Clipping and Scan Conversion

Connelly Barnes

CS 4810: Graphics

Acknowledgment: slides by Jason Lawrence, Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin
3D Rendering Pipeline (for direct illumination)

- 3D Primitives
- 3D Modeling Coordinates
- Modeling Transformation
- 3D World Coordinates
- Camera Transformation
- 3D Camera Coordinates
- Lighting
- 3D Camera Coordinates
- Projection Transformation
- 2D Screen Coordinates
- Clipping
- 2D Screen Coordinates
- Viewport Transformation
- 2D Image Coordinates
- Scan Conversion
- Image

3D Model

2D Image
Transformations

\[ p(x,y,z) \]

Modeling Transformation

\[ 3D \text{ Object Coordinates} \]

3D World Coordinates

Camera Transformation

\[ 3D \text{ Camera Coordinates} \]

Projection Transformation

\[ 2D \text{ Screen Coordinates} \]

Window-to-Viewport Transformation

\[ 2D \text{ Image Coordinates} \]

\[ p'(x',y') \]

3D Object Coordinates

3D World Coordinates
Transformations

\[ p(x,y,z) \]

3D Object Coordinates

Modeling Transformation

\[ \text{Transform} = M \]

M := local to world transform

3D World Coordinates

Camera Transformation

3D Camera Coordinates

Projection Transformation

2D Screen Coordinates

Window-to-Viewport Transformation

2D Image Coordinates

\[ p'(x',y') \]
Transformations

\[ p(x,y,z) \]

3D Object Coordinates

Modeling Transformation

3D World Coordinates

Camera Transformation

3D Camera Coordinates

Projection Transformation

2D Screen Coordinates

Window-to-Viewport Transformation

2D Image Coordinates

\[ p'(x',y') \]

Transform \( = M \)

\[ M := \text{local to world transform} \]
Transformations

\[ p(x,y,z) \]

Modeling Transformation

\[ p'(x',y') \]

Projection Transformation

Window-to-Viewport Transformation

Transformations

\[ p'(x',y') \]

Camera Right

Camera Up

Camera Back

Transform = \( C^{-1}M \)

\[ C := \text{camera transform} \]

\[
C = \begin{bmatrix}
R_x & U_x & B_x & E_x \\
R_y & U_y & B_y & E_y \\
R_z & U_z & B_z & E_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Transformations

\[ p(x,y,z) \]

3D Object Coordinates

Modeling Transformation

3D World Coordinates

Camera Transformation

3D Camera Coordinates

Projection Transformation

2D Screen Coordinates

Window-to-Viewport Transformation

2D Image Coordinates

\[ p'(x',y') \]

\[ \text{Transform} = PC^{-1}M \]

\[ P := \text{projection transform} \]

\[
P_o = \begin{bmatrix}
1 & 0 & L \cos\phi & 0 \\
0 & 1 & L \sin\phi & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
P_p = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & \frac{1}{D} & 0
\end{bmatrix}
\]
Transformations

\[ p(x, y, z) \]

Modeling Transformation

3D Object Coordinates

Camera Transformation

3D World Coordinates

Projection Transformation

3D Camera Coordinates

Window-to-Viewport Transformation

2D Screen Coordinates

Normalized Coords

Transform = \( VPC^{-1}M \)

\[ V := \text{viewport transform} \]

Image

Viewport

Window

2D Image Coordinates

\[ p'(x', y') \]
Transformations

- **Transformations**
  - **Modeling Transformation**
    - 3D Object Coordinates
  - **Camera Transformation**
    - 3D World Coordinates
  - **Projection Transformation**
    - 3D Camera Coordinates
  - **Window-to-Viewport Transformation**
    - 2D Screen Coordinates
    - 2D Image Coordinates

\[ p(x, y, z) \]

\[
\begin{align*}
p'(x', y') &= \text{Window-to-Viewport Transformation} \\
&= VPC^{-1}M \\
V &= \begin{bmatrix}
1 & 0 & 0 & v_x^1 \\
0 & 1 & 0 & v_y^1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
v_x^2 - v_x^1 \\
w_x^2 - w_x^1 \\
v_y^2 - v_y^1 \\
w_y^2 - w_y^1 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 & -w_x^1 \\
0 & 0 & 1 & -w_y \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\end{align*}
\]
3D Rendering Pipeline (for direct illumination)

\[ p(x, y, z) \]

- 3D Object Coordinates
- Modeling Transformation
- 3D World Coordinates
- Camera Transformation
- 3D Camera Coordinates
- Projection Transformation
- 2D Screen Coordinates
- Window-to-Viewport Transformation
- 2D Image Coordinates
- \[ p'(x', y') \]
3D Rendering Pipeline (for direct illumination)

