Homework 4 - Due 14 September 2011
Math 1140 Financial Mathematics

Collaboration Policy: You are encouraged to collaborate with your fellow students on this homework. You must turn in individual solutions and you are not allowed to use any written, typed, or recorded artifact from the meeting with your classmates.

Pledge: On my honor, I pledge that I have neither given nor received unauthorized aid on this assignment.

Name(use block letters):

Signature:

For full credit you must show your work and your calculations for all the problems. I am not asking for the presentation of silly arithmetic!

Problem 1

Alice takes a 10% simple interest loan from Bob with a one year term. After n days Bob has some financial trouble and he sells the loan to Katie. Since Katie is a nice and generous person she is not trying to take advantage of Bob's difficult situation and she asks for the same interest rate of 10%. How does the amount Bob gets from Katie compare with amount Bob would make if he would give Alice a 10% simple interest loan with term n days?
Your answer should be in terms of n, where $1 \le n \le 364$.

Problem 2

Modular arithmetic is a fancy name for the type of addition and subtraction you do when you calculate time.
Please read from http://mathdude.quickanddirtytips.com/what-is-modular-arithmetic.aspx about it (or any other resource).
a) Use modular arithmetic to calculate the term, in days, of a loan taken out on 22 January 2011 with a due date 5 March 2012?
b) Use modular arithmetic to calculate the term, in months, of a loan taken out on 2 December 2011 with a due date 2 March 2012?

Problem 3

Exercise 14, page 29

Problem 4

Exercise 12, page 32

Problem 5

Exercise 14, page 33

Problem 6

Exercise 18, page 45

Problem 7

Exercise 16, page 47

Problem 8

Exercise 18, page 47

Problem (Bonus)

In the lecture one notes I presented an example of interest calculation using the average daily balance and daily accrual. I obtained an interest of \$33.94 using average daily balance and \$34.09 using daily accrual.

Next, I will present a proof claiming that the interest calculated using average daily balance and the interest calculated using daily accrual are equal.

Let n be the number of days in the cycle. Let B_k be the daily balance of the credit card for the kth day of the cycle.

First, I will calculate the interest using the average daily balance, denoted by I_{ADB}. The average daily balance is the sum of the balance of each day, divided by the number of days in the cycle, that is $\frac{1}{n} \sum_{k=1}^{n} B_k$. Thus

$$I_{ADB} = \left(\frac{1}{n} \sum_{k=1}^{n} B_k \right) \frac{APR}{365} n = \left(\sum_{k=1}^{n} B_k \right) \frac{APR}{365}$$

Second, I will calculate the interest using the daily accrual method, denoted by I_{DA}. The interest for the kth day, denoted by I_k, is $I_k = B_k \frac{APR}{365}$. The interest using daily accrual is the sum of daily accrual.

$$I_{DA} = \sum_{k=1}^{n} I_k = \sum_{k=1}^{n} \left(B_k \frac{APR}{365} \right)$$

Since the real numbers are distributive, then $\sum_{k=1}^{n} \left(B_k \frac{APR}{365} \right) = \left(\sum_{k=1}^{n} B_k \right) \frac{APR}{365}$.

Thus, $I_{ADB} = I_{DA}$ In words, the interest calculated using average daily balance is mathematically equal to the interest calculated daily accrual, when applied to the same data.

This seems to contradict the example from lecture 1.

Did I make a mistake in the calculation? Did I make a mistake in the proof? If both the calculations from the notes and the proof are correct, how do you explain the seemingly contradictory results?