Applied Algorithms and Optimization

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UVa Computer Science

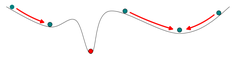
"Make everything as simple as possible, but not simpler."
- Albert Einstein (1879-1955)

Algorithmics

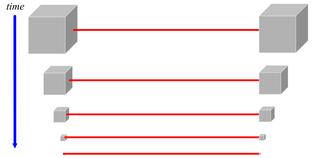
Solution

Speed	exact	approximate
fast	Short & sweet	Quick & dirty
slow	Slowly but surely	Too little, too late

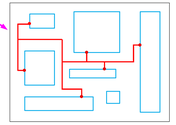
Complexity



Trends in Interconnect

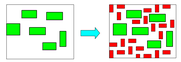


Placement & Routing



VLSI Design

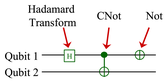
Enhanced Manufacturability



Quantum Computing

Quantum register - the "ket"
Superposition of states:

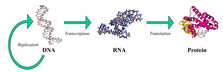
$$\sum_{i=0}^{2^N-1} a_i |s_i\rangle \quad \text{Where:} \quad \sum_{i=0}^{2^N-1} |a_i|^2 = 1$$



$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \rightarrow$$

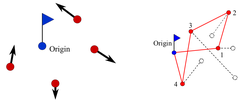
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

Bioinformatics



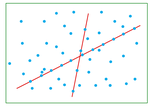
Evolution

Moving-Target Traveling Salesperson

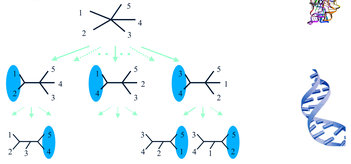


Theorem: "waiting" can never help
Algorithms:
- efficient exact solution for 1-dimension
- efficient heuristics for other variants

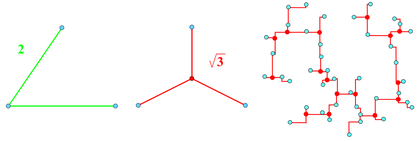
Landmine Detection



Phylogenetic Tree Reconstruction

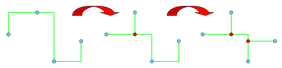


Steiner Trees



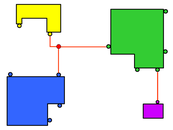
Iterated 1-Steiner Algorithm

Q: Given pointset S, which point p minimizes |MST(S ∪ p)| ?
Algorithmic idea: Iterate!



Theorem: Solutions cost < 3/2 • OPT
Theorem: Solutions cost ≤ 4/3 • OPT for "difficult" pointsets
In practice: Solution cost is within 0.5% of OPT on average

Group Steiner Problem



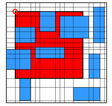
Theorem: o(log # groups) • OPT approximation is NP-hard
Theorem: efficient solution with cost = O((# groups)/) • OPT ∀ ε>0

Density Analyses

Input:
- n×n layout
- k rectangles
- w×w window

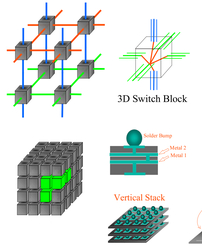
Output: all extremal density w×w windows

Theorem: extremal density windows all lie on Hanan grid

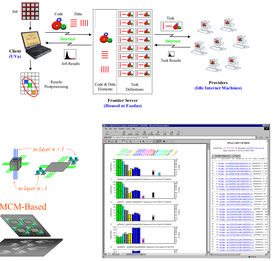


Algorithms:
$O(n^2)$ time
$O(k^2)$
$O\left(\binom{n}{2}^2 \times k \log k + \binom{n+2}{4}^2\right)$

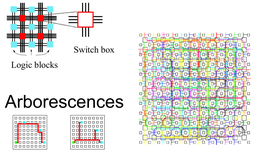
3D FPGAs



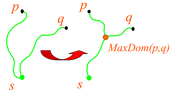
Gene Identification Tools



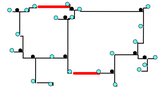
FPGA Routing

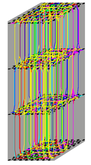


Arborescences



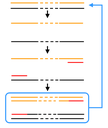
Non-Tree Routing



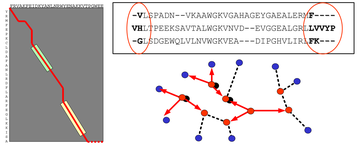


Protein Discovery

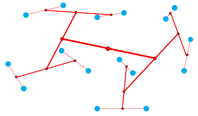
Polymerase Chain Reaction (PCR)



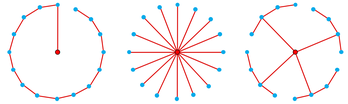
Tree-Based Multiple Sequence Alignment



Low-Skew Clock Trees



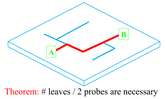
Bounded Radius Trees



Algorithm:
Input:
- points / graph
- any ε > 0

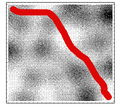
Output: tree T with
- radius(T) ≤ (1+ε)·OPT
- cost(T) ≤ (1+2/ε)·OPT

Circuit Testing

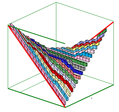


Theorem: # leaves / 2 probes are necessary
Theorem: # leaves / 2 probes are sufficient
Algorithm: linear time

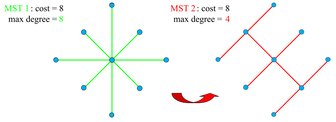
Robust Paths



Minimum Surfaces

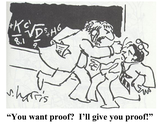


Low-Degree Spanning Trees



Theorem: max degree 4 is always achievable in 2D
Theorem: max degree 14 is always achievable in 3D

Computational Geometry



"You want proof? I'll give you proof!"

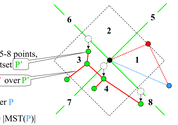
Proof: Low-Degree Minimum Spanning Trees in 2D & 3D

Input: pointset P
Find: MST(P)
- Perturb region 5-8 points, yielding pointset P'
- Compute MST' over P'

Output: MST' over P

Idea: |MST'(P')| = |MST(P)|

Theorem: max MST degree ≤ 4

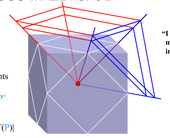


Partition space:
- 6 square pyramids
- 8 triangular pyramids

Input: 3D pointset P
Find: MST(P)
- Perturb boundary points to yield pointset P'
- Compute MST' over P'
- Output: MST' over P

Idea: |MST'(P')| = |MST(P)|

Theorem: max MST degree in 3D is ≤ 6 + 8 = 14
Theorem: lower bound on max MST degree in 3D is ≥ 13



"I think you should be more explicit here in step two."

Primer Selection Problem

Input: set of DNA sequences
Output: minimal set of covering primers

Theorem: NP-complete
Theorem: Ω(log # sequences)·OPT within P-time
Heuristic: O(log # sequences)·OPT solution

"Gabe aiming to solve a tough problem"
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