

Math 1140 Financial Mathematics

Lecture 38
Options

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Last Time

Bonds rating and junk bonds.
The book value of a bond.

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The **credit rating** of a bond is a financial indicator to potential investors.

The credit rating of a bond is assigned by a **credit rating agency** registered with the Securities and Exchange Commission (SEC).

Examples: Standard & Poor's, Moody's.

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Moody's	S&P	Capacity of the issuer to meet its obligations
Aaa	AAA	Extremely strong
Aa1	AA+	Very strong
Aa2	AA	
Aa3	AA-	
A1	A+	Strong
A2	A	
A3	A-	
Baa1	BBB+	Adequate
Baa2	BBB	
Baa3	BBB-	

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Junk Bonds

Moody's	S&P	Capacity of the issuer to meet its obligations
Ba1	BB+	Less vulnerable
Ba2	BB	
Ba3	BB-	
B1	B+	More vulnerable
B2	B	
B3	B-	
Caa	CCC	Currently vulnerable
Ca	CC	Highly vulnerable
	C	Highly vulnerable (a bankruptcy petition may have been filled)
C	D	Failed to pay one or more of its financial obligations when it became due.

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The Book Value

Consider a bond with face value, F , coupon rate, r , yield, i , and n coupon payments.

The book value of the bond after the k^{th} payment is the value of the remaining coupons plus the redeeming value right after the k^{th} payment.

$$R(1+i)^{-(n-k)} + Fr \frac{1-(1+i)^{-(n-k)}}{i}$$

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An annuity will make 60 quarterly payments (the first a year from now) of \$3200 each. If the yield rate is 8% convertible quarterly, what is the price of the annuity?

$m = 60$
 $R = \$3,200$
 $i = 0.08/4 = 0.02$
 $P = R \frac{1 - (1+i)^{-n}}{i}$

A) $\$3,200 \frac{1 - (1+0.02)^{-60}}{0.02}$
 B) $\$3,200 \frac{1 - (1+0.02)^{-240}}{0.02}$
 C) $\$3,200 \frac{1 - (1+0.08)^{-60}}{0.08}$
 D) $\$3,200 \frac{1 - (1+0.08)^{-240}}{0.08}$

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A 10-year bond with a face value of \$1000, redeemable at par, earns interest at 9% convertible semi-annually. Describe the payments to the holder of the bond.

$7\% (2)$
 $r = 0.07/2$
 coupon payments are $P \cdot r$

A) $0.09 \times \$1,000$ twice a year for 10 years and \$1,000 in 10 years
 B) $0.045 \times \$1,000$ twice a year for 10 years and \$1,000 in 10 years
 C) $0.09 \times \$1,000$ twice a year for 10 years
 D) $0.045 \times \$1,000$ twice a year for 10 years
 E) \$1,000 in 10 years

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A 10-year bond with a face value of \$1000, redeemable at par, earns interest at 9% convertible semi-annually. Find the price to yield an investor 7% convertible semi-annually.

$F = R = \$1,000$
 $r = 0.09/2$
 $i = 0.07/2$
 $n = 20$

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Upcoming Deadlines

Wednesday, Nov 30 – final project report due
 Nov 30, Dec 2, Dec 5 – project presentations
 Friday, Dec 2 – extra extra credit HW 13 due

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Options

A **call option** gives the holder the right to *buy* a specified asset at a certain date for a specific price.
 A **put option** gives the holder the right to *sell* a specified asset at a certain date for a specific price.
 The specific price is called **exercise price** or **strike price**.

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Expiration Date

The date in the definitions of options is called **expiration date** or **maturity date**.

All options expire on the *third Friday* of the expiration month, so all we need to know about the expiration date is the month.

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American Option

An **American option** is an option that allows the holder to exercise the option at any time up to the expiration date.

What does this mean?

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European Option

An **European option** is an option that can be exercised only on the expiration date itself.

