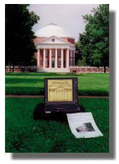
Quantum Algorithms for the Moving-Target Traveling Salesperson Problem

Jennifer Mifflin and Gabriel Robins

{jmm6ad, robins}@cs.virginia.edu



Department of **Computer Science**

School of Engineering and Applied Science

University of Virginia

(434) 982-2207 www.cs.virginia.edu

Schroedinger's Cat

Like Schroedinger's Cat, which is in a superposed alive/dead state, a qubit (quantum bit) can be both 0 and 1 simultaneously!

Quantum Computing Notation

Quantum register - the "ket": $|x\rangle$

Superposition of states:

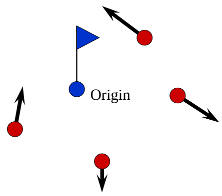
$$\sum_{i=0}^{2^N-1} a_i|s_i\rangle \quad \text{Where: } \sum_{i=0}^{2^N-1} |a_i|^2 = 1$$

Superposition of Qubits

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

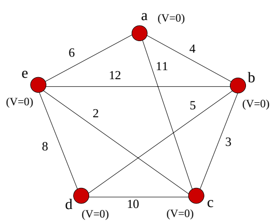
Moving-Target Traveling Salesperson Problem



Given N moving targets with constant velocities, find a min-time path that intercepts all targets

Moving-Target TSP is Intractable

Proof: NP-Complete TSP is a special case of Moving-Target TSP (with zero velocities)



Moving-Target TSP is NP-Complete

Contained in NP:

Decision version:

∃ path with time < T?

- Non-deterministically travel all paths
- Check each path length against T

Optimization version:

What is min-time path?

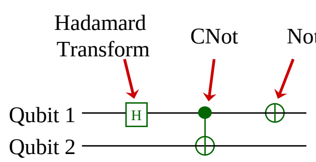
- Upper-bound T w/initial random path
- Binary search the range using T/2, T/4, etc. to find optimal min-time path

The Ubiquitous CNot Gate

The CNot gate negates the second bit only if the control bit is 1 (true)

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix} = \begin{bmatrix}a\\b\\d\\c\end{bmatrix}\begin{pmatrix}00\\01\\10\\11\end{pmatrix}$$

Visual Representation



$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \rightarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

Matrix Representation

2^Nx1 matrix represents N qubits

(x-1)th row represents amplitude of x value

Example, 1 qubit:

$|0\rangle$ is represented by: $\begin{bmatrix}1\\0\end{bmatrix}$

$|1\rangle$ is represented by: $\begin{bmatrix}0\\1\end{bmatrix}$

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ by: $\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}$

Unitary Transformation

Unitary Transform changes states

- Conjugate Transpose = Inverse

$$\bar{A}^T = A^{-1}$$

Example: Hadamard Transform

$$\begin{bmatrix}\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Hamiltonian Path Solution: Traverse Every Possible Path

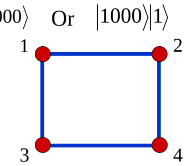
- For N nodes, use N^2+N qubits
- Separate into N+1 equal registers
- First register
  - parity of times a node has been visited
- Last N registers track path

Start State

$|1000\rangle|1000\rangle|0000\rangle|0000\rangle|0000\rangle$ Or $|1000\rangle|1\rangle$

First Step

$|1100\rangle|1\rangle|2\rangle + |1010\rangle|1\rangle|3\rangle$



Second Step

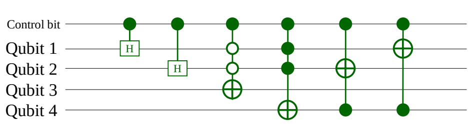
$|0100\rangle|1\rangle|2\rangle|1\rangle + |1101\rangle|1\rangle|2\rangle|4\rangle + |0010\rangle|1\rangle|3\rangle|1\rangle + |1011\rangle|1\rangle|3\rangle|4\rangle$

After N+1 steps:
1's in parity register imply Hamiltonian Path

Third Step

$|0000\rangle|1\rangle|2\rangle|1\rangle|2\rangle + |0110\rangle|1\rangle|2\rangle|1\rangle|3\rangle + |1001\rangle|1\rangle|2\rangle|4\rangle|2\rangle + |1111\rangle|1\rangle|2\rangle|4\rangle|3\rangle +$
$|0110\rangle|1\rangle|3\rangle|1\rangle|2\rangle + |0000\rangle|1\rangle|3\rangle|1\rangle|3\rangle + |1111\rangle|1\rangle|3\rangle|4\rangle|2\rangle + |1001\rangle|1\rangle|3\rangle|4\rangle|3\rangle$

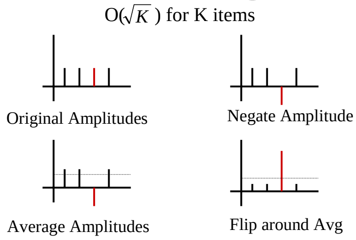
Transition for 2^J Adjacent Nodes



- At each step, this transition is applied
- Places 2^J qubits into equal superposition

Grover's Search Algorithm

O($\sqrt{K}$) for K items



Extend to (Moving-Target) TSP

Extend to TSP:
- Another register:
  - Large enough to hold longest path
- Each iteration, add edge weight to register

Extend to Moving-Target TSP:
- Add time register
- Track the total time elapsed

Final Steps

- Obtain superposition of all paths
- Grover's algorithm for Hamiltonian paths
- Grover's algorithm for min-time path

Summary

- O($2^{N/2}$) time complexity
- Superposition achieved in linear time
- Need logarithmic search algorithm to achieve linear time complexity
- Implementation prototype

Future Work

- Improve (Moving-Target) TSP Complexity
- Improve search algorithm
- P=NP for quantum computer?