Financial Mathematics: Homework #5

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Problem 10 on page 47

Find the coupon equivalence of a 12-week bank discount rate of 8%.

Answer:

The coupon equivalence is the interest rate of an equivalent simple interest loan, that is, one that will have the same interest charges for the same term, same money borrowed (P), same money paid back (S). The interest charges for the simple interest loan are $I = Pit$ and the interest charges for the discount interest loan are $D = Sdt$. We also know that $P = S(1-dt)$. Thus

$$S(1-dt)it = Sdt$$

Therefore $i = \frac{d}{1-dt}$. In our case $i = \frac{0.08}{1-0.08\cdot\frac{12}{52}}$.

Problem 12 on page 47

Find the bank discount rate equivalent to a 2.5 year simple interest loan at 8%.

Answer:

To solve this problem we can solve the previous equation $S(1-dt)it = Sdt$ for d and get the formula $d = \frac{i}{1+it}$. In our case, $d = \frac{0.08}{1+0.08\cdot 2.5}$.

Another method is repeat the procedure from the previous problem, this time considering the simple interest loan data as given.The bank discount rate equivalent to a given simple interest loan is the discount rate of a loan with the proceeds equal to the principal, the amount equal to the maturity value, and same term. The interest charges are again equal (i.e the interest is equal to the discount). Since in this problem, the simple interest loan is given, we use the formula $S = P(1+it)$. We have

$$Pit = P(1+it)dt$$

Thus $d = \frac{i}{1+it}$.

Problem 8 on page 50

Ivan has a \$100 CD that matures with interest at 6% in 2 years and another noninterest bond for \$2,000 that matures in 3 years. Find the cash value of both at a bank discount rate of 10%.

Answer:

The problem does not state explicitly at which date to find the cash value of the two investments. I will calculate it at the present.

The cash value of both investments at the present at a bank discount rate of 10% is the sum of the present value at a bank discount rate of 10% of each investment. In other words, we need to solve the two simpler problems for each investment:
Problem 1: Ivan has a \$100 CD that matures with interest at 6% in 2 years. Find the cash value of the \$100 CD at a bank discount rate of 10%.
Problem 2: Ivan has a noninterest bond for \$2,000 that matures in 3 years. Find the cash value of the noninterest bond for \$2,000 at a bank discount rate of 10%.

First, I will calculate the present value at a bank discount rate of 10% of the \$100 CD. The \$100 investment in 2 years brings returns of $\$100(1+0.06\cdot 2)$. Here, I used the formula $S = P(1+it)$. The present value of those returns at a bank discount rate of 10% is calculated using the formula $P = S(1-dt)$ and we obtain $\$100(1+0.06\cdot 2)(1-0.1\cdot 2)$.

Second, I will calculate the the present value at a bank discount rate of 10% of the noninterest bond for \$2,000. The \$2,000 is the value of the bond 3 years from now. To calculate its present value at a a bank discount rate of 10% we use again the formula $P = S(1-dt)$ and we obtain $\$2,000(1-0.1\cdot 3)$.

The cash value of both investments at a bank discount rate of 10% is

$$\$100(1+0.06\cdot 2)(1-0.1\cdot 2)+\$2,000(1-0.1\cdot 3)$$

Note: You must be careful in reading the problem. Sometimes you are given different and independent investments as above, other times you are given partial payments of one loan as in the partial payments problems, or the problem refers to an initial investment and subsequent expenses and returns as in the net present value problems.

Problem 14 on page 50

Akhil has made the following investments:
First is a 2-year, \$10,000, 9% CD bought on 4/1/05
Second is an 18-month, \$3,000, 7% CD bought on 6/1/05
Third is 6-month, \$20,000, non-interest-bearing bond bought on 7/1/06
Find Akhil's proceeds on 3/1/06 for all three at a bank discount rate of 12%.

Answer:

Since the problem does not specify the type of time and interest, and all the dates are the first of the month, we will work with months.

As in the previous problem the proceeds for all three investments is the sum of the proceeds of each investment.

The first investment, a 2-year, $\$10,000$, 9% CD bought on 4/1/05, matures on 4/1/07 to $\$10,000(1+0.09\cdot 2)$. The value on 3/1/06 at a bank discount rate of 12% is calculated using the formula $P = S(1-dt)$ and it is $\$10,000(1+0.09\cdot 2)(1-0.12\cdot \frac{13}{12})$.