What does this mean?

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Suppose that you hold an American call option on Apple Computer stock with a strike price of \$325 that expires in July.

Call option means the holder has the right to buy the Apple Computer stock.

American option means the holder has the right to exercise the option, that is to buy the stock, any time before July.

The *strike price* of \$325 means that the holder can buy the stock at \$325.

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If the stock is selling for *more* than the strike price of \$325, then the option is said to be **in the money**.

If the stock is selling for *less* than the strike price of \$325, then the option is said to be **out the money**.

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Suppose that you hold an American call option on Apple Computer stock with a strike price of \$325 that expires in July.

If the current price is \$375 should you exercise the option now?

- A) Yes, I am making good money.
- B) No, I am loosing money now.

When would you want to wait and exercise your option later?

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Suppose that you hold an American call option on Apple Computer stock with a strike price of \$325 that expires in July.

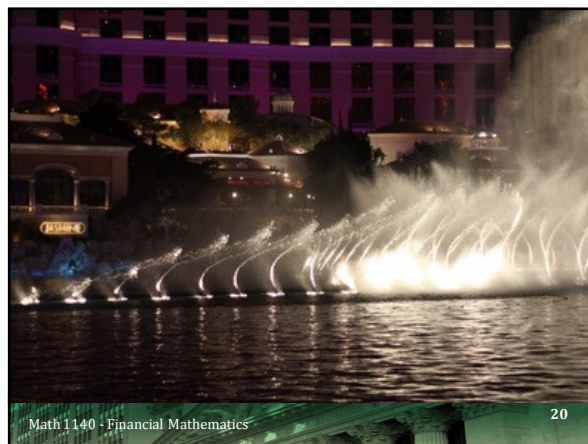
If the current price is \$300 should you exercise the option now?

- A) Yes, I am making good money.
B) No, I am losing money now.

Does your option have any value?

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Why would anyone want to sell (write) options?

Options are sold on the open market, with the price determined by a variety of factors, including the strike price, the current stock price, and the expiration date.

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Commitment of Parties

The *holder* has the right to exercise the option, but is not obligated to do so.

If the holder wants to exercise an option, the *writer (seller)* must comply.

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Suppose that a September \$65 call on Univision stock is sold for \$7. Show graphs of the profit pattern for both the buyer and seller at expiration.

Stock Price (\$)	50	55	60	65	70	75	80	85	90
Holder's Profit (P)	-7	-7	-7	-7	-2	3	8	13	18
Writer's Profit	7	7	7	7	2	-3	-8	-13	-18

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Holder's Profit

If the stock is under the strike price \$65, then the holder should not exercise his option. He has a loss equal to the price of the option.

If the stock is equal to the strike price, will the holder make a profit?

If the stock is equal or above the strike price, the holder should exercise the option. He will have a profit of:

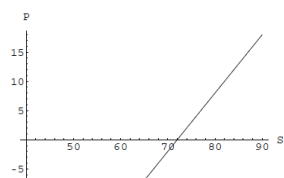
(stock price – strike price) – option price.

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Holder's Profit

$$P = \begin{cases} -7, & S \leq 65 \\ S - 72, & S > 65 \end{cases}$$

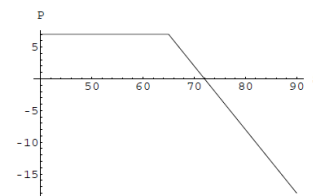


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Writer's Profit

$$P = \begin{cases} 7, & S \leq 65 \\ 72 - S, & S > 65 \end{cases}$$



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What is the worst thing that can happen to the buyer?

The option expires worthless, so that the price of the option is lost.

Is there a limit on how much the writer (seller) can lose?

No, there is no limit. There is no limit on how much the stock price can go up.

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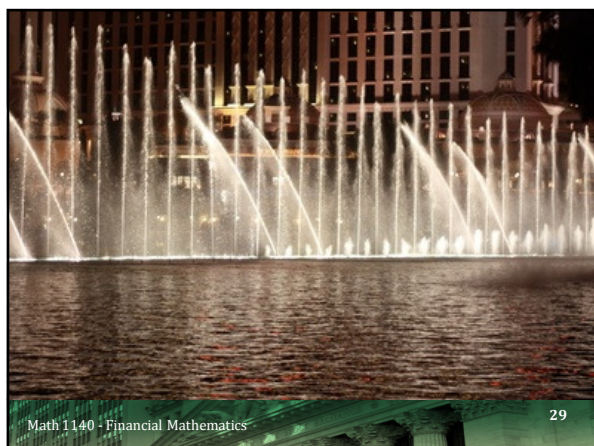
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S denotes the price of the stock at expiration and P the profit. We also let K denote the strike price of the option and C the price of the option.

Holder's Profit for a **call** option:

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Suppose that an October \$90 put on Microsoft stock is sold for \$4. Show graphs of the profit pattern for both the buyer and seller at expiration. A *put option* means that the holder of the option has the right to sell the stock at the strike price of \$90.

What does this mean for the writer?

The writer is obligated to buy the stock at \$90 if the holder chooses to exercise the option.

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Suppose that an October \$90 put on Microsoft stock is sold for \$4. Show graphs of the profit pattern for both the buyer and seller at expiration.

What is the holder hoping for?

A) The stock price increases.
B) The stock prices decreases.

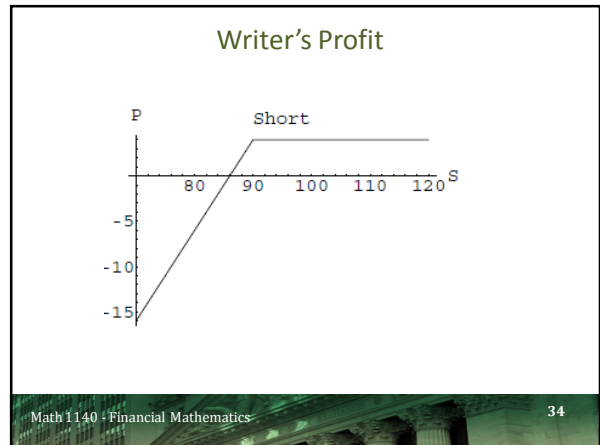
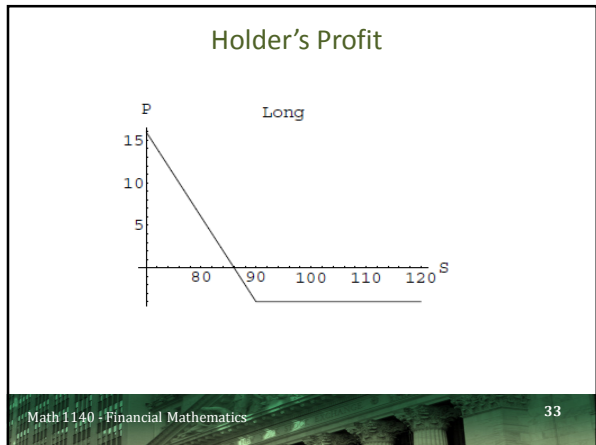
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Suppose that an October \$90 put on Microsoft stock is sold for \$4. Show graphs of the profit pattern for both the buyer and seller at expiration.

Stock Price (S)	70	75	80	85	90	95
Holder's Profit (P)						
Writer's Profit						

$$P = \begin{cases} 86 - S, & S < 90 \\ -4, & S \geq 90 \end{cases} \quad P = \begin{cases} S - 86, & S < 90 \\ 4, & S \geq 90 \end{cases}$$

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S denotes the price of the stock at expiration and P the profit. We also let K denote the strike price of the option and C the price of the option.

Holder's Profit for a **put** option:

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Happy Thanksgiving!

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