3D Primitives -> 3D Modeling Coordinates
   Modeling Transformation -> 3D World Coordinates
      Camera Transformation -> 3D Camera Coordinates
         Lighting -> 3D Camera Coordinates
            Projection Transformation -> 2D Screen Coordinates
               Clipping -> 2D Screen Coordinates
                  Viewport Transformation -> 2D Image Coordinates
                     Scan Conversion -> 2D Image Coordinates
                        Image
Clipping

• Avoid drawing parts of primitives outside window
  > Window defines part of scene being viewed
  > Must draw geometric primitives only inside window
Clipping

• Avoid drawing parts of primitives outside window
  ■ Points
  ■ Line Segments
  ■ Polygons
Point Clipping

• Is point \((x, y)\) inside the clip window?
Point Clipping

• Is point \((x, y)\) inside the clip window?

\[
\text{inside} = (x \geq wx1) \land (x \leq wx2) \land (y \geq wy1) \land (y \leq wy2);
\]
Clipping

- Avoid drawing parts of primitives outside window
  - Points
  - Line Segments
  - Polygons
Line Segment Clipping

- Find the part of a line inside the clip window

Before Clipping
Line Segment Clipping

- Find the part of a line inside the clip window

After Clipping
Cohen-Sutherland Line Clipping

- Use simple tests to classify easy cases first
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test

Bit 1
- 0000
- 0001
- 0100
- 0101

Bit 2
- 0010
- 0110

Bit 3
- 1000
- 1010

Bit 4
- 1001
- 0101

Points:
- P5
- P6
- P7
- P8
- P9
- P10
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

• If both outcodes are 0, line segment is inside
• If AND of outcodes not 0, line segment is outside
• Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

• If both outcodes are 0, line segment is inside
• If AND of outcodes not 0, line segment is outside
• Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test

Note: both lines have the same bit codes!
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- If both outcodes are 0, line segment is inside
- If AND of outcodes not 0, line segment is outside
- Otherwise clip and test
Cohen-Sutherland Line Clipping

- Before clipping
Cohen-Sutherland Line Clipping

• After clipping

```plaintext
Bit 1  Bit 2  Bit 3  Bit 4

0000  0100  1000  
0001  0101  1001  
0010  0110  1010  

P' 8  
P' 7  
P 4  
P 3  
P 5  
P 6  
P 8  

Bit 4

1001
1000
1010

Bit 3

0001
0010
0100

Bit 2

0101
0110
0110

Bit 1
```
Clipping

- Avoid drawing parts of primitives outside window
  - Points
  - Line Segments
  - Polygons
Polygon Clipping

• Find the part of a polygon inside the clip window
Polygon Clipping

- Find the part of a polygon inside the clip window

After Clipping
Sutherland-Hodgeman Clipping

- Clip to each window boundary one at a time
Sutherland-Hodgeman Clipping

- Clip to each window boundary one at a time
Sutherland-Hodgeman Clipping

• Clip to each window boundary one at a time
Sutherland-Hodgeman Clipping

• Clip to each window boundary one at a time
Sutherland-Hodgeman Clipping

- Clip to each window boundary one at a time
Sutherland-Hodgeman Clipping

- How do we clip a polygon with respect to a line?
Sutherland-Hodgeman Clipping

• Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary
Sutherland-Hodgeman Clipping

• Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
- Insert new points when cross window boundary,
- Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
- Insert new points when cross window boundary,
- Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence.
- Insert new points when cross window boundary.
- Remove points outside window boundary.
Sutherland-Hodgeman Clipping

- Do inside test for each point in sequence,
  Insert new points when cross window boundary,
  Remove points outside window boundary

Window Boundary

P' P’’

P_1

P_2

Inside

Outside
3D Rendering Pipeline (for direct illumination)

3D Primitives

- 3D Modeling Coordinates
  - Modeling Transformation
  - 3D World Coordinates
  - Camera Transformation
  - 3D Camera Coordinates
  - Lighting
  - 3D Camera Coordinates
  - Projection Transformation
  - 2D Screen Coordinates
  - Clipping
  - 2D Screen Coordinates
  - Viewport Transformation
  - 2D Image Coordinates
  - Scan Conversion
  - 2D Image Coordinates
  - Image

3D Model

2D Screen

2D Window
2D Rendering Pipeline

3D Primitives

2D Primitives

Clipping

Scan Conversion

Image

Clip portions of geometric primitives residing outside the window

Fill pixels representing primitives in screen coordinates
Overview

• Scan conversion
  o Figure out which pixels to fill

• Shading
  o Determine a color for each filled pixel

• Depth test
  o Determine when the color of a pixel should be overwritten
Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

```c
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle
Scan Conversion

• Render an image of a geometric primitive by setting pixel colors

```c
void SetPixel(int x, int y, Color rgba)
```

• Example: Filling the inside of a triangle
Triangle Scan Conversion

- Properties of a good algorithm
  - MUST BE FAST!
  - No cracks between adjacent primitives
Simple Algorithm

• Color all pixels inside triangle

