



Math 1140 Financial Mathematics

Lecture 36
Bonds

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Plan for the rest of the semester

Nov 14, Nov 16, Nov 18 – Bonds

Nov 21, Nov 28 – Introduction to Options

Nov 30, Dec 2, Dec 5 – Project Presentations

Upcoming Deadlines

Friday, Nov 18 – HW 12 due

Monday, Nov 21 – presentation sign-up

two groups signed up for Dec 2

one group signed up for Dec 5

Nov 30 – final report and evaluations

Nov 30, Dec 2, Dec 5 – presentations and evaluations

Dec 2 – HW 13 (extra extra credit)

When do I know my final grade?

I'll try to grade HW11 and HW12 by Nov 24.

Friday, the day after Thanksgiving, I will email you current grade (does not include the project report and presentation).

You'll know then if need to solve HW13.

A+ will be given to students having over 113 points, without the final.

Final

Simple Interest – sections 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9

Discount Interest – sections 2.1, 2.2, 2.3, 2.4, 2.5

Compound Interest – sections 3.1, 3.2, 3.4, 3.5, 3.6, 3.7

Ordinary Annuities – sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.6

Other Annuities – sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.6

Debt Retirement Methods – sections 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7

Bonds – handout and practice problems, and sections 7.1 and 7.3

Last Time

The **rate of return** is the interest rate earned or lost in an investment.

The *rate of return* is calculated by using an equation of value and solving for the interest rate.

The **yield rate** of an investment is the rate of return.

Alice loans \$10,000 at 7% effective to Carl, to be repaid with 8 annual payments, the first a year from now. Immediately after Carl makes his third payment, Alice sells the right for the remaining payments to Betty at a price to yield 9% effective.

What is Alice's rate of return?

Alice made an agreement with Carl.

When Alice sells the rights to the last payments to Betty, **Carl's payments can not change.**

The **sell price** is the present value of the remaining payments at the interest rate Betty asks for.

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Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate on the remaining payments is 7.5% effective.

How large are the new payments?

Step 1: Calculate the outstanding balance after the 5th payment

Step 2: Calculate the new payment using present value of an annuity formula with values

P = outstanding balance from step 1

$i = 7.5\%$

$n = 4$

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Suppose that you have a loan at an effective rate of 6% that requires you to make 12 annual payments of \$5000 each.

Immediately after making the 5th payment, you contact the lender and ask to repay the loan with 4 more larger payments.

The lender agrees, but only if the yield rate is 7.5% effective for the entire loan.

How large are the new payments?

What changes from the previous problem?

The outstanding value is calculated differently.

We have to use the retrospective method.

First, we calculate the present value of the payments at 6%.

$$P = \$5,000 \frac{1 - (1 + 0.06)^{-12}}{0.06}$$

$$P = \$41,919.22$$

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Next, we calculate the outstanding balance using the retrospective method.

$$OB = P(1+i)^k - R \frac{(1+i)^k - 1}{i}$$

$$OB = \$41,919.22(1 + 0.075)^5 - \$5,000 \frac{(1 + 0.075)^5 - 1}{0.075}$$

$$OB = \$31,138.51$$

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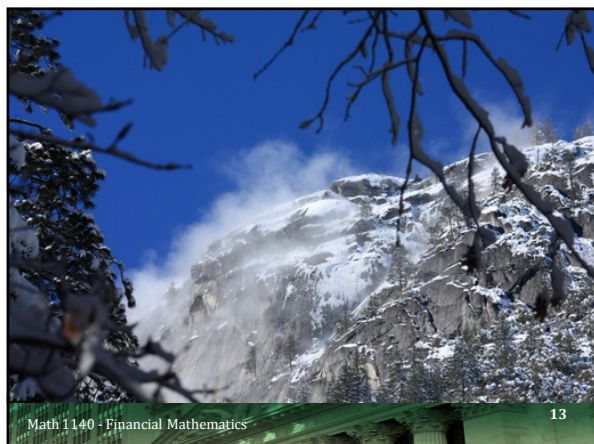
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We calculate the new payment as in the previous exercise:

$$R = \$9,296.95$$

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Today

Bonds definitions and yield rates

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- A) I plan to come to office hours today 10:30-11:30.
 B) I plan to come to office hours today 3-4.
 C) I don't plan to come to office hours.