The second investment, an 18-month, $\$3,000$, 7% CD bought on 6/1/05, matures on 12/1/06 to $\$3,000(1+0.07\cdot \frac{22}{12})$. The value on 3/1/06 at a bank discount rate of 12% is calculated using the formula $P = S(1-dt)$ and it is $\$3,000(1+0.07\cdot \frac{22}{12})(1-0.12\cdot \frac{8}{12})$.

The third investment, 6-month, $\$20,000$, non-interest-bearing bond bought on 7/1/06, matures to $\$20,000$ on 1/1/07. The value on 3/1/06 at a bank discount rate of 12% is calculated using the formula $P = S(1-dt)$ and it is $\$20,000(1-0.12\cdot \frac{10}{12})$.

Akhil's proceeds on 3/1/06 for all three investments at a bank discount rate of 12% is

$$\$10,000(1+0.09\cdot 2)(1-0.12\cdot \frac{13}{12})+\$3,000(1+0.07\cdot \frac{22}{12})(1-0.12\cdot \frac{8}{12})+\$20,000(1-0.12\cdot \frac{10}{12})$$

Problem 14 on page 54

On September 25, 2004 a 180-day $\$100,000$ Treasury bill is purchased with a bid of 98.5. Find the rate of return on this investment using discount interest. Also find the coupon equivalent.

Answer:

The purchase price of the bill is 98.5% of the face value ($\$100,000$), $\$98,500$. Since $D = Sdt = S-P$, then the rate of return on this investment using discount interest is

$$d = \frac{S-P}{St} = \frac{\$100,000-\$98,500}{\$100,000\cdot \frac{180}{360}}$$

The coupon equivalent is the simple interest rate of an equivalent investment. Thus $I = Pit = S-P$ gives us

$$i = \frac{S-P}{Pt} = \frac{\$100,000-\$98,500}{\$98,500\cdot \frac{180}{360}}$$

Note: In class we defined a similar notion called investment yield. The definition of investment yield specifies that exact interest should be used and we divided by 365. In this problem we are asked to calculate the coupon equivalent of the discount yield (rate of return on this investment using discount interest) so we use the same type of interest, ordinary interest, as when we calculated the discount yield. This is the reason why I divide by 360.

Problem 8 on page 56

A 90-day corporate note was purchased on July 14, 2004, at a discount rate of 5.4%; the note matures to $\$2,000,000$. If the buyer discounted the note in the secondary market on Spetember

28, 2004, at a discount rate of 7.5%, what was her return on a simple interest basis? What was the new owner rate of return if he holds the note to maturity? Use exact time and exact interest.

Answer:

This problem is an example of *discounting the note* discussed in sections 1.6 and 2.5.

To determine the return on the investment we need to determine the price paid for the note on July 14, 2004 and the price the note sold on September 28, 2004. Again we solve two easier problems.

First, we calculate the price paid for the note on July 14, 2004. This is an easy application of the discount proceeds formula. The corporate note has a maturity value of $\$2,000,000$, a discount rate of 5.4%, and a term of 90 days. We use the discount proceeds formula

$$P = S(1-dt) = \$2,000,000(1-0.054\cdot\frac{90}{365}) = \$1,973,369.86$$

Second, we calculate the the price the note sold on September 28, 2004. From July 14, 2004 to September 28, 2004, there are $271-195=76$ days. Since the note matures in 90 days, the third-party buying the note on the secondary market has the note for $90-76=14$ days and has to pay

$$P = S(1-dt) = \$2,000,000(1-0.075\cdot\frac{14}{365}) = \$1,994,246.57$$

The first owner's return on the investment is the price she sells the note minus the price paid for the note $\$1,994,246.57-\$1,973,369.86=\$20,876.71$. The rate of return on a simple interest basis is

$$i = \frac{I}{Pt} = \frac{\$20,876.71}{\$1,973,369.86\frac{76}{365}}$$

The term is only 76 days this time because the original owner has the note from July 14, 2004 to September 28, 2004.

The rate of return of the new owner on a discount interest basis is, clearly, the rate of discount given in the problem 7.5%. The rate of return of the new owner on a simple interest basis is

$$i = \frac{S-P}{Pt} = \frac{\$2,000,000-\$1,994,246.57}{\$1,994,246.57\frac{14}{365}}$$

Note: I believe that discounting the note is one the most important concepts in this class and a problem like this will be on tests and exams until everybody in the class will be able to solve it.

