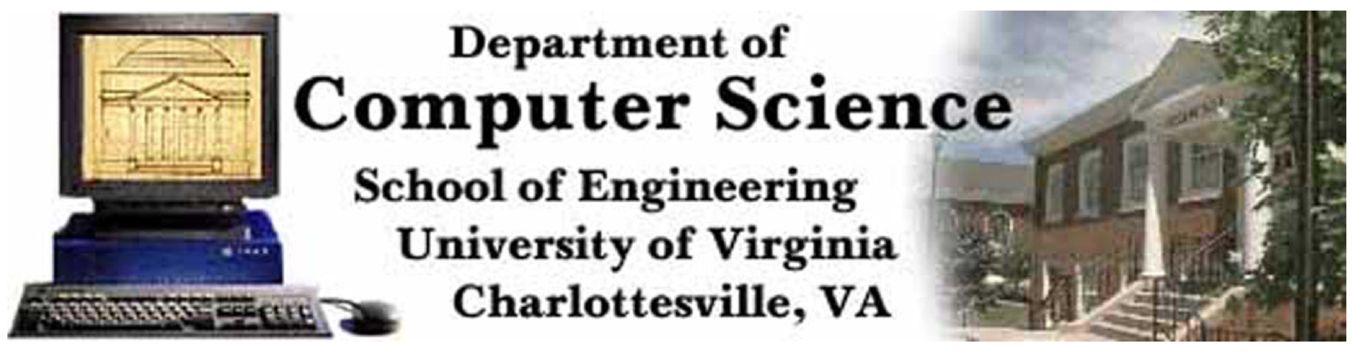
Improved Approximation Bounds for the Group Steiner Problem



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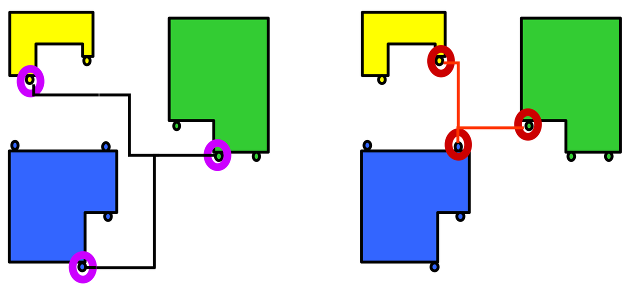
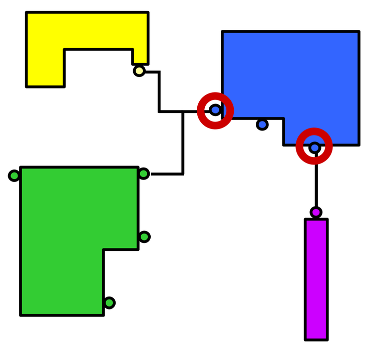
Outline

- VLSI CAD Problem
- Mathematical Model
- Approximation Algorithm
- Practical Enhancement
- Experimental Results
- Java Implementation

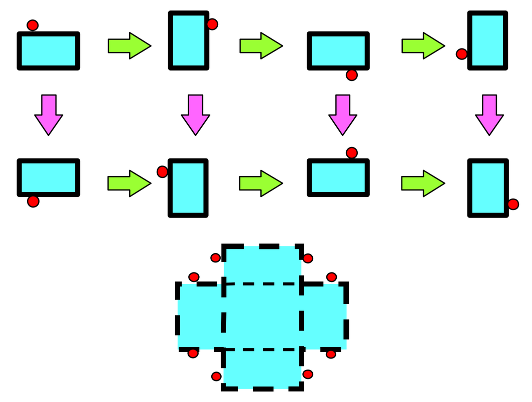
Motivation: Multi-Port Terminals

- Terminals have size / geometry
- Port choice is significant
- ≥ 1 port per terminal is involved
- Interconnection is a forest



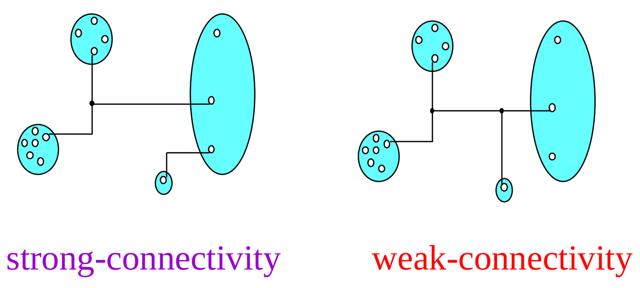


- Modules can be rotated / flipped
  - Each terminal ⇒ 8 "virtual" ports



- Pin Assignment Problem

- Strong connectivity variant:
  - All nodes of a group implicitly connected to each other
- Weak connectivity variant:
  - Intra-group edges must be explicitly part of the solution



