Global Illumination

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CS 4810: Graphics

Acknowledgment: slides by Jason Lawrence, Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin
Overview

• Direct Illumination
  - Emission at light sources
  - Direct light at surface points

• Global illumination
  - Shadows
  - Transmissions
  - Inter-object reflections
Shadows

• Shadow term tells if light sources are blocked
  • Cast ray towards each light source $L_i$. If the ray is blocked, do not consider the contribution of the light.
Shadows

• Shadow term tells if light sources are blocked
  - Cast ray towards each light source $L_i$
  - $S_i = 0$ if ray is blocked, $S_i = 1$ otherwise

\[
I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) I_L S_L
\]
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Ray Casting

• Trace primary rays from camera
  - Direct illumination from unblocked lights only
Recursive Ray Tracing

• Also trace secondary rays from hit surfaces
  o Consider contributions from:
    1. Reflected Rays
    2. Refracted Rays
Mirror Reflections

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  - Consider contributions from:
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I = I_E + K_A I_A + \sum_L (K_D (N \cdot L) + K_S (V \cdot R)^n) I_L S_L + K_S I_R
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Transparent Refraction

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Transparency and Shadow

- Problem:
  - If a surface is transparent, then rays to the light source may pass through the object

Over-shadowing
Transparency and Shadow

• Problem:
  o If a surface is transparent, then rays to the light source may pass through the object.
  o Need to modify the shadow term so that instead of representing a binary (0/1) value, it gives the fraction of light passing through.

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Transparency and Shadow

• Problem:
  o If a surface is transparent, then rays to the light source may pass through the object
  o Need to modify the shadow term so that instead of representing a binary (0/1) value, it gives the fraction of light passing through.
  o Accumulate transparency values as the ray travels to the light source.
Transparency and Shadow

• Accumulate transparency values as the ray travels to the light source.

\[ I = I_E + K_A I_A + \sum_L \left( (K_D (N \cdot L) + K_S (V \cdot R)^n) I_L S_L + K_S I_R + K_T I_T \right) \]
Transparency and Shadow

- Accumulate transparency values as the ray travels to the light source.

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Transparency and Shadow

- Accumulate transparency values as the ray travels to the light source.
Transparent Refraction

- When a ray of light passes through a transparent object it can bend.
Transparent Refraction

- When a ray of light passes through a transparent object, the ray of light can bend, ($\theta_i \neq \theta_r$).
Snell’s Law

• The way that light bends is determined by the indices of refraction of the internal and external materials $\eta_i$ and $\eta_r$:

$$\eta_r \sin \theta_r = \eta_i \sin \theta_i$$
Snell’s Law

• The way that light bends is determined by the indices of refraction of the internal and external materials $\eta_i$ and $\eta_r$:

$$\eta_r \sin \theta_r = \eta_i \sin \theta_i$$

$$T = \left( \frac{\eta_i}{\eta_r} \cos \theta_i - \cos \theta_r \right) \frac{1}{N} - \frac{\eta_i}{\eta_r} L$$
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Snell’s Law and Shadows

• Problem:
  - If a surface is transparent, then rays to the light source may not travel in a straight line
Snell’s Law and Shadows

• Problem:
  o If a surface is transparent, then rays to the light source may not travel in a straight line
  o This is difficult to address with ray-tracing
General Issue

• How do we determine when to stop recursing?
General Issue

• How do we determine when to stop recursing?
  o Depth of iteration
    » Bounds the number of times a ray will bounce around the scene
  o Cut-off value
    » Ignores contribution from bounces that contribute very little
Putting it all Together

Image RayCast(Camera camera, Scene scene, int width, int height) 
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(scene, ray, hit);
        }
    }
    return image;
}
Putting it all Together

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        }
    }
    return image;
}

With Illumination
Putting it all Together

```cpp
Pixel GetColor(scene, ray, depth, cutoff) {
    Pixel p(0,0,0)
    Ray reflect, refract
    Intersection hit = FindIntersection(ray, scene);
    if (hit) {
        p += GetSurfaceColor(hit.position);

        reflect.direction = Reflect(ray.direction, hit.normal);
        reflect.position = hit.position + reflect.direction * ε
        if (depth > 0 && hit.kSpec > cutoff)
            p += GetColor(scene, reflect, depth-1, cutoff / hit.kSpec) * hit.kSpec

        refract.direction = Refract(ray.direction, hit.normal, hit.ir);
        refract.position = hit.position + refract.direction * ε
        if (depth > 0 && hit.kTran > cutoff)
            p += GetColor(scene, refract, depth-1, cutoff / hit.kTran) * hit.kTran
    }
    return p
}
```
Putting it all Together

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Why do we need the ε terms?
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        if( depth >0 && hit.kTran>cutOff)
            p += GetColor(scene, refract, depth-1, cutOff/hit.kTran)
    }
    return p
}
Illumination Examples

• Ray casting (direct illumination)
Illumination Examples

• Soft Shadows

Courtesy Henrik Wann Jensen
Illumination Examples

• Caustics

Courtesy Henrik Wann Jensen
Illumination Examples

• Full Global Illumination
Recursive Ray Tracing

- \texttt{GetColor} is a recursive function
Summary

• Ray casting (direct Illumination)
  o Usually use simple analytic approximations for light source emission and surface reflectance

• Recursive ray tracing (global illumination)
  o Incorporate shadows, mirror reflections, and pure refractions

All of this is an approximation so that it is practical to compute