Homework 10 - Solutions

Math 1140 Financial Mathematics

Collaboration Policy: You are encouraged to collaborate with your fellow students on this homework. You must turn in individual solutions and you are not allowed to use any written, typed, or recorded artifact from the meeting with your classmates. You are allowed to use any resources **except for the Appendix D in the textbook (the solutions to the odd-numbered exercises)**.

Pledge: On my honor, I pledge that I have neither given nor received unauthorized aid on this assignment.

Name(use block letters):

Signature:

For full credit you must show your work and your calculations for all the problems. I am not asking for the presentation of silly arithmetic!

Problem7. A small business makes $\$15,000$ quarterly payments into an equity fund that earns an average return of $8\%(12)$. The fund was started June 1, 1999 and payments will continue until September 1, 2010. Suppose the fund is reinvested on September 1, 2010 at $9\%(4)$, and starting on October 1, 2012 monthly payments will be made from the account to help cover emplyees' benefits. With payments starting on October 1, 2012, how much is available monthly if
a) payments continue until August 1, 2020?
b) payments continue indefinitely?

Answer:

Step 1: Calculate the interest rate convertible quarterly equivalent to $8\%(12)$.

The term of the equivalent loan is 3 months. The maturity value of $\$1$ at x per quarter, in three months, is $1+x$. The maturity value of $\$1$ at $8\%(12)$, in three months, is $(1+\frac{0.08}{12})^3$. To determine x we set the two maturity values equal to each other and get $x = 0.020134$.

Step 2: Calculate the value, S_1, of the $\$15,000$ quarterly payments at the time of the last payment, September 1, 2010.

The number of payments is $n = 46$. Using the maturity value formula for an ordinary annuity we have

$$S_1 = R\frac{(1+i)^n - 1}{i} = \$15,000\frac{(1+0.020134)^{46} - 1}{0.020134} = \$1,118,766.31$$

Step 3: Calculate the interest rate convertible monthly equivalent to $9\%(4)$.

The term of the equivalent loan is 3 months. The maturity value of $\$1$ at x per month, in three months, is $(1+x)^3$. The maturity value of $\$1$ at $9\%(4)$, in three months, is $(1+\frac{0.09}{4})^1$. To determine x we set the two maturity values equal to each other and get $x = 0.007444$.

Step 4: Calculate the value, S_2, of the \$15,000 quarterly payments on September 1, 2012.

There are no payments from September 1, 2010 until September 1, 2012 and money accrue interest. Thus

$$S_2 = S_1(1+i)^n = \$1,118,766.31(1+\frac{0.09}{4})^8 = \$1,336,736.83$$

Part a:

From October 1, 2012 trough August 1, 2020 there are $7 \cdot 12 + 11 = 95$ months. To determine the payments we use the present value formula of an ordinary annuity

$$P = R\frac{1-(1+i)^{-n}}{i}$$

$$R = \frac{iP}{1-(1+i)^{-n}} = \frac{0.007444 \cdot \$1,336,736.83}{1-(1.007444)^{-95}}$$

Part b:

Since the payments continue indefinitely, the series of payments is an annuity. Since we calculated the value of the account one month before the first payment, we can use the present value formula for an ordinary annuity. We have

$$R = iP = 0.007444 \cdot \$1,336,736.83$$