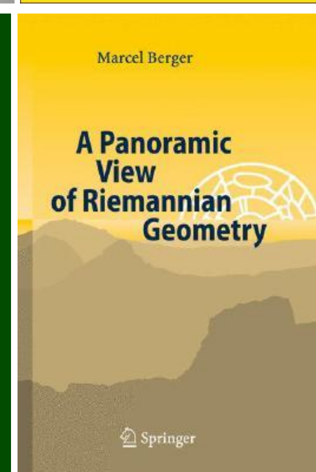
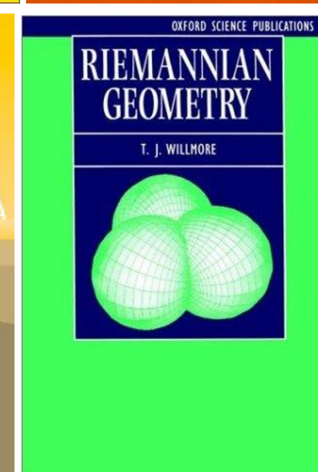

Marcel Berger
A Panoramic
View
of Riemannian
Geometry
Springer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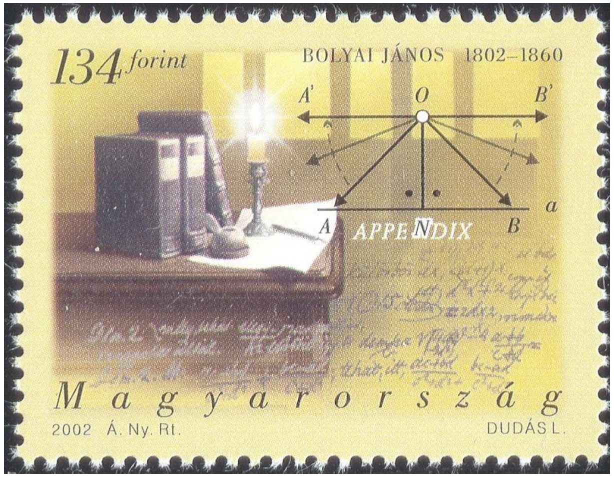
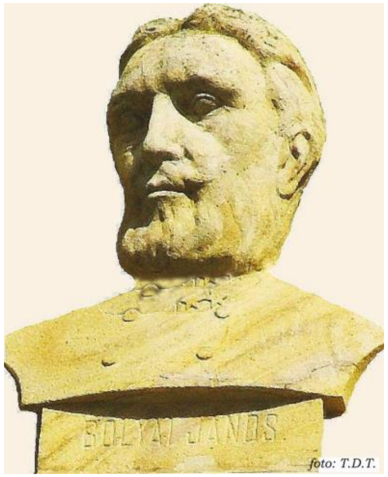
Founders of Non-Euclidean Geometry

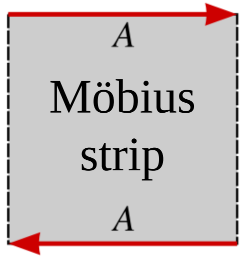
János Bolyai (1802-1860)

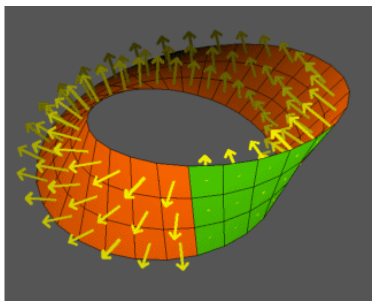


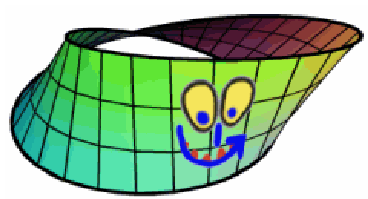


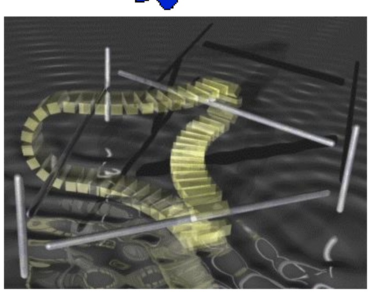
Nikolai Ivanovich Lobachevsky (1792-1856)

