

# Homework 7 - Due 14 October 2011

## Math 1140 Financial Mathematics

**Collaboration Policy:** You are encouraged to collaborate with your fellow students on this homework. You must turn in individual solutions and you are not allowed to use any written, typed, or recorded artifact from the meeting with your classmates.

**Pledge:** On my honor, I pledge that I have neither given nor received unauthorized aid on this assignment.

**Name(use block letters):**

**Signature:**

**For full credit you must show your work and your calculations for all the problems.** I am not asking for the presentation of silly arithmetic!

### Problem 1

Dilbert makes annual deposits of \$5000 on the first day of each year for 20 years. If the effective rate of interest is 7%, how much is in the account immediately after the last deposit?

### Problem 2

On January 1, 2004 Ralph opens an IRA with a \$2,000 deposit. He continues to deposit \$2,000 at the beginning of each year until January 1, 2044, when he makes the final deposit. If the account earns an effective rate of interest of 9%, how much is in the account on the day of the last deposit?

### Problem 3

Ralph deposits \$1000 on January 1 of each year from 1990 to 2015 into an account paying an effective rate of 6%. If there are no further deposits, how much is in the account on January 1, 2030?

### Problem 4

Luke makes deposits of \$50,000 on January 1 of years 2000, 2005, 2010, 2015, and 2020 into an account paying 10% interest convertible quarterly. How much is the account worth on January 1, 2040?

### Problem 5

Suppose that an investment will make 20 annual payments of \$12,000, the first coming a year from now. Assuming an effective rate of 8%, what is the present value of the investment?

### Problem 6

Suppose that an annuity pays \$1200 once per year for 15 years, with the first payment coming one year from now. If the effective rate of interest is 9%, what is the present value?

## Problem 7

An annuity will provide a sequence of 40 quarterly payments of \$5000, the first coming 7 years from now. If we assume a nominal rate of 6% convertible monthly, what is the present value of the annuity?

## Problem Bonus 1

Compare the Merchant's Rule and the US Rule for a compound interest loan, when one partial payment is made after a number of conversions periods.

*Hint:* Your solutions should have the following structure:

Step 1: Give names (letter labels) to all quantities in the problem: e.g the principal, the partial payment, the number of conversion periods until the partial payment, the number of conversion periods from the first payment to the due date.

Step 2: Calculate the balance using the Merchant's Rule.

Step 3: Calculate the balance using the US Rule.

Step 4: Compare the quantities at step 2 and 3. Might be easier if you subtract one from the other.

## Problem (Bonus 2)

Consider an annuity for which the payments are made at the beginning of the rent period. Suppose the annuity has  $n$  payments. Calculate the value of the annuity at the end of the last rent period.

*Hint:* Given the number of payments  $n$ , the rent or payment  $R$ , the interest rate per rent period  $i$ . Your answer should be a formula containing these symbols.

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## Problem (Bonus 4)

Assume Alice has saved \$5,000 and wants to open a retirement account. She has two options:

(1) pay  $x\%$  tax now, and not pay tax when she withdraws money

(2) not pay tax now, deposit the entire \$5,000, but pay  $y\%$  tax on the money she withdraws.

Assuming that both accounts will earn interest with an effective rate of 6%, discuss which option is more advantageous for Alice. Your answer should depend on whether  $x$  or  $y$  is bigger.