

## Solution to Problem (Bonus 2)

Sigmund and Karl each borrowed an identical amount from Ludwig at a nominal rate of discount of 5.4% convertible quarterly. Sigmund repays his loan by making payments of \$2000 at the end of each year for six years. Karl makes payments of \$3200 at four equally spaced times  $T$ ,  $2T$ ,  $3T$ , and  $4T$ . Find  $T$ .

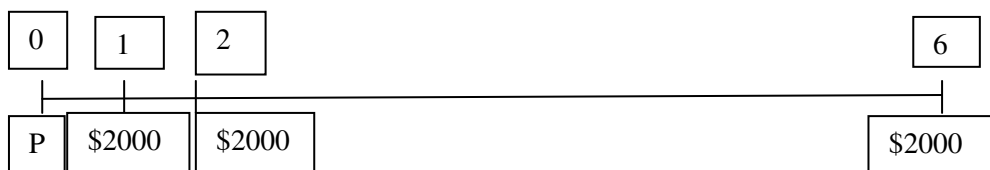
**Solution:**

-Dealing with ordinary annuity (payments made at the end of rent periods)

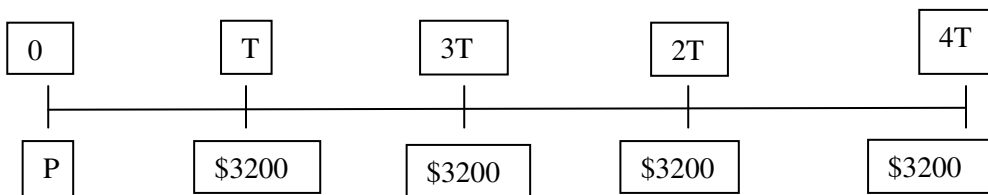
-Let  $P$  = present value (at the beginning of the 1<sup>st</sup> rent period)

→ Same for both Sigmund and Karl because they both borrow an identical amount from Ludwig

Sigmund Timeline:



Karl Timeline:



1. Find the nominal interest rate for Sigmund from the given nominal discount rate (5.4% (4)) compounded quarterly.

$$\$1(1 + i)^1 = \$1(1 - .054/4)^4$$

$$i = 1.05587 - 1 = .055873$$

2. Find the present value of an ordinary annuity (same for Sigmund and Karl as explained above)

Using Sigmund's information,  $R = \$2000$ ,  $n = 6$ , and  $i = .05587$

$$P = R (1 - (1 + i)^{-n})/i = 2000 (1 - (1 + .05587)^{-4})/.00587 = \$6996.21$$

3. The equivalent nominal interest rates (from the given nominal discount rate compounded quarterly) are different because the rent periods for Sigmund and Karl are different. Therefore, if we find the equivalent nominal interest rate for Karl we can find T.

Let  $k$  = equivalent nominal interest rate for Karl

$P = \$6996.21$ ,  $R = \$3200$ ,  $n = 4$ ,  $k$  = interest rate/T (T is the rent period for Karl)

$$P = R (1 - (1 + k)^{-n})/k$$

$$\$6996.21 = \$3200 (1 - (1 + k)^{-4})/k$$

→Using Wolfram alpha,  $k = 0.294513$

4. Solve for T using the same method as Step #1 (as  $j$  is the interest rate per T period).

$$\$1 (1 + k) = \$1(1 - .054/4)^{-4}$$

$$\$1(1 + .294513/T)^1 = \$1(1 - .054/4)^{-4}$$

$$(1 + .294513/T) = 1.05587$$

$$.294513T = .05587$$

$$T = .189713 \text{ quarters}$$