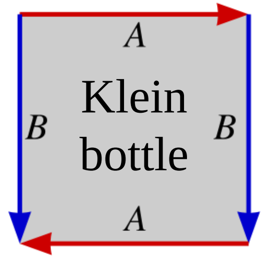
Non-Euclidean Non-Orientable Surfaces

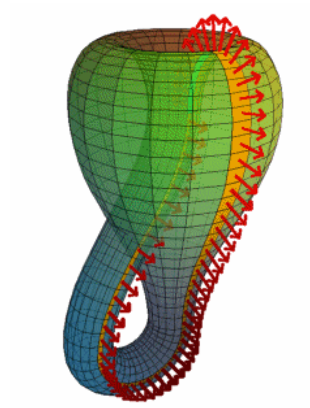


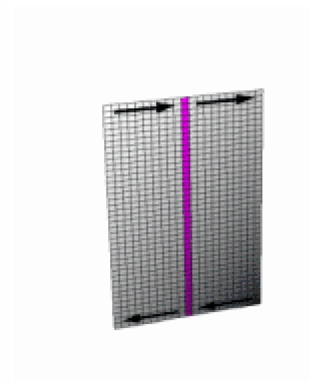


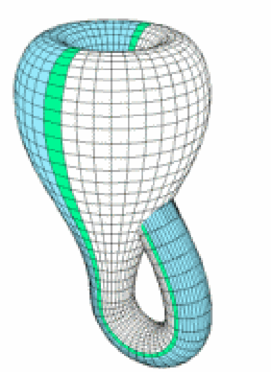


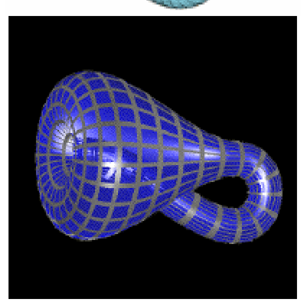




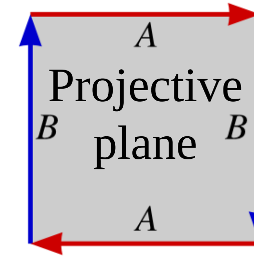


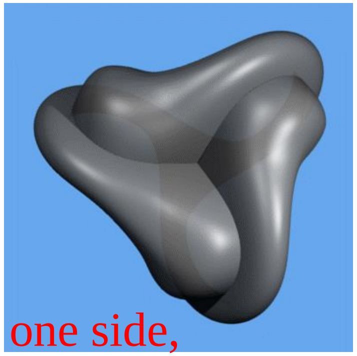






one side,
no boundary!





