

Math 1140 Financial Mathematics

Lecture 35
Yield Rates

Ana Nora Evans
403 Kerchof
AnaNEvans@virginia.edu
<http://people.virginia.edu/~ansSk>

Plan for the rest of the semester

Nov 14, Nov 16, Nov 18 – Bonds
Nov 21, Nov 28 – Introduction to Options
Nov 30, Dec 2, Dec 5 – Project Presentations

- A) I like the plan.
- B) I prefer two classes about bonds and have one on stocks.
- C) I want some other topic covered.

Upcoming Deadlines

Nov 18 – HW 12 due
Nov 21 – presentation sign-up
Nov 30 – final report and evaluations
Nov 30, Dec 2, Dec 5 – presentations and evaluations
Dec 2 – HW 13 (extra credit)

- A) I want HW13 to be mandatory.
- B) I want HW13 to be optional to increase my homework score.
- C) I want HW13 to be optional to increase my extra credit score.
- D) There should be no HW 13.

Advice

The students that must take the final should start studying for it very seriously.

Read the class notes and the textbook starting from the beginning.

Come to office hours and ask questions about what is unclear.

Solve the homework problems again, without looking at the textbook or solutions.

Final

Simple Interest – sections 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9
 Discount Interest – sections 2.1, 2.2, 2.3, 2.4, 2.5
 Compound Interest – sections 3.1, 3.2, 3.4, 3.5, 3.6, 3.7
 Ordinary Annuities – sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.6
 Other Annuities – sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.6
 Debt Retirement Methods – sections 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7
 Bonds – handout

Last Time

Comparison of amortization and sinking funds.

The sinking fund is **more expensive** than amortization when the interest rate earned by the fund is smaller than the interest rate paid for the loan. ($j < i$)

The sinking fund is **less expensive** than amortization when the interest rate earned by the fund is bigger than the interest rate paid for the loan. ($i < j$)

Today

Rate of Return
 Yield Rates

Rate of Return

The **rate of return** is the interest rate earned or lost in an investment.

The *rate of return* is calculated by using an equation of value and solving for the interest rate.

Simple Interest

Alice buys a note today for \$1,000 earning 8% simple interest for 3 years. After 1 year Alice sells the note to Bob who asks for 10% simple interest on his investment.

What is Alice's rate of return at simple interest?

The maturity value of Alice's note is:

$$\$1,000(1+0.08 \times 3)$$

The sell price of the note is:

$$\frac{\$1,000(1+0.08 \times 3)}{1+0.1 \times 2}$$

Have:

The amount invested:
\$1,000.

The amount earned by this investment in 1 year:

$$\frac{\$1,000(1+0.08 \times 3)}{1+0.1 \times 2} = \$1,033.33$$

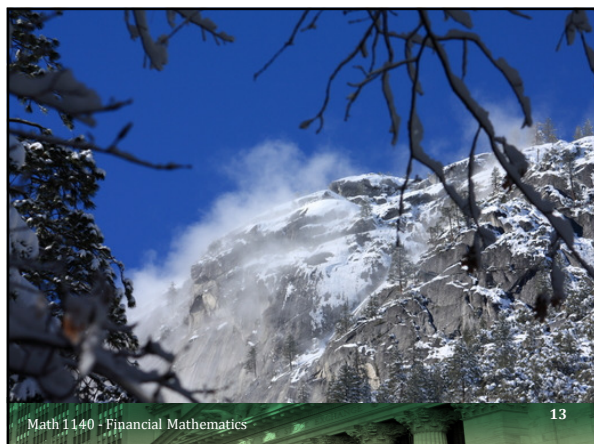
Want:

The simple interest rate.

$$i = \frac{I}{Pt} = \frac{S - P}{Pt}$$

$$i = \frac{\$1,033.33 - \$1,000}{\$1,000 \times 1}$$

$$i = 3.33\%$$



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Compound Interest

Alice buys a note today for \$1,000 earning an effective interest rate of 8% for 3 years. After 1 year Alice sells the note to Bob who asks for an effective interest rate of 10% on his investment.

What is Alice's rate of return?

The maturity value of Alice's note is:

$$\$1,000(1+0.08)^3$$

The sell price of the note is:

$$\$1,000(1+0.08)^3(1+0.1)^{-2}$$

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Have:

The amount invested:
\$1,000.

The amount earned by this investment in 1 year:

$$\$1,000(1+0.08)^3(1+0.1)^{-2}$$

Want:

The compound interest rate.

