Problem 6.

Joseph pays $100 per month into a savings account earning 5%(4), with the first deposit on May 1, 1999 and the last one August 1, 2003. The money sits until October 1, 2005 when Joseph takes out his first semiannual retirement payment. Find the semiannual retirement payment if

a) the last one it on April 1, 2010
b) the retirement payments are to last indefinitely

Answer:

First we must find the equivalent interest. We do this by calculating the maturity value of $1 for one year, first using the known interest rate 5%(4), and second using the unknown interest rate per month, x. Then we set the two maturity values equal to each other and we solve for x.

The maturity value of $1 for one year at 5%(4) is $1(1 + 0.0125)^4 = $1.0509453. The maturity of $1 for one year at interest rate x per month compounded monthly is $1(1 + x)^{12}.

Now we solve the equation for x $1.050945337 = $1(1 + x)^{12}. This results in x = 0.0041494.

Now that we have the appropriate interest rate we can find the amount Joseph has on October 1, 2005. To do this, we use the equation for a forborne annuity

$$S = R\frac{(1+i)^n - 1}{i}(1+i)^p = \$100\frac{(1+0.0041494)^{18} - 1}{0.0041494}(1+0.0041494)^8 = \$1{,}927.73$$

Now we have found the present value of Joseph's account as of October 1, 2005. From this, we can now find the semiannual retirement payment.

a) First we must find the equivalent interest. We do this by calculating the maturity value of $1 for one year, first using the known interest rate 5%(4), and second using the unknown interest rate on a semiannual basis, x. Then we set the two maturity values equal to each other and we solve for x.

The maturity value of $1 for one year at 5%(4) is $1(1 + 0.0125)^4 = $1.0509453. The maturity of $1 for one year at interest rate x compounded semiannually is $1(1 + x)^2.

Now we solve the equation for x $1.0509453 = $1(1 + x)^2. This results in x = 0.0251563.

Now that we have the appropriate interest rate we can find the amount of Joseph's semiannual payment by solving for R. To do this, we use the equation for the present value of an ordinary annuity

$$\$1{,}927.73 = R\frac{1-(1+i)^{-n}}{i} = R\frac{1-(1+0.0251563)^{-10}}{0.0251563}$$

R = $220.44

b) This part asks for Joseph's payment amount it they were to continue indefinitely. To do this we use the equation for perpetuity due and solve for R

$$\$1{,}927.73 = R\left(\frac{1}{i} + 1\right) = R\left(\frac{1}{0.0251563} + 1\right)$$

R = $47.30