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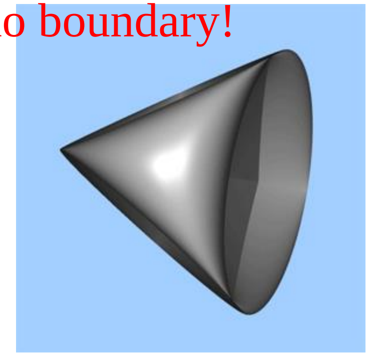
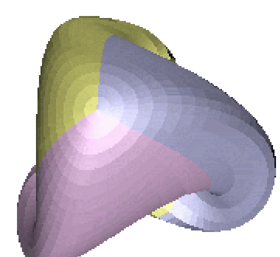
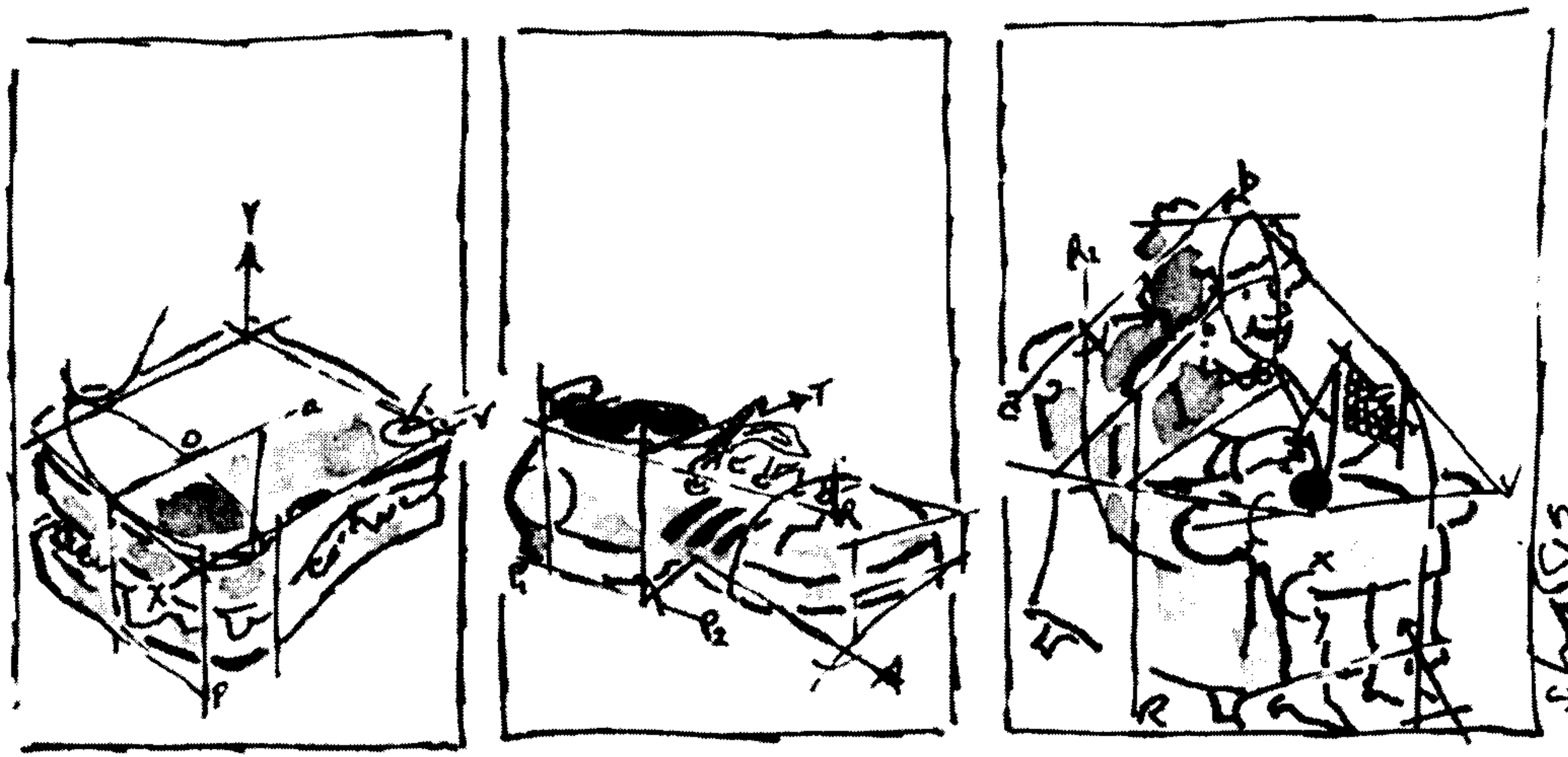
no boundary!



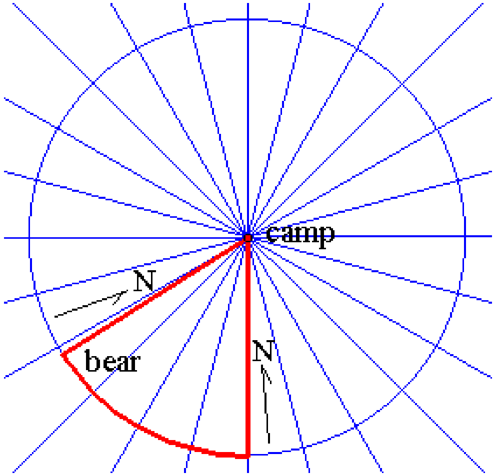


THE GEOMETRY OF EVERYDAY LIFE



TUNA SANDWICH　　SNEAKER　　GRANDMA

Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?



Problem: Is the house location unique?