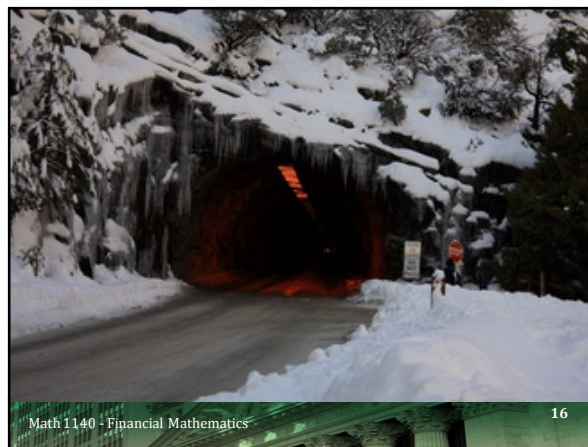
$$i = \sqrt[n]{\frac{S}{P}} - 1$$

$$i = \sqrt{\frac{\$1,000(1+0.08)^3(1+0.1)^{-2}}{\$1,000}} - 1$$

$$i = 4.11\%$$

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Alice loans \$10,000 at 7% effective to Carl, to be repaid with 8 annual payments, the first a year from now. Immediately after Carl makes his third payment, Alice sells the right for the remaining payments to Betty at a price to yield 9% effective.

What is Alice's rate of return?

Step 1: Calculate the annual payment

We use the present value formula of an ordinary annuity.

$$R = \frac{iP}{1 - (1+i)^{-n}}$$

$$R = \frac{0.07 \times \$10,000}{1 - (1+0.07)^{-8}} = \$1,674.68$$

Alice loans \$10,000 at 7% effective to Carl, to be repaid with 8 annual payments, the first a year from now. Immediately after Carl makes his third payment, Alice sells the right for the remaining payments to Betty at a price to yield 9% effective.

What is Alice's rate of return?

Step 2: Calculate the price paid by Betty

We use the present value formula of an ordinary annuity.

$$P = R \frac{1 - (1+i)^{-n}}{i}$$

$$P = \$1,674.68 \frac{1 - (1+0.09)^{-5}}{0.09}$$

$$P = \$6,513.92$$

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Have:	Want:
The amount invested:	The interest rate.
$PV = \$10,000.$	
3 yearly payments of:	$PV = R \frac{1 - (1+i)^{-3}}{i} + S(1+i)^{-3}$
$R = \$1,674.68$	
Sell price of the loan:	$i = 5.77\%$
$S = \$6,866.92$	

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Does this equation look familiar?

A) NPV
B) IRR
C) Ordinary Annuity
D) Never seen something like this before

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The **yield rate** of an investment is the rate of return.

The **rate of return** is the interest rate earned or lost in an investment.

The *rate of return* is calculated by using an equation of value and solving for the interest rate.

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Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate on the remaining payments is 7.5% effective.

How large are the new payments?

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The yield rate (aka interest rate) of the last 4 payments is 7.5%.

What is the present value of these payments?

A) $12 \times \$5,000$
B) The outstanding balance after the 5th payment
C) $7 \times \$5,000$
D) I can't calculate it

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Plan

Step 1: Calculate the outstanding balance after the fifth payment

Step 2: Calculate the new payment

Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate on the remaining payments is 7.5% effective.

How large are the new payments?

Step 1: Calculate the outstanding balance after the 5th payment

Which method is the easiest to use?

$$OB = R \frac{1 - (1+i)^{-(n-k)}}{i}$$

$$OB = \$5,000 \frac{1 - (1+0.06)^{-7}}{0.06}$$

$$OB = \$27,911.91$$

Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate on the remaining payments is 7.5% effective.

How large are the new payments?

Step 2: Calculate the new payment

What do the remaining payments are?

$$P = \$27,911.91$$

$$i = 7.5\%$$

$$n = 7$$

$$R = \$8,333.59$$



Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate is 7.5% effective for the entire loan.

How large are the new payments?

What changes from the previous problem?

The outstanding value is calculated differently.

We have to use the retrospective method.

First, we calculate the present value of the payments at 6%.

$$P = \$5,000 \frac{1 - (1+0.06)^{-12}}{0.06}$$

$$P = \$41,919.22$$

Next, we calculate the outstanding balance using the retrospective method.

$$OB = P(1+i)^k - R \frac{(1+i)^k - 1}{i}$$

$$OB = \$41,919.22(1+0.075)^5 - \$5,000 \frac{(1+0.075)^5 - 1}{0.075}$$

$$OB = \$31,138.51$$

We calculate the new payment as in the previous exercise:

$$R = \$9,296.95$$

Questions?

Upcoming Deadlines

Friday, Nov 18 – HW 12 due

Monday, Nov 21 – presentation sign-up

Nov 30 – final report and evaluations

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