Problem 28 on page 73

If home values are growing at 3.5% per year, how much will a $\$100,000$ home be worth in 20 years?

Answer:

The phrase 'growing at 3.5% per year' indicates you should be thinking about compound interest. A simple interest problem would have been something like: *How much will a* $100,000 *home be worth in* 20 *years at* 3.5%*?*

This problem is an application of the compound amount formula, $S = P(1+i)^n$ where $P = \$100,000$ is the cost of the home, $i = 3.5\%$, and $n = 20$. The home values are growing at 3.5% per year means that the conversion period is one year and the interest rate per conversion period is 3.5%.

Problem 32 on page 73

Suppose your savings account is paying 4%(1). On February 1, 2002 you have $382.50 in your account. How much remains in your savings account on February 1, 2010 when you withdraw $500?

Answer:

This problem is essentially asking you to determine how much money are in the account on February 1, 2010. The $500 withdraw means you need to subtract the $500 from the balance.

First we need to calculate how much is in the account on February 1, 2004. Since the money stay in the account for eight years and there is one conversion period per year, the total number of conversion periods is 8. We apply the compound amount formula

$$S = P(1+i)^n = \$382.50(1+0.04)^8 = \$523.47$$

To obtain the final result you can type $382.50(1+0.04)^8$ in Wolfram Alpha.

Problem 12 on page 76

Gerald has two notes coming due. The first is a $5,000 note due on 1/1/02. The second is a $2,000 note due on 5/1/04. Find the equivalent worth of these debts on 1/1/03 discounting the $2,000 at 9%(12) and letting the $5,000 note accrue interest at 12%(2).

Answer:

The equivalent of the two debts is the sum of the two debts calculated on 1/1/03. This problem can be split into two easier problems.

Problem 1: Gerald has a debt of $5,000 on 1/1/02. If he pays 12%(2) interest, how much does he owe on 1/1/03?
From 1/1/2 to 1/1/03 there is one year, thus $n = 2$. On 1/1/03 Gerald owes

$$S = P(1+i)^n = \$5,000(1+0.12/2)^2 = \$7,969.24$$

Problem 2: Gerald owes $2,000 on 5/1/04. How much does he has to pay on 1/1/03 to settle the debt?

From 1/1/03 to 5/1/04 there are 2 years and 4 months, or 28 months. Since we are given the maturity value, we need to determine the principal. Thus

$$P = S(1+i)^{-n} = \$2,000(1+0.09/12)^{-28} = \$1,622.43$$

The equivalent of the two debts on 1/1/03 is $\$7,969.24 + \$1,622.43$.

Problem 14 on page 76

A man owes a note for $\$6,000$ in 3 years. A buyer desiring $9\%(4)$ on her money should pay what for this note? What is the compound discount?

Answer:

This is a example of *discounting the note*.

The first sentence of the problem, *A man owes a note for* $\$6,000$ *in 3 years.* tells us that the maturity value of the note is $\$6,000$. It also tells us that the due date of the note is 3 years from now.

The second sentence *A buyer desiring* $9\%(4)$ *on her money should pay what for this note?* tells us that the note is sold today and the third-party, the buyer, wants to make $9\%(4)$. The $9\%(4)$ interest tells us we need to use compound interest with 4 conversion periods per year and nominal interest rate $i(4) = 9\%$. The buyer will hold the note for 3 years, thus the total number of conversion periods is $n = 3 \cdot 4 = 12$. The maturity value is $S = \$6,000$. We apply the compound present value formula:

$$P = S(1+i(m)/m)^{-n} = \$6,000(1+0.09/4)^{-12}$$

Note: The first sentence is a little ambiguous, we might not know if the $\$6,000$ is the maturity value or the original price paid for the note. Since there is no other information about the note, like when it was bought or the interest, the only logical conclusions are that the problem is incomplete, or that the $\$6,000$ represent the maturity value. However, since you are told the man **owes** the $\$6,000$ in three years, suggests that the $\$6,000$ is the maturity value.
If the problem said *the man bought a* $\$6,000$, then the $\$6,000$ would be the price paid for the note, and would be a present value.