Problem Formulation and Bounds

- Strong Connectivity Version
  - Assign 0-cost to intra-group edges ⇒ Steiner Tree Problem
- The Steiner Tree Problem
  - Given: weighted graph $G=(V,E)$ and node subset $M \subseteq V$
  - Find: min-cost tree spanning M
- NP-hard
- 11/6 · OPT approximation

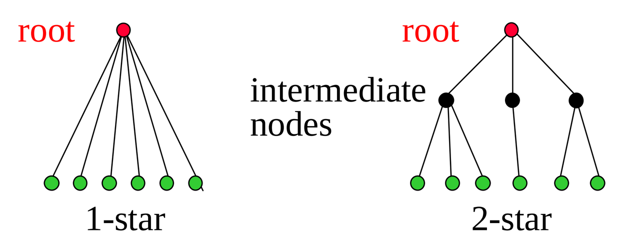
- The Group Steiner Tree Problem:
  - Given: weighted graph $G=(V,E)$ and k disjoint groups $N_1, \ldots, N_k \subseteq V$
  - Find: min-cost tree containing at least one node from each group N_i
- Even $O(\ln k)$ approx. is NP-hard
- Best previous bound: $O(k)$
- Our heuristics:

  sub-linear performance bounds:

$$O(k^{\varepsilon}) \quad \forall \varepsilon > 0$$

- A greedy framework:
  - Find any feasible solution
  - Greedily improve current solution while it is possible
- Our goal is to specify:
  - First bounded feasible solution
  - Local improvements of solution
  - Measure local improvement quality

- *depth* ≡ max #edges on root-leaf paths
- *d-star* ≡ rooted tree of depth *d*



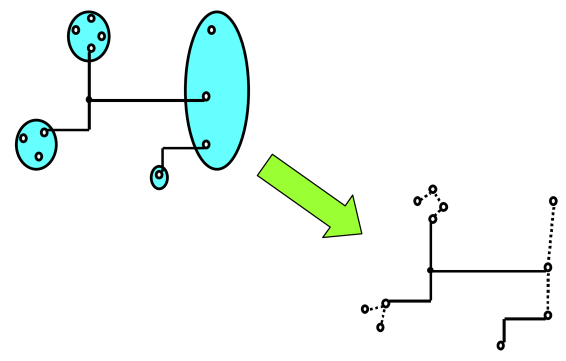
Optimal 1-star rooted at center costs at most $\frac{1}{2}\#groups \cdot OPT$

Optimal d-star rooted at center costs at most $2d \cdot (\#groups)^{1/d} \cdot OPT$

Strong Connectivity

- Cost of intra-group edges = 0

  ⇒ Steiner Problem



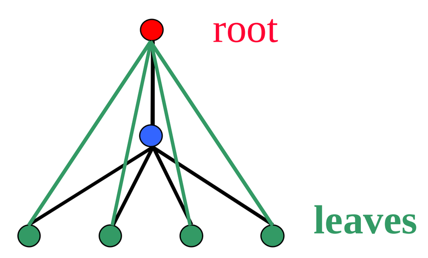
- 11/6 · OPT approximation

d-Star Approximation

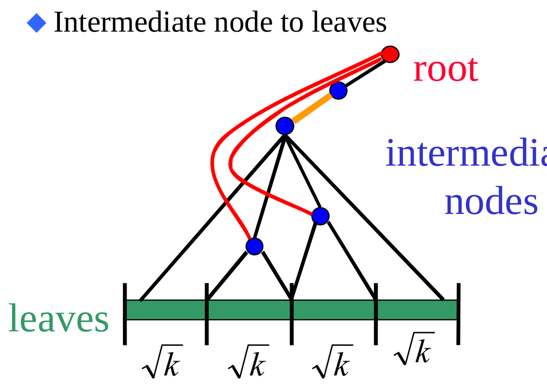
Theorem: 2-Stars cost no more than $O(k^{1/2})$ times the cost of the optimal group Steiner tree.

- Proof requires concept of "edge reuse"
- Green edge cost ≤ corresponding path
- Middle edge "reused" 4 times ⇒

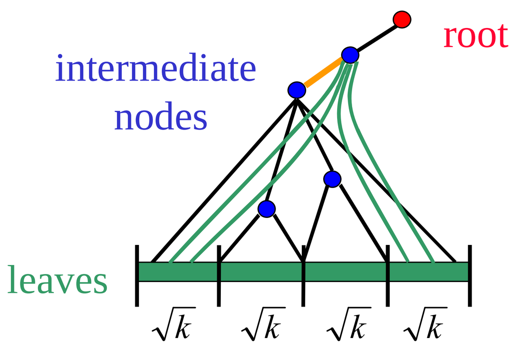
  Green tree cost ≤ 4 x (black tree cost)



