Hidden Markov Models

- An attempt to understand Markov Processes
  - We know the state of the system at an instant
    - state \( x_1, x_2, \ldots, x_n \) at times \( t_1, t_2, \ldots, t_n \)
  - Transitions to new states are only dependent on the current state
    - Use a matrix, \( A \) to represent transitions
  - The transitions between states are well understood
    - All elements of \( j \) are \( \geq 0 \) and \( \leq 1 \)
    - Parameters are time independent

Transition model

- A matrix called \( A \)
  - \( a_{(i,j)} = P(\text{system in state } j | \text{system was in state } i) \)

Transitions Matrix

- What if states aren’t observable?
  - \( b_{(j,k)} = \text{Probability} (k \text{ is observed } | \text{system in state } j) \)
  - Use seaweed as an indicator of weather
    - Seaweed is dry, dryish, damp, soggy
    - New matrix is
      \[
      B = \begin{pmatrix}
      0.6 & 0.2 & 0.15 & 0.05 \\
      0.25 & 0.25 & 0.25 & 0.25 \\
      0.05 & 0.1 & 0.35 & 0.5 \\
      \end{pmatrix}
      \]

What’s the hidden part?

- There is a disconnect between the states you’ve created and the true states you are modeling
  - The state of seaweed may or may not be well correlated to tomorrow’s weather
  - If it works, it works!
HMM questions

- given a model and a sequence of observations, what is the probability that the model actually generated those observations
- if we had two models $\lambda_1 = (\pi_1, A_1, B_1)$ and $\lambda_2 = (\pi_2, A_2, B_2)$, which one better describes a sequence of given observations
- Can we automatically improve a model to better fit observations?
  * adjust model parameters $\lambda = (\pi, A, B)$ to maximize $P(O | \lambda)$

Speech Recognition

- Understanding Spoken Language
  - Input is a signal (frequency over time)
  - Output is a sequence of words

HMM for speech

- Words are made of phonemes
  - Well-defined categorization of sounds
  - English has 45 +/- 4 phonemes
  - English has 600 ways to spell these 45 sounds
  - Could these be the hidden states behind predicting what words are pronounced?

An HMM for each word?

- Build a sequence of states that model a transition from saying nothing to saying “had your”
  - Segmentaion is a tough issue
  * silence, end/beginning of words, end/beginning of phonemes