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A **bond** is a legal promise to pay the owner of the bond, regular payments, ending at a specified date in the future.

On the bond are specified the **face value (par value)**, F , and the **coupon rate (bond rate)**, r .

The **coupon period** is the time interval between two payments.

The coupon is, usually, semiannual.

The coupon rate is given per coupon period.

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The regular payment, called **coupon**, is the interest on the face value of the bond for the duration between two payments at the coupon rate.

The coupon is equal to Fr .

The last payment is called the **maturity value (redemption value)**, R .

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The **price** of the bond is the amount paid by the owner of the bond to the issuer ~~for the rights~~.

The price is calculated using the desired yield rate per coupon period.

$$P = R(1+i)^{-n} + Fr \frac{1 - (1+i)^{-n}}{i}$$

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Suppose that Cavalier Enterprises issues 8-year bonds, redeemable at par, with a face value of \$1000 and a coupon rate of 7% convertible semi-annually.

If you purchase one of these bonds, here's what you'll get:

1. Interest payments twice a year of $\$1,000(0.07/2) = \35 for 8 years.
2. The face value of \$1,000.

"Redeemable at par" this means that you'll get the face value of \$1000 in 8 years (at the same time as the final coupon payment) when the bond matures.

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Suppose that Cavalier Enterprises issues 8-year bonds, redeemable at par, with a face value of \$1000 and a coupon rate of 7% convertible semi-annually.

The face value is $F = \$1,000$.

The coupon rate is $r = 0.07/2$.

The redeeming value is $R = \$1,000$.

The number of coupon payments is $n = 8 \times 2 = 16$.

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Suppose that Cavalier Enterprises issues 8-year bonds, redeemable at par, with a face value of \$1000 and a coupon rate of 7% convertible semi-annually.

If the yield rate is 9%(2), calculate the price of one bond.

The yield rate is $i = 0.09/2 = 0.045$ *0.045*

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$$P = R(1+i)^{-n} + Fr \frac{1-(1+i)^{-n}}{i}$$

$$P = \$1,000(1+0.045)^{-16} + \$1,000 \times \underbrace{0.035}_{r = \frac{0.07}{2}} \frac{1-(1+0.045)^{-16}}{0.045}$$

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Suppose that a \$5000 12-year par-value bond pays coupons at 8% convertible semi-annually. If the yield rate is 6% convertible semi-annually, what is the price of the bond?

$$\begin{aligned} F &= \$5,000 \\ R &= \$5,000 \\ r &= 0.08/2 = 0.04 \\ i &= 0.06/2 = 0.03 \\ n &= 12 \times 2 = 24 \\ P &= \$5,846.7771 \end{aligned}$$



Suppose that a 9-year, \$1000 face value bond pays semi-annual coupons and is redeemable at twice par.

If the price of the bond is \$1343.75 at a yield rate of 7.6% convertible semi-annually, what is the coupon rate? (Give the nominal rate convertible twice per year.)

$$\begin{aligned} F &= \$1,000 \\ R &= \$2,000 \\ P &= \$1,343.75 \\ i &= 0.076/2 \\ n &= 9 \times 2 = 18 \\ r &= ? \end{aligned}$$

$$P = R(1+i)^{-n} + Fr \frac{1-(1+i)^{-n}}{i}$$

$$\$1,343.75 = \$2,000(1.038)^{-18} + \$1,000 \times r \frac{1-(1.038)^{-18}}{0.038}$$

$$r = 0.025$$

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