Problem (Bonus 3)

Starting on his 25th birthday and continuing through his 60th birthday, Fred deposits $7,500 each year on his birthday into a retirement fund earning an annual effective rate of 5%. Immediately after the last deposit, the accumulated value of the fund is transferred to a fund earning an annual effective rate of *j*. Five years later, a twenty-five year annuity-due paying $5,800 each month is purchased with the funds. The purchase price of the annuity was determined using an annual effective rate of interest 4%. Find *j*.

Answer: We use the given rent payment for the first term at an interest rate of 5%. If we choose to set up the problem as an ordinary annuity, we must use 36 rent periods since the term would start one period before the first deposit. We now have the formula and resulting equation for finding the future value of the first annuity, which gives the value of the annuity on Fred's 60th birthday:

$$S = R((1 + i)^n - 1) / i \quad = \quad \$7500((1 + .05)^{36} - 1) / .05$$

$$S = \$718{,}772.42$$

Because the value of S is located at Fred's 65th birthday, we can now use that value as the present value of the fund compounded for 5 years. The future value of this fund, which will later be equated to the present value of the annuity-due, is given by the equation:

$S = P(1 + j)^n$ where i=j and n=5 so…

$$S = \$718{,}772.42(1 + j)^5$$

Now we calculate the present value of the annuity-due and equate it to the equation just give. For the annuity-due, went have rent payments of $5,800 each and an effective interest rate of 4%. Because payments occur each month and the annuity-due lasts for 25 years, we have (25*12) periods, or 300 periods. Further, we must calculate the new interest rate, given by the following equation:

$(1 + .04)^1 = (1 + i(12)/12)^{12}$ Therefore… $i(12)/12 = 0.00327$

Now we calculate the present value of the annuity-due:

$P = R(1 + i)(1 - (1 + i)^{-n}) / i$

$P = \$5800(1 + .00327)(1 - (1 + .00327)^{300}) / .00327$

$P = \$1,111,979.84$

Finally, we equate our earlier equation with this new present value:

$\$1,111,979.84 = \$718,772.42(1 + j)^5$

Therefore j = 9.09%