Financial Mathematics: Homework #3

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Problem 1(Exercise 26, page 13)

On 5 February 2008 Pierce has $\$10,000$ in savings. The next date that interest is posted (added) to his account is 1 June 2008. If Pierce withdraws his money early, he will lose all the interest on the amount withdrawn. On 8 April 2008, Pierce needs $\$8,000$ for medical bills. If he can get a loan at 10% interest, should Pierce withdraw the money from his savings account or should he get a loan? Use exact time with ordinary interest.

Answer:

Since Pierce does not lose the interest on the amount left in the savings account, then we need not to worry about the $\$2,000$ left in the account. They will earn the same interest in both cases.

First, I will calculate the interest lost if Pierce withdraws $\$8,000$ from his checking account. Since 2008 is a leap year, the serial number of 1 June 2008 is 153 and the serial number of 5 February 2008 is 36. Thus, the term for the simple interest is $153-36=117$ days. We use the simple interest with ordinary interest formula:

$$I = Pit = \$8,000 \cdot 0.05 \cdot \frac{117}{360} = \$130$$

Next, I will calculate the interest Pierce would pay on the 10% loan. Since 2008 is a leap year, then the serial number for 8 April 2008 is 99. Then the term of the loan is $153-99=54$ days. We use the simple interest with ordinary interest formula:

$$I = Pit = \$8,000 \cdot 0.1 \cdot \frac{54}{360} = \$120$$

If Pierce does not withdraw $\$8,000$ from his savings account he has $\$130-\$120=\$10$ left after he pays his debt. If Pierce withdraws his money he has \$0 left. We conclude that Pierce should take the loan.

Problem 2(Exercise 12, page 17)

Brittany owes $\$50,000$ on December 2003 and $\$40,000$ on March 2004. Find the single value in February 2005 that she must pay to repay both amounts if money is worth 7% simple interest.

Answer:

First, we will calculate the interest for the $\$50,000$ loan taken out in December 2003 and paid in February 2005. The main issue is to calculate the term of this loan. We calculate the number of months as follows. February is the second month and December is the 12^{th} month, thus, the number of months between them is $2-12+12=2$. Therefore, the term is $1\frac{2}{12}=\frac{7}{6}$. Thus the interest Brittany pays in February 2005, on the $\$50,000$ loan taken out in December 2003 is

$$I = Pit = \$50,000 \cdot 0.07 \cdot \frac{7}{6} = \$4083.33$$

Second, we will calculate the interest for the $\$40,000$ loan taken out in March 2004 and paid in February 2005. February is the second month and March is the third month, thus the number of months between them is $2-3+12=11$. Therefore, the term is $\frac{11}{12}$. Thus the interest Brittany pays in February 2005, on the $\$40,000$ loan taken out in March 2004 is

$$I = Pit = \$40,000 \cdot 0.07 \cdot \frac{11}{12} = \$2566.66$$

Therefore, in February 2005, Brittany will pay the interest and principal of both loans

$$\$50,000 + \$4083.33 + \$40,000 + \$2566.66$$

Problem 3(Exercise 16, page 17)

If Aaron buys $\$5,520$ worth of building materials for his basement renovations and he receives an invoice with 3% discount if the bill is paid within 10 days and the net due in 60 days. What rate of interest will he earn if he makes his payment on the 10^{th} day?

Answer:

I will use ordinary interest
If Aaron pays on the 10^{th} day, the he has a 3% discount. This means he pays 97% of the bill, $\$5,520$.
To calculate the interest rate we use the formula

$$i = \frac{S-P}{Pt}$$

In this case, the principal is 97% of $\$5,520$ and the maturity value is $\$5,520$. The term is $60-10=50$ days. Therefore

$$i = \frac{0.03 \cdot \$5,520}{0.97\$5,520\frac{50}{360}} = 22.27\%$$

Observe that the value of the bill does not matter.

Problem 4(Exercise 12, page 22)

A tax free municipal security is bought in March 2005 and matures to $40,000 in September 2005. In July 2005 this security is sold to a third party requiring 10%. Find the amount the third person paid for the municipal security. Find the actual interest the original holder lost on his investment. Why would the original holder ever sell?

Answer:

The term in months from July 2005 to September 2005 is $9-7=2$ months. We use the present value formula to calculate the amount the third person paid for the municipal security

$$P=\frac{S}{1+it}=\frac{\$40,000}{1+0.1\frac{2}{12}}=\$39,344.26$$

The actual interest the original holder lost on his investment if $\$40,000-\$39,344.26$.

A reason the original holder would sell is he needed money.

Problem 5(Exercise 14, page 26)

Randall buys a $500 chocolate lab puppy in June 2004 and agrees with the breeder to make three equal payments in December 2004, June 2005, and December 2005. If the breeder charges 10% for credit, find the amount of the three payments. Put the focal date at December 2005.

Answer:

The term from June 2004 to December 2005 is $12-6=6$ months and $2005-2004=1$ year. Thus, $t=\frac{3}{2}$ years.
The maturity value of the $500 in December 2005 is

$$S=\$500(1+0.1\frac{3}{2})=\$575$$

Let Q denote the amount of each payment.
First we need to bring the December 2004 payment to December 2005. The term, in this case, is 1 year. Then Q moved to the focal date is

$$Q(1+0.1\cdot 1)=1.1Q$$

First we need to bring the June 2005 payment to December 2005. The term, in this case, is 0.5 year. Then Q moved to the focal date is

$$Q(1+0.1\cdot 0.5)=1.05Q$$

The last payment is already at the focal date. To determine Q we need to solve the equation

$$1.1Q+1.05Q+Q=\$575$$

Thus, $Q=\$182.54$.

Problem 6(Exercise 12, page 29)

A&R Plumbing contracts for a new home plumbing job. The contract stipulates that the homeowner will pay A&R $\$15,400$ in six months if the job is complete. Supplies are estimated to cost A&R $\$11,200$ now. Assuming the job is done on time, find the NPV at 70% and the IRR.

Answer:

To calculate the net present value we need to bring the payment to the present using the present value formula:

$$P = \frac{S}{1+it} = \frac{\$15,400}{1+0.7\cdot\frac{6}{12}} = \$11,407.41$$

The net present value is the algebraic sum of income(positive) and expenses(negative) moved to the present

$$NPV_{@70\%} = -\$11,200 + \$11,407.41$$

The internal return rate, IRR is the rate of interest that makes NPV zero. To calculate IRR we need to repeat the calculations above with the unknown interest rate replacing the 70% from above.

$$P = \frac{S}{1+it} = \frac{\$15,400}{1+IRR\cdot\frac{6}{12}}$$

The net present value is the algebraic sum of income(positive) and expenses(negative) moved to the present

$$NPV_{@IRR} = -\$11,200 + \frac{\$15,400}{1+IRR\cdot\frac{6}{12}}$$

By the definition of IRR, $NPV_{@IRR} = 0$. Therefore, to determine IRR we need to solve the equation

$$0 = -\$11,200 + \frac{\$15,400}{1+IRR\cdot 0.5}$$

Thus $IRR = 75\%$.