- Sort leaves in depth-first order
- Partition leaves into $\sqrt{k}$ blocks of $\sqrt{k}$
- Intermediate nodes are least common ancestors of blocks
- 2-stars have two types of paths:
  - Root to intermediate nodes
  - Intermediate node to leaves



- $\sqrt{k}$ root-to-intermediate-node paths
- Left-most and right-most blocks not completely contained in subtree ⇒ edge reused $2\sqrt{k}$ times in such paths
- Total 2-star cost ≤ $3\sqrt{k}$ times original

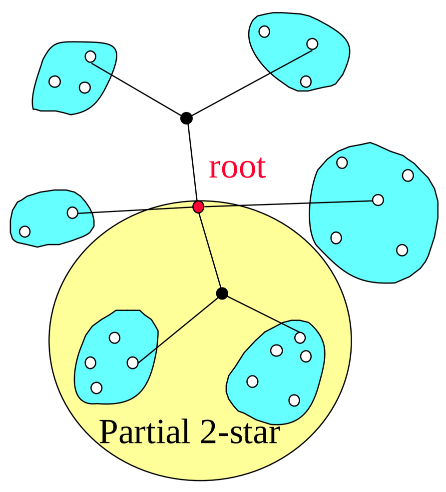


Corollary: d-stars cost no more than $O(k^{1/d})$ times optimal

Low-Norm Partial d-Stars

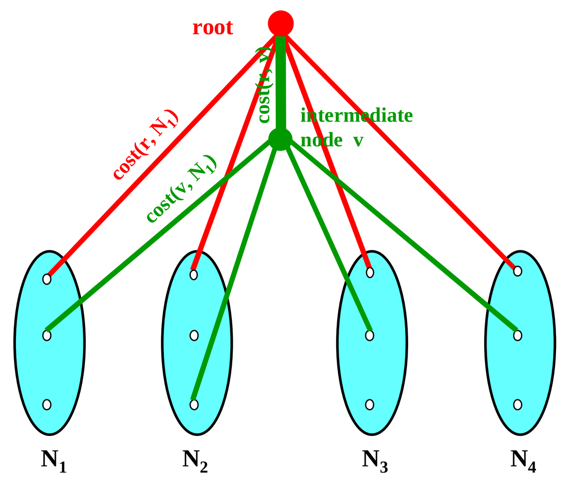
- *Partial d-star*
  - Potential subtree of the solution
- *Corresponding 1-star*
  - Same root and same groups
- *norm(d-star)* = cost(d-star)/cost(1-star)
  - Relative gain of d-star over 1-star

- d-stars are composed of partial d-stars
- Partial d-stars consist of:
  - The root
  - d-1 levels of intermediate nodes
  - Leaves



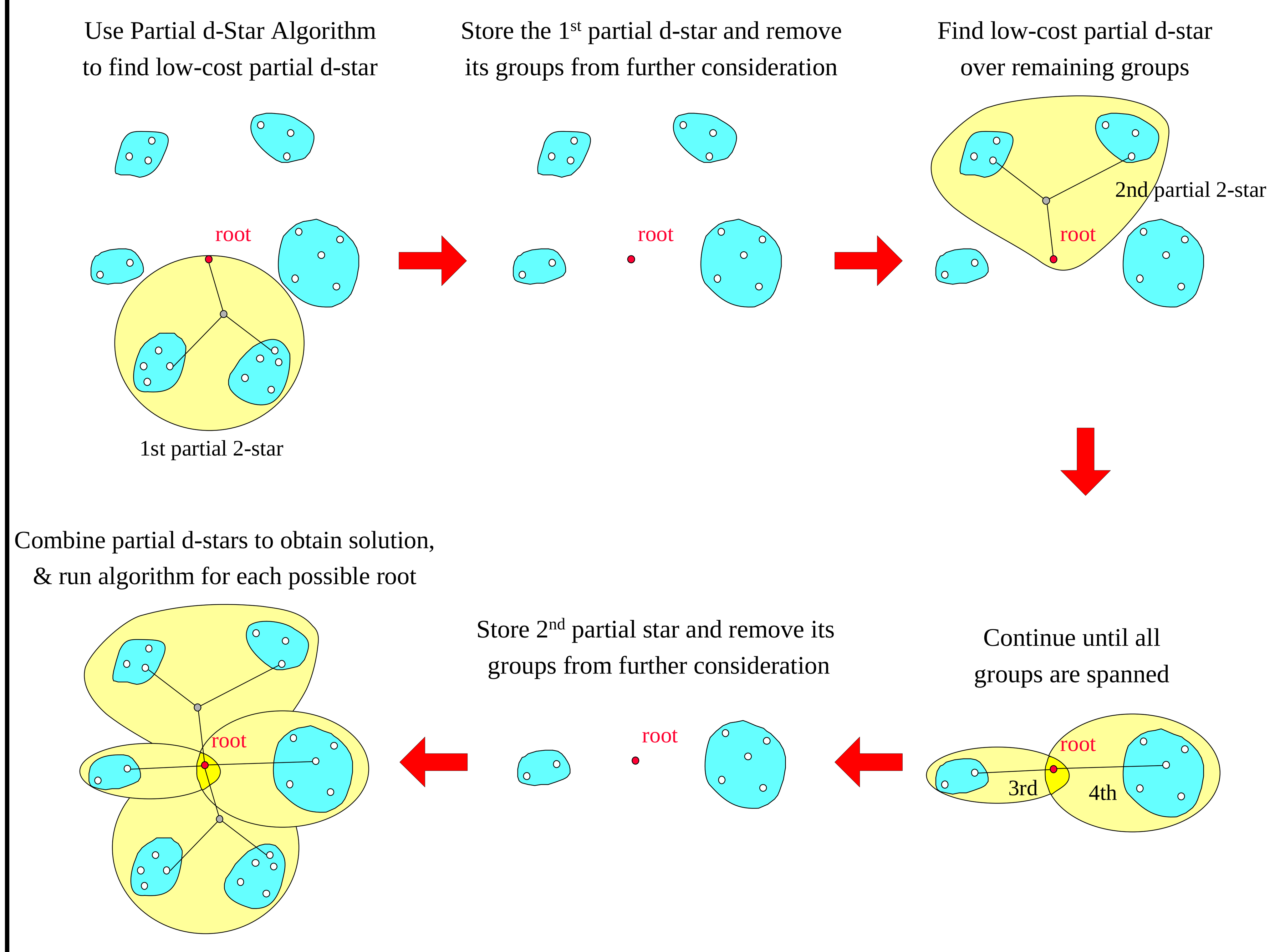
- Algorithm is recursive:

