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Homework 10 Solution – 1,2,3 and 4

1. The essence of all perpetuities is that you can distribute the **payments** forever if you do not touch the **principal**.

   To find the essence of all perpetuities, it is crucial to understand the definition of Perpetuity. Perpetuity is an annuity with infinitely many payments. The principal never has to be paid, since the period payments are made on the principal from the previous rent period in the form of the coupon, or interest. However, this is only when you do not touch the principal. Otherwise, the interest earned from the previous periods will diminish which will lead to finite payment.

2.
   a) The present value of the payments from 2005 to 2015 is $R * \frac{1-(1+i)^{-21}}{i}(1+i)$ which equals to $1000000\left(1+\left(\frac{0.07}{2}\right)^4\right)$. Thus to find R, we have to set up an equation.

   $1000000(1+\left(\frac{0.07}{2}\right)^4) = R\left(\frac{1-\left(1+\frac{0.07}{2}\right)^{-21}}{\frac{0.07}{2}}\right)(1+\left(\frac{0.07}{2}\right))$

   To find R, $R = \frac{1000000(1+\left(\frac{0.07}{2}\right)^4)}{\left(\frac{1-\left(1+\frac{0.07}{2}\right)^{-21}}{\frac{0.07}{2}}\right)(1+\left(\frac{0.07}{2}\right))} = \$65735.931$

   b) The present value of a perpetuity due is $P = R(\frac{1}{i}+1)$. We also need to set up an equation to solve for R. $1000000\left(1+\frac{0.07}{2}\right)^4 = R\left(\frac{1}{\frac{0.07}{2}}+1\right)$

   To find R, $R = \frac{1000000\left(1+\frac{0.07}{2}\right)^4}{\left(\frac{1}{\frac{0.07}{2}}+1\right)} = \$38805.126$

3. To find the equivalent interest rates, we use $S = (1+i)^n$. Also to make equation simpler, we assume that P=1.
   a) $\left(1+\frac{0.08}{4}\right)^4 = \left(1+\frac{x}{12}\right)^{12}$ You calculate the left side and expand the equation

   $1.0824 = \left(1+\frac{x}{12}\right)^{12}$

   $\ln(1.0824) = 12\ln\left(1+\frac{x}{12}\right)$ Take the natural log of both sides of the equation to bring the exponent to the front of the equation.

   $0.0792 = 12\ln\left(1+\frac{x}{12}\right)$

   $\frac{0.0792}{12} = \ln\left(1+\frac{x}{12}\right)$, Divided both side by 12.

   $0.0066 = \ln\left(1+\frac{x}{12}\right)$

$e^{0.0066} = e\wedge \ln\left(1+\frac{x}{12}\right)$, e and natural log eliminates each other.

$1.0066 = 1+\frac{x}{12}$

$0.0066 = \frac{x}{12}$

X=0.0795=7.95%

b) $\left(1+\frac{0.09}{12}\right)^{12} = \left(1+\frac{x}{2}\right)^{2}$ You calculate the left side and expand the equation

$1.0938 = \left(1+\frac{x}{2}\right)^{2}$

$\ln(1.0938) = 2\ln\left(1+\frac{x}{2}\right)$ Take the natural log of both sides of the equation to bring the exponent to the front of the equation.

$0.0897 = 2\ln\left(1+\frac{x}{2}\right)$

$\frac{0.0897}{2} = \ln\left(1+\frac{x}{2}\right)$, Divided both side by 2.

$0.0448 = \ln\left(1+\frac{x}{2}\right)$

$e^{0.0448} = e\wedge \ln\left(1+\frac{x}{2}\right)$, e and natural log eliminates each other.

$1.0458 = 1+\frac{x}{2}$

$0.0458 = \frac{x}{2}$

X=0.0917=9.17%

c) $\left(1+\frac{0.068}{2}\right)^{1} = \left(1+\frac{x}{4}\right)^{2}$ You calculate the left side and expand the equation $1.068 = \left(1+\frac{x}{4}\right)^{4}$

$\ln(1.068) = 4\ln\left(1+\frac{x}{4}\right)$ Take the natural log of both sides of the equation to bring the exponent to the front of the equation.

$0.0658 = 4\ln\left(1+\frac{x}{4}\right)$

$\frac{0.0658}{4} = \ln\left(1+\frac{x}{4}\right)$, Divided both side by 4.

$0.0164 = \ln\left(1+\frac{x}{4}\right)$

$e^{0.0164} = e\wedge \ln\left(1+\frac{x}{4}\right)$, e and natural log eliminates each other.

$1.0165 = 1+\frac{x}{4}$

$0.0165 = \frac{x}{4}$

X=0.0663=6.63%

d) $\left(1+\frac{0.052}{4}\right)^4 = \left(1+\frac{x}{2}\right)^2$ You calculate the left side and expand the equation $1.053 = \left(1+\frac{x}{2}\right)^2$

$\ln(1.053) = 2\ln\left(1+\frac{x}{2}\right)$ Take the natural log of both sides of the equation to bring the exponent to the front of the equation.

$0.0516 = 2\ln\left(1+\frac{x}{2}\right)$

$\frac{0.0516}{2} = \ln\left(1+\frac{x}{2}\right)$, Divided both side by 2.

$0.0258 = \ln\left(1+\frac{x}{2}\right)$

$e^{0.0258} = e\text{^} \ln\left(1+\frac{x}{2}\right)$, e and natural log eliminates each other.

$1.0261 = 1+\frac{x}{2}$

$0.0261 = \frac{x}{2}$

X=0.0532=5.23%

4. Since the money earns 9% compounded monthly(12), we can set up an equation and calculate the interest rate per quarter which is $1+i = \left(1+\frac{0.09}{12}\right)^3$

When we solve for i, $i = \left(1+\frac{0.09}{12}\right)^3 - 1 = 0.02267$

Using$P = \$200000, R = \$10000, i = 0.02267$, we can calculate n by using $P = R\frac{1-(1+i)^n}{i}(1+i)$.

$$200000 = 10000\frac{1-(1+0.02267)^n}{0.02267}(1+0.02267)$$

We simplify equation,

$$\frac{200000}{(1+0.02267)} = 10000\frac{1-(1+0.02267)^n}{0.02267}$$
$$\frac{195566.507}{10000} = \frac{1-(1+0.02267)^n}{0.02267}$$
$$19.5566507(0.02267) = 1-(1+0.02267)^n$$
$$0.443349 - 1 = -(1+0.02267)^n$$
$$1 - 0.443349 = (1+0.02267)^n$$
$$0.556651 = (1+0.02267)^n$$

Use natural log to simplify equation.

$$\ln(0.556651) = nln(1+0.02267)$$
$$n = \frac{\ln(0.556651)}{ln(1+0.02267)} = -26.1329$$

We then use the result to find the number of full payments. Since the present value of all the payments is $R * \left(\frac{1-(1+i)^{-n}}{i}\right)(1+i) + x(1+i)^{-n}$, and this equals to \$200000, we can find x by solving $200000 = 10000 * \left(\frac{1-(1+0.02267)^{26}}{0.02267}\right)(1 + 0.02267) + x(1 + 0.02267)^{26}$.

$$200000 - 10000 * \left(\frac{1 - (1 + 0.02267)^{26}}{0.02267}\right)(1 + 0.02267) = x(1 + 0.02267)^{26}$$

$$199999.99 = x(1 + 0.02267)^{26}$$

$$\frac{199999.99}{(1 + 0.02267)^{26}} = x$$

$$x = \$111662$$