```c
void ScanTriangle(Triangle T, Color rgba){
    for each pixel P at (x,y){
        if (Inside(T, P))
            SetPixel(x, y, rgba);
    }
}
```
Line defines two halfspaces

• Test: use implicit equation for a line
  - On line: $ax + by + c = 0$
  - On right: $ax + by + c < 0$
  - On left: $ax + by + c > 0$
Inside Triangle Test

- A point is inside a triangle if it is in the positive half-space of all three boundary lines
  - Triangle vertices are ordered counter-clockwise
  - Point must be on the left side of every boundary line
Inside Triangle Test

Boolean Inside(Triangle T, Point P) {
    for each boundary line L of T {
        Scalar d = L.a*P.x + L.b*P.y + L.c;
        if (d < 0.0) return FALSE;
    }
    return TRUE;
}
Simple Algorithm

• What is bad about this algorithm?

```c
void ScanTriangle(Triangle T, Color rgba){
    for each pixel P at (x,y){
        if (Inside(T, P))
            SetPixel(x, y, rgba);
    }
}
```
Triangle Sweep-Line Algorithm

• Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order

• Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally
Triangle Sweep-Line Algorithm

void ScanTriangle(Triangle T, Color rgba) {
    for both edge pairs {
        initialize $x_L$, $x_R$;
        compute $dx_L/dy_L$ and $dx_R/dy_R$;
        for each scanline at $y$
        for (int $x = x_L$; $x <= x_R$; $x++$)
            SetPixel($x$, $y$, rgba);
            $x_L += dx_L/dy_L$;
            $x_R += dx_R/dy_R$;
    }
}
Polygon Scan Conversion

- Will this method work for convex polygons?
Polygon Scan Conversion

• Will this method work for convex polygons?
  o Yes, since each scan line will only intersect the polygon at two points.
Polygon Scan Conversion

- How about these polygons?
Polygon Scan Conversion

- How about these polygons?
Polygon Scan Conversion

• Fill pixels inside a polygon
  ○ Triangle
  ○ Quadrilateral
  ○ Convex
  ○ Star-shaped
  ○ Concave
  ○ Self-intersecting
  ○ Holes

What problems do we encounter with arbitrary polygons?
Polygon Scan Conversion

- Need better test for points inside polygon
  - Triangle method works only for convex polygons
Inside Polygon Rule

• What is a good rule for which pixels are inside?

Concave  Self-Intersecting  With Holes
Inside Polygon Rule

- Odd-parity rule
  - Any ray from P to infinity crosses odd number of edges

Concave  |  Self-Intersecting  |  With Holes
Polygon Sweep-Line Algorithm

- Incremental algorithm to find spans, and determine “insideness” with odd parity rule
  - Takes advantage of scan line coherence
Polygon Sweep-Line Algorithm

```c
void ScanPolygon(Polygon P, Color rgba){
    sort edges by max y
    make empty "active edge list"
    for each scanline (top-to-bottom) {
        insert/remove edges from "active edge list"
        update x coordinate of every active edge
        sort active edges by x coordinate
        for each pair of active edges (left-to-right)
            SetPixels(x_i, x_{i+1}, y, rgba);
    }
}
```
Hardware Scan Conversion

• Convert everything into triangles
  • Scan convert the triangles
Scan Conversion

• What about pixels on edges?
  
  If we set them either “on” or “off” we get aliasing or “jaggies”
Scan Conversion

- What about pixels on edges?
  - If we set them either “on” or “off” we get aliasing or “jaggies”

This amounts to using a “nearest” interpolation filter!
Antialiasing Techniques

• Display at higher resolution
  o Corresponds to increasing sampling rate
  o Not always possible (fixed size monitors, fixed refresh rates, etc.)

• Modify pixel intensities
  o Vary pixel intensities along primitive boundaries for antialiasing
  o Must have more than bi-level display
Scan Conversion

• What about pixels on edges?
  • If we set them either “on” or “off” we get aliasing or “jaggies”
  • Vary pixel intensities along primitive boundaries for antialiasing
Antialiasing

- Method 1: Area sampling (aka prefiltering)
  - Calculate percent of pixel covered by primitive
  - Multiply this percentage by desired intensity/color
  - Set resulting pixel to closest available display level
Antialiasing

• Method 2: Supersampling (aka postfiltering)
  o Sample as if screen were higher resolution
  o Average multiple samples to get final intensity
Antialiasing

- Method 2: Supersampling (aka postfiltering)
  - Sample as if screen were higher resolution
  - Average multiple samples to get final intensity

This amounts to using a “bilinear” interpolation filter!
Antialiasing

- Method 2: Supersampling (aka postfiltering)
  - Sample as if screen were higher resolution
  - Average multiple samples to get final intensity

This amounts to using a “bilinear” interpolation filter!

Can use other filters (e.g. Gaussian for better interpolation)
Scan Conversion

- Example:

No Anti-Aliasing

4 x Anti-Aliasing

Images courtesy of NVIDIA