  Find d-stars by first finding (d-1) -stars
- Base case: d = 2
- Sort groups by potential gain
- Include groups while norm decreases



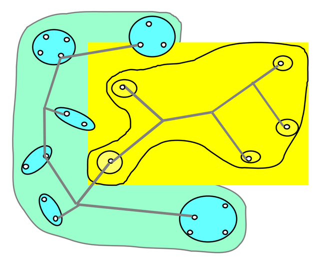
$$\frac{cost(v, N_i)}{cost(r, N_i)} \le \frac{cost(v, N_{i+1})}{cost(r, N_{i+1})}$$

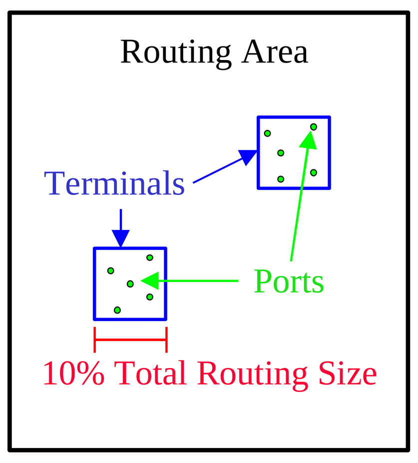
Group Steiner Heuristic

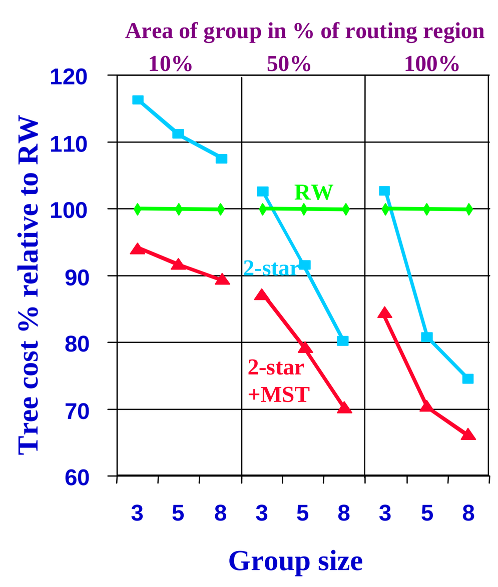


- Degenerate (size-1) Groups
  - Find usual Steiner tree for size-1 groups
  - Find Group Steiner tree for m groups of size >1 groups + one degenerate group
  - Combine into one tree
- Performance bound:

$$\frac{11}{6} + 2d \cdot (2 + \ln(2m+2))^{d-1} \cdot (m+1)^{\frac{1}{d}}$$







Summary

- New approach for low-cost routing of multi-port terminals
  - → Depth-bounded Steiner trees
- First heuristics with sub-linear performance bounds:

$$O\left((\#groups)^{\varepsilon}\right) \quad \forall \varepsilon > 0$$

- Promising experimental results
- Web Java implementation

  (thanks to Doug Bateman)

http://www.cs.virginia.edu/~robins/GroupSteiner/