CS 202 section 2 (Fall ’04): Midterm 1

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Signature: ____

There are 50 minutes for this exam and 100 points on the test; don’t spend too long on any one question!

All work must be on these three exam pages.

Short answer questions (5 points each): these questions only require a sentence or two for full credit.

Question 1 (5 points)
Define and explain the difference between a Boolean variable and a proposition.

Answer:

A proposition is a statement that is either true or false. A Boolean variable is a letter that represents a proposition so that manipulation of the propositions is easier.

Question 2 (5 points)
Define and explain the difference between a proposition and a propositional function.

Answer:

A propositional function is a function that can be determined to be true or false when it's variables are either assigned specific values, or when the variables are quantified. Once that happens, the propositional function becomes a proposition.

Question 3 (5 points)
What is the power set of {1, 2, 3}?

Answer:

$P(\{1,2,3\} = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Question 4 (10 points)

Encode the following statement using quantifiers and propositional functions (of one or two variables). Clearly label what your propositional functions and variables represent.

a) “There is a building on campus of some college in the United States in which every room is painted white.”

Answer:

Let $W(r)$ mean room r is painted white. Let $I(r,b)$ mean that room r is in building b. Let $L(b,u)$ mean that building b is on the campus of US university u. The statement is $\exists u \exists b \left(L(b,u) \wedge \forall r \left(I(r,b) \rightarrow W(r) \right) \right)$.

(From Rosen, chapter 1 supplementary exercises, questions 19 and 20, page 115)

Question 5 (25 points)
A pirate famous for his bizarre sense of humor and love of logic puzzles left the following clues as to the location of the treasure. The treasure can only be in one place.

a) If the house is next to a lake, then the treasure is in the kitchen
b) If the house is not next to a lake or the treasure is buried under the flagpole, then the tree in the front yard is an elm and the tree in the back yard is not an oak
c) If the treasure is in the garage, then the tree in the back yard is not an oak
d) If the treasure is not buried under the flagpole, then the tree in the front yard is not an elm
e) The treasure is not in the kitchen

Using rules of inference, determine where the treasure is hidden. Clearly state what your Boolean variables represent.

Answer:

Let p = “The house is next to a lake”
Let q = “The treasure is in the kitchen”
Let r = “The tree in the front yard is an elm”
Let s = “The tree in the back yard is an oak”
Let t = “The treasure is in the garage”
Let u = “The treasure is buried under the flagpole”

Our clues are:

1. $p \to q$
2. $\neg p \vee u \to r \wedge \neg s$
3. $t \to \neg s$
4. $\neg u \to \neg r$
5. $\neg q$

Our steps with the rules of inference are:

1.	$p \to q$	1st hypothesis
2.	$\neg q$	5th hypothesis
3.	$\neg p$	Modus tollens on steps 1 and 2
4.	$\neg p \vee u$	Addition on step 3
5.	$\neg p \vee u \to r \wedge \neg s$	2nd hypothesis
6.	$r \wedge \neg s$	Modus ponens on steps 4 and 5
7.	r	Simplification on step 6
8.	$\neg u \to \neg r$	4th hypothesis
9.	u	Modus tonens on steps 8 and 9

The treasure is buried under the flagpole.

(From Epp, section 1.3, question 36, page 40)

Question 6 (25 points)

For all sets A, B, and C, prove using set identities that $(A \cup B) - (C - A) \equiv A \cup (B - C)$. Recall that $A - B \equiv A \cap \overline{B}$. Label each step with the name of the set identity that was used.

Answer:

$(A \cup B) - (C - A) \equiv A \cup (B - C)$	Original statement
$(A \cup B) \cap \overline{(C \cap \overline{A})} \equiv A \cup (B \cap \overline{C})$	Definition of difference
$(A \cup B) \cap (\overline{C} \cup \overline{\overline{A}}) \equiv A \cup (B \cap \overline{C})$	DeMorgan's law
$(A \cup B) \cap (\overline{C} \cup A) \equiv A \cup (B \cap \overline{C})$	Complentation law
$(A \cup B) \cap (A \cup \overline{C}) \equiv A \cup (B \cap \overline{C})$	Commutative law
$A \cup (B \cap \overline{C}) \equiv A \cup (B \cap \overline{C})$	Distributive law

(From Epp, section 5.2, question 29, page 257)

Question 7 (25 points)
Consider the statement: "The sum of any even integer and any odd integer is odd"

a) Restate this (in English) as a conditional

Answer:

Given two numbers m and n, if m is even and n is odd, then $m+n$ is odd

b) Prove it via an indirect proof XOR a proof by contradiction

Answer (indirect proof):

We want to prove the contrapositive: If $m+n$ is even, then m is odd OR n is even. Note that it doesn't matter which of m or n is odd – just that either one of them is odd or one of them is even. The only way an or statement can be false is if both are false – namely that m is even and n is odd. However, since we don't care which of m or n is odd and even, we can swap them and fulfill the or statement. (There are more formal ways to prove this, which will get credit, but an informal explanation is allowed here).

Answer (proof by contradiction):

Assume that the implication is false, namely that the antecedent is true (namely, that m is even and n is odd), and that the consequence is false (namely, that m+n is even). Let $m = 2k$, and let $n = 2l + 1$, for some integers k and l

$$m + n = 2k + 2l + 1$$
$$= 2(k + l) + 1$$

As $m+n$ is 2 times an integer plus one (that integer is $k+l$), $m+n$ must be odd. This contradicts our assumption